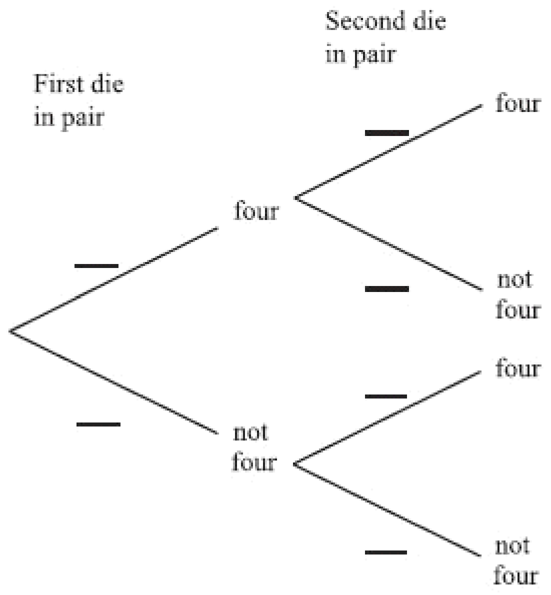
1. A pair of fair dice is thrown.

(a) Copy and complete the tree diagram below, which shows the possible outcomes.



(3)

Let E be the event that **exactly** one four occurs when the pair of dice is thrown.

(b) Calculate P(E).

(3)

The pair of dice is now thrown five times.

(c) Calculate the probability that event E occurs **exactly** three times in the five throws.

(3)

(d) Calculate the probability that event E occurs **at least** three times in the five throws.

(3)
(Total 12 marks)

2. The probability of obtaining heads on a biased coin is 0.18. The coin is tossed seven times.

(a) Find the probability of obtaining **exactly** two heads.

(2)

(b) Find the probability of obtaining **at least** two heads.

(3)
(Total 5 marks)

3. A test has five questions. To pass the test, at least three of the questions must be answered correctly.

The probability that Mark answers a question correctly is $\frac{1}{5}$. Let X be the number of questions that Mark answers correctly.

(a) (i) Find E(X).

(ii) Find the probability that Mark passes the test.

(6)

Bill also takes the test. Let Y be the number of questions that Bill answers correctly.
The following table is the probability distribution for Y.

y	0	1	2	3	4	5
P($Y = y$)	0.67	0.05	$a + 2b$	$a - b$	$2a + b$	0.04

(b) (i) Show that $4a + 2b = 0.24$.

(ii) Given that E(Y) = 1, find a and b.

(8)

(c) Find which student is more likely to pass the test.

(3)
(Total 17 marks)

4. Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a) Find the probability that he wins exactly four games.

(2)

For game B, the probability that Evan wins is p. He plays game B seven times.

(b) Write down an expression, in terms of p, for the probability that he wins exactly four games.

(2)

(c) Hence, find the values of p such that the probability that he wins exactly four games is 0.15.

(3)
(Total 7 marks)

5. A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.
90 % of the boxes made by machine B pass inspection.

A box is selected at random.

(a) Find the probability that it passes inspection.

(3)

(b) The company would like the probability that a box passes inspection to be 0.87.
Find the percentage of boxes that should be made by machine B to achieve this.

(4)
(Total 7 marks)

6. In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.

(a) (i) Find the number of boys who play both sports.

(ii) Write down the number of boys who play only rugby.

(3)

(b) One boy is selected at random.

(i) Find the probability that he plays only one sport.

(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby.

(4)

Let *A* be the event that a boy plays football and *B* be the event that a boy plays rugby.

(c) Explain why *A* and *B* are **not** mutually exclusive.

(2)

(d) Show that *A* and *B* are **not** independent.

(3)
(Total 12 marks)

7. The following table shows the probability distribution of a discrete random variable X.

x	-1	0	2	3
$P(X = x)$	0.2	$10k^2$	0.4	$3k$

(a) Find the value of k.

(4)

(b) Find the expected value of X.

(3)
(Total 7 marks)

8. Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.

Score on second die

Score on first die	1	2	3	4	5	6
1	•	•	•	•	•	•
2	•	•	•	•	•	•
3	•	•	•	•	•	•
4	•	•	•	•	•	•
5	•	•	•	•	•	•
6	•	•	•	•	•	•

Let X be the sum of the scores on the two dice.

(a) Find

(i) $P(X = 6)$;

(ii) $P(X > 6)$;

(iii) $P(X = 7 \mid X > 5)$.

(6)

(b) Elena plays a game where she tosses two dice.

If the sum is 6, she wins 3 points.
If the sum is greater than 6, she wins 1 point.
If the sum is less than 6, she **loses** k points.

Find the value of k for which Elena's expected number of points is zero.

(7)
(Total 13 marks)