1. José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.
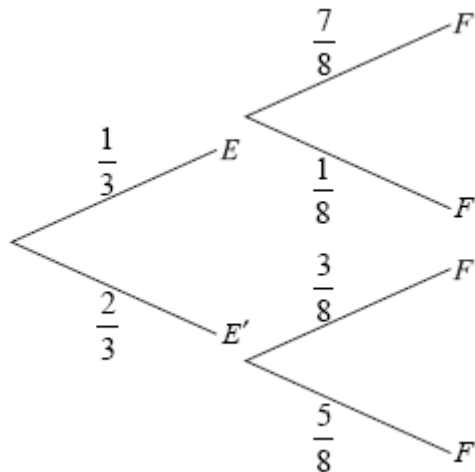
If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.

If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.

Let E be the event "he misses his bus" and F the event "he is late for school".
The information above is shown on the following tree diagram.

(a) Find

    (i)   P($E \cap F$);

    (ii)   P(F).

(4)

(b) Find the probability that

    (i)   José misses his bus and is not late for school;

    (ii)   José missed his bus, given that he is late for school.

(5)

The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday.

(c) **Copy** and complete the probability distribution table.

X (cost in euros)	0	3	6
P (X)	$\dfrac{1}{9}$		

(3)

(d) Find the expected cost for José for both days.

(2)
(Total 14 marks)

2. The probability distribution of a discrete random variable X is given by

$$P(X = x) = \frac{x^2}{14}, \ x \in \{1, 2, k\}, \text{ where } k > 0.$$

(a) Write down $P(X = 2)$.

(1)

(b) Show that $k = 3$.

(4)

(c) Find $E(X)$.

(2)
(Total 7 marks)

3. A fisherman catches 200 fish to sell. He measures the lengths, l cm of these fish, and the results are shown in the frequency table below.

Length l cm	$0 \leq l < 10$	$10 \leq l < 20$	$20 \leq l < 30$	$30 \leq l < 40$	$40 \leq l < 60$	$60 \leq l < 75$	$75 \leq l < 100$
Frequency	30	40	50	30	33	11	6

(a) Calculate an estimate for the standard deviation of the lengths of the fish.

(3)

(b) A cumulative frequency diagram is given below for the lengths of the fish.
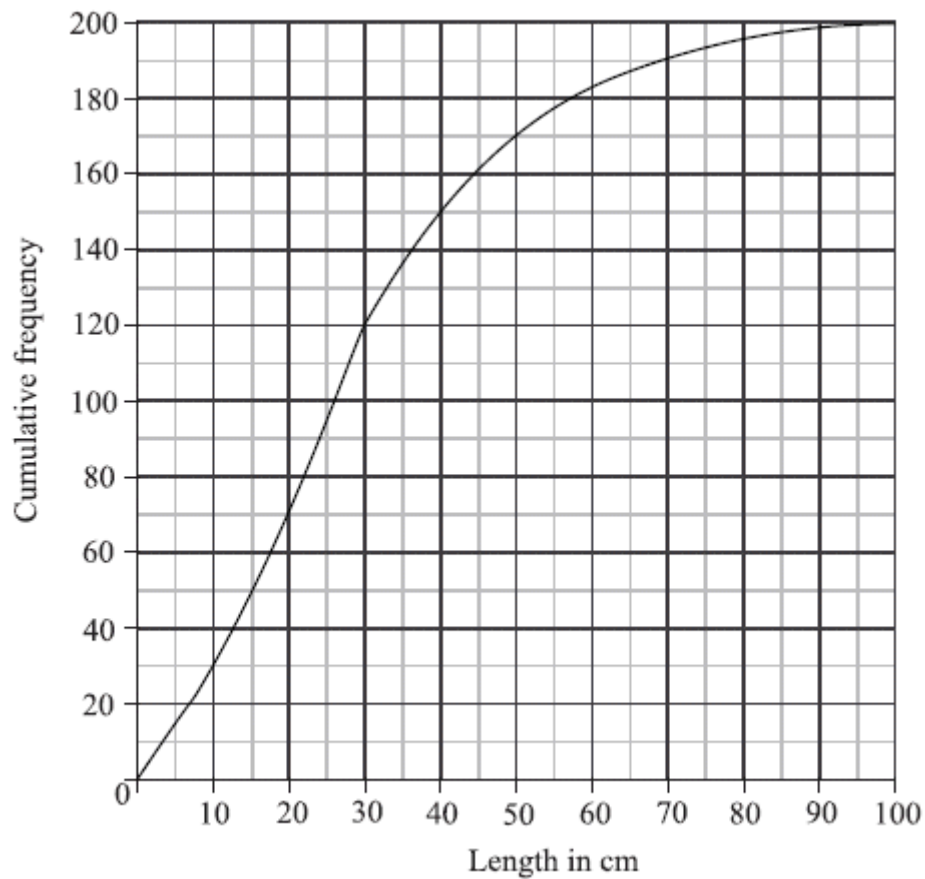
Use the graph to answer the following.

(i) Estimate the interquartile range.

(ii) Given that 40 % of the fish have a length more than k cm, find the value of k.

(6)

In order to sell the fish, the fisherman classifies them as small, medium or large.

Small fish have a length less than 20 cm.
Medium fish have a length greater than or equal to 20 cm but less than 60 cm.
Large fish have a length greater than or equal to 60 cm.

(c)   Write down the probability that a fish is small.

(2)

The cost of a small fish is $4, a medium fish $10, and a large fish $12.

(d)   Copy and complete the following table, which gives a probability distribution for the cost X.

Cost X	4	10	12
P($X = x$)		0.565	

(2)

(e)   Find E(X).

(2)
(Total 15 marks)

4.   A multiple choice test consists of ten questions. Each question has five answers.
Only one of the answers is correct. For each question, Jose randomly chooses one of the five answers.

(a)   Find the expected number of questions Jose answers correctly.

(1)

(b)   Find the probability that Jose answers exactly three questions correctly.

(2)

(c)   Find the probability that Jose answers more than three questions correctly.

(3)
(Total 6 marks)