Name:

1. (11 points)

   (a) By successive differentiation find the first five non-zero terms in the Maclaurin series for $f(x) = (x+1)\ln(1+x)$ [9]

   (b) Hence estimate the value of $2\ln 2$. Write your estimate as a fraction in lowest terms. [2]

2. (6 points)

Let the Maclauring series for $\tan x$ be

$$\tan x = a_1 x + a_3 x^3 + a_5 x^5 + ...$$

where a_1, a_3 and a_5 are constants.

(a) Find series for $\sec^2 x$, in terms of a_1 , a_3 and a_5 , up to and including the x^4 term by: [3]

(i) differentiating the above series for $\tan x$;

(ii) by using the relationship $\sec^2 x = 1 + \tan^2 x$.

(b) Hence, by comparing the two series, determine the values of a_1, a_3 and a_5. [3]

3. (10 points) Consider the differential equation

$$\frac{dy}{dx} = x(1+y)e^x$$

with $y = 0$ when $x = 1$.

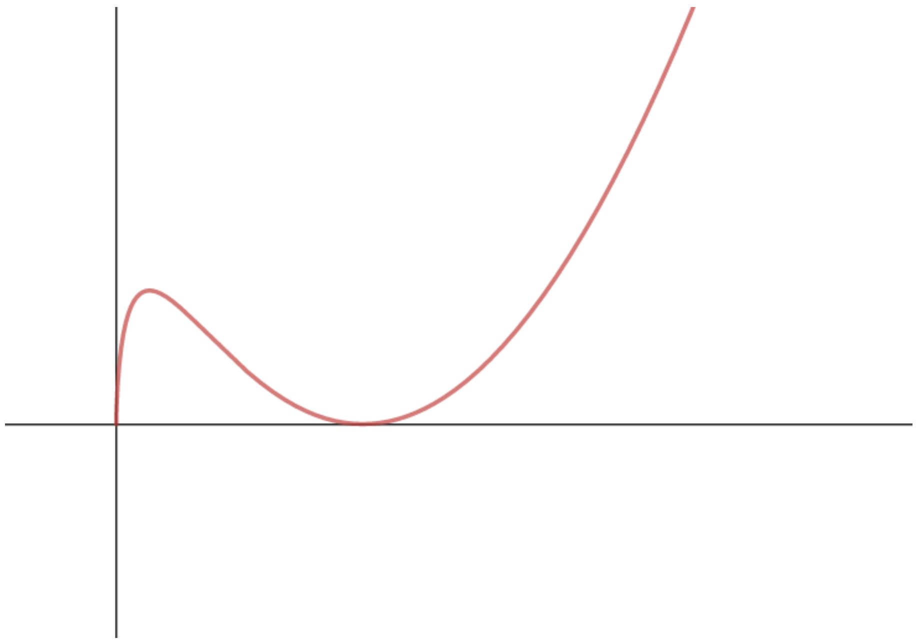
(a) Use Euler's method with step size of 0.01 to approximate the value of y when $x = 2$. [3]

(b) Solve the differential equation and hence find the exact value of y when $x = 2$. [6]

(c) Calculate the percentage error of your estimate in part (a). [1]

4. (13 points)

Part of the graph of $f(x) = x(\ln x)^2$ is shown below.



(a) By applying l'Hospital rule show that

$$\lim_{x \to 0^+} f(x) = 0$$

[3]

(b) Find $f'(x)$ and hence find the exact coordinates of the stationary points of the graph $y = f(x)$. [5]

(c) Using integration by parts find the area enclosed by the graph of $y = f(x)$ and the x-axis. [5]