Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Mathematics

Higher level

Paper 1

17 pages

International Baccalaureate®
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Section A

1. attempt to substitute $x=-1$ or $x=2$ or to divide polynomials *(M1)*
$1-p-q+5=7$, $16+8p+2q+5=1$ or equivalent ***A1A1***
attempt to solve their two equations ***M1***
$p=-3$, $q=2$ ***A1***

[5 marks]

2. (a) attempt at chain rule or product rule ***(M1)***

$$\frac{\mathrm{d}y}{\mathrm{d}\theta}=2\sin\theta\cos\theta$$ ***A1***

[2 marks]

(b) $2\sin\theta\cos\theta=2\sin^2\theta$

$\sin\theta=0$ ***(A1)***
$\theta=0, \pi$ ***A1***
obtaining $\cos\theta=\sin\theta$ ***(M1)***
$\tan\theta=1$ ***(M1)***
$\theta=\frac{\pi}{4}$ ***A1***

[5 marks]

Total [7 marks]

3. (a) $a=\frac{3}{16}$ and $b=\frac{5}{16}$ ***(M1)A1A1***

[3 marks]

Note: Award ***M1*** for consideration of the possible outcomes when rolling the two dice.

continued

Question 3 continued

(b) $\mathrm{E}(T) = \frac{1+6+15+28}{16} = \frac{25}{8}\left(= 3.125\right)$ ***(M1)A1***

Note: Allow follow through from part (a) even if probabilities do not add up to 1.

[2 marks]

Total [5 marks]

4. (a) $\int_{-2}^{0} f(x)\mathrm{d}x = 10 - 12 = -2$ ***(M1)(A1)***

$\int_{-2}^{0} 2\,\mathrm{d}x = \left[2x\right]_{-2}^{0} = 4$ ***A1***

$\int_{-2}^{0} \left(f(x)+2\right)\mathrm{d}x = 2$ ***A1***

[4 marks]

(b) $\int_{-2}^{0} f(x+2)\mathrm{d}x = \int_{0}^{2} f(x)\mathrm{d}x$ ***(M1)***

$= 12$ ***A1***

[2 marks]

Total [6 marks]

5. $(\ln x)^2 - (\ln 2)(\ln x) - 2(\ln 2)^2 \left(= 0\right)$

EITHER

$\ln x = \frac{\ln 2 \pm \sqrt{(\ln 2)^2 + 8(\ln 2)^2}}{2}$ ***M1***

$= \frac{\ln 2 \pm 3\ln 2}{2}$ ***A1***

OR

$(\ln x - 2\ln 2)(\ln x + \ln 2)\left(= 0\right)$ ***M1A1***

THEN

$\ln x = 2\ln 2$ or $-\ln 2$ ***A1***

$\Rightarrow x = 4$ or $x = \frac{1}{2}$ ***(M1)A1***

Note: ***(M1)*** is for an appropriate use of a log law in either case, dependent on the previous ***M1*** being awarded, ***A1*** for both correct answers.

solution is $\frac{1}{2} < x < 4$ ***A1***

[6 marks]

6. if $n = 1$

$\text{LHS} = 1\,;\text{RHS} = 4 - \dfrac{3}{2^0} = 4 - 3 = 1$ ***M1***

hence true for $n = 1$

assume true for $n = k$ ***M1***

Note: Assumption of truth must be present. Following marks are not dependent on the first two ***M1*** marks.

so $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + k\left(\frac{1}{2}\right)^{k-1} = 4 - \dfrac{k+2}{2^{k-1}}$

if $n = k+1$

$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + k\left(\frac{1}{2}\right)^{k-1} + (k+1)\left(\frac{1}{2}\right)^k$

$= 4 - \dfrac{k+2}{2^{k-1}} + (k+1)\left(\dfrac{1}{2}\right)^k$ ***M1A1***

finding a common denominator for the two fractions ***M1***

$= 4 - \dfrac{2(k+2)}{2^k} + \dfrac{k+1}{2^k}$

$= 4 - \dfrac{2(k+2)-(k+1)}{2^k} = 4 - \dfrac{k+3}{2^k}\left(= 4 - \dfrac{(k+1)+2}{2^{(k+1)-1}}\right)$ ***A1***

hence if true for $n = k$ then also true for $n = k+1$, as true for $n = 1$, so true (for all $n \in \mathbb{Z}^+$) ***R1***

Note: Award the final ***R1*** only if the first four marks have been awarded.

[7 marks]

7. (a) $y = \arccos\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = -\frac{1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}} \left(= -\frac{1}{\sqrt{4-x^2}}\right)$ ***M1A1***

Note: ***M1*** is for use of the chain rule.

[2 marks]

(b) attempt at integration by parts ***M1***

$u = \arccos\left(\frac{x}{2}\right) \Rightarrow \frac{du}{dx} = -\frac{1}{\sqrt{4-x^2}}$

$\frac{dv}{dx} = 1 \Rightarrow v = x$ ***(A1)***

$\int_0^1 \arccos\left(\frac{x}{2}\right) dx = \left[x \arccos\left(\frac{x}{2}\right)\right]_0^1 + \int_0^1 \frac{x}{\sqrt{4-x^2}} dx$ ***A1***

using integration by substitution or inspection ***(M1)***

$\left[x \arccos\left(\frac{x}{2}\right)\right]_0^1 + \left[-\left(4-x^2\right)^{\frac{1}{2}}\right]_0^1$ ***A1***

Note: Award ***A1*** for $-\left(4-x^2\right)^{\frac{1}{2}}$ or equivalent.

Note: Condone lack of limits to this point.

attempt to substitute limits into their integral ***M1***

$= \frac{\pi}{3} - \sqrt{3} + 2$ ***A1***

[7 marks]

Total [9 marks]

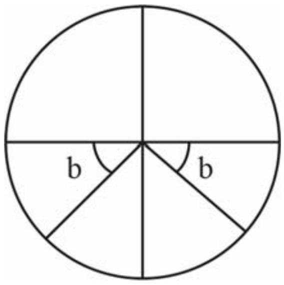
8. $\sin 2x = -\sin b$

EITHER

$\sin 2x = \sin(-b)$ or $\sin 2x = \sin(\pi + b)$ or $\sin 2x = \sin(2\pi - b)\ldots$ ***(M1)(A1)***

Note: Award ***M1*** for any one of the above, ***A1*** for having final two.

OR



(M1)(A1)

Note: Award ***M1*** for one of the angles shown with b clearly labelled, ***A1*** for both angles shown. Do not award ***A1*** if an angle is shown in the second quadrant and subsequent ***A1*** marks not awarded.

THEN

$2x = \pi + b$ or $2x = 2\pi - b$ ***(A1)(A1)***

$x = \frac{\pi}{2} + \frac{b}{2}$, $x = \pi - \frac{b}{2}$ ***A1***

[5 marks]

Section B

9. (a) attempt to differentiate ***(M1)***

$f'(x) = -3x^{-4} - 3x$ ***A1***

Note: Award ***M1*** for using quotient or product rule award ***A1*** if correct derivative seen even in unsimplified form, for example $f'(x) = \dfrac{-15x^4 \times 2x^3 - 6x^2(2-3x^5)}{(2x^3)^2}$.

$-\dfrac{3}{x^4} - 3x = 0$ ***M1***

$\Rightarrow x^5 = -1 \Rightarrow x = -1$ ***A1***

$\mathrm{A}\left(-1, -\dfrac{5}{2}\right)$ ***A1***

[5 marks]

(b) (i) $f''(x) = 0$ ***M1***

$f''(x) = 12x^{-5} - 3(=0)$ ***A1***

Note: Award ***A1*** for correct derivative seen even if not simplified.

$\Rightarrow x = \sqrt[5]{4}\left(= 2^{\frac{2}{5}}\right)$ ***A1***

hence (at most) one point of inflexion ***R1***

Note: This mark is independent of the two ***A1*** marks above. If they have shown or stated their equation has only one solution this mark can be awarded.

$f''(x)$ changes sign at $x = \sqrt[5]{4}\left(= 2^{\frac{2}{5}}\right)$ ***R1***

so exactly one point of inflexion

continued

Question 9 continued

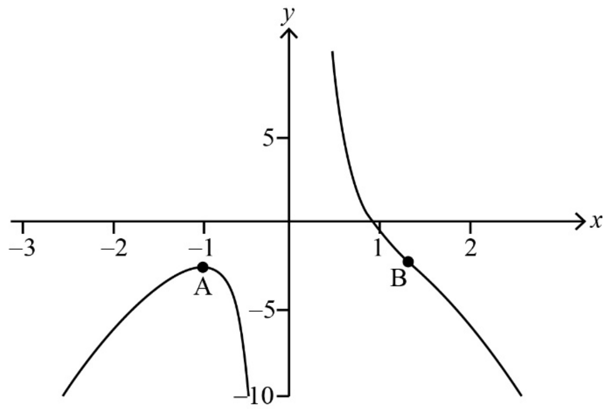
(ii) $x = \sqrt[5]{4} = 2^{\frac{2}{5}} \left(\Rightarrow a = \frac{2}{5}\right)$ ***A1***

$f\left(2^{\frac{2}{5}}\right) = \frac{2 - 3 \times 2^2}{2 \times 2^{\frac{6}{5}}} = -5 \times 2^{-\frac{6}{5}} \ (\Rightarrow b = -5)$ ***(M1)A1***

[8 marks]

Note: Award ***M1*** for the substitution of their value for x into $f(x)$.

(c)



A1A1A1A1

A1 for shape for $x < 0$
A1 for shape for $x > 0$
A1 for maximum at A
A1 for POI at B.

Note: Only award last two ***A1***s if A and B are placed in the correct quadrants, allowing for follow through.

[4 marks]

Total [17 marks]

10. (a) recognising normal to plane or attempting to find cross product of two vectors lying in the plane ***(M1)***

for example, $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AD}} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ ***(A1)***

$\Pi_1 : x + z = 1$ ***A1***

[3 marks]

(b) **EITHER**

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 1 = \sqrt{2}\sqrt{2}\cos\theta$ ***M1A1***

OR

$\left| \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right| = \sqrt{3} = \sqrt{2}\sqrt{2}\sin\theta$ ***M1A1***

Note: ***M1*** is for an attempt to find the scalar or vector product of the two normal vectors.

$\Rightarrow \theta = 60^\circ \left(= \frac{\pi}{3} \right)$ ***A1***

angle between faces is $120^\circ \left(= \frac{2\pi}{3} \right)$ ***A1***

[4 marks]

(c) $\overrightarrow{\mathrm{DB}} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ or $\overrightarrow{\mathrm{BD}} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ ***(A1)***

$\Pi_3 : x + y - z = k$ ***(M1)***

$\Pi_3 : x + y - z = 0$ ***A1***

[3 marks]

continued

Question 10 continued

(d) **METHOD 1**

line $\text{AD}:(\mathbf{r}=)\begin{pmatrix}0\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\-1\end{pmatrix}$ ***M1A1***

intersects Π_3 when $\lambda-(1-\lambda)=0$ ***M1***

so $\lambda=\frac{1}{2}$ ***A1***

hence P is the midpoint of AD ***AG***

METHOD 2

midpoint of AD is $(0.5, 0, 0.5)$ ***(M1)A1***

substitute into $x+y-z=0$ ***M1***

$0.5+0-0.5=0$ ***A1***

hence P is the midpoint of AD ***AG***

[4 marks]

(e) **METHOD 1**

$\text{OP}=\frac{1}{\sqrt{2}}$, $\text{O}\hat{\text{P}}\text{Q}=90^\circ$, $\text{O}\hat{\text{Q}}\text{P}=60^\circ$ ***A1A1A1***

$\text{PQ}=\frac{1}{\sqrt{6}}$ ***A1***

area $=\frac{1}{2\sqrt{12}}=\frac{1}{4\sqrt{3}}=\frac{\sqrt{3}}{12}$ ***A1***

continued

Question 10 continued

METHOD 2

line $\text{BD}:(\mathbf{r}=)\begin{pmatrix}1\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}-1\\-1\\1\end{pmatrix}$

$\Rightarrow \lambda = \frac{2}{3}$ ***(A1)***

$\vec{\text{OQ}} = \begin{pmatrix}\frac{1}{3}\\\frac{1}{3}\\\frac{2}{3}\end{pmatrix}$ ***A1***

area $= \frac{1}{2}\left|\vec{\text{OP}} \times \vec{\text{OQ}}\right|$ ***M1***

$\vec{\text{OP}} = \begin{pmatrix}\frac{1}{2}\\0\\\frac{1}{2}\end{pmatrix}$ ***A1***

Note: This ***A1*** is dependent on ***M1***.

area $= \frac{\sqrt{3}}{12}$ ***A1***

[5 marks]

Total [19 marks]

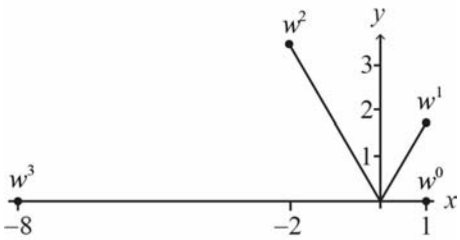
11. (a) (i) $w^2 = 4cis\left(\frac{2\pi}{3}\right)$; $w^3 = 8\text{cis}(\pi)$ ***(M1)A1A1***

Note: Accept Euler form.

Note: ***M1*** can be awarded for either both correct moduli or both correct arguments.

Note: Allow multiplication of correct Cartesian form for ***M1***, final answers must be in modulus-argument form.

(ii)



A1A1

[5 marks]

(b) use of area $= \frac{1}{2}ab\sin C$ ***M1***

$\frac{1}{2}\times 1\times 2\times \sin\frac{\pi}{3} + \frac{1}{2}\times 2\times 4\times \sin\frac{\pi}{3} + \frac{1}{2}\times 4\times 8\times \sin\frac{\pi}{3}$ ***A1A1***

Note: Award ***A1*** for $C = \frac{\pi}{3}$, ***A1*** for correct moduli.

$= \frac{21\sqrt{3}}{2}$ ***AG***

Note: Other methods of splitting the area may receive full marks.

[3 marks]

(c) $\frac{1}{2}\times 2^0\times 2^1\times \sin\frac{\pi}{n} + \frac{1}{2}\times 2^1\times 2^2\times \sin\frac{\pi}{n} + \frac{1}{2}\times 2^2\times 2^3\times \sin\frac{\pi}{n} + \ldots + \frac{1}{2}\times 2^{n-1}\times 2^n\times \sin\frac{\pi}{n}$

M1A1

Note: Award ***M1*** for powers of 2, ***A1*** for any correct expression including both the first and last term.

$= \sin\frac{\pi}{n}\times\left(2^0 + 2^2 + 2^4 + \ldots + 2^{2n-2}\right)$

identifying a geometric series with common ratio $2^2 (=4)$ ***(M1)A1***

$= \frac{1-2^{2n}}{1-4}\times\sin\frac{\pi}{n}$ ***M1***

Note: Award ***M1*** for use of formula for sum of geometric series.

$= \frac{1}{3}\left(4^n - 1\right)\sin\frac{\pi}{n}$ ***A1***

[6 marks]

Total [14 marks]