Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Wednesday 2 May 2018 (afternoon)

2 hours

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2218 – 7203

12 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Let $f(x) = x^4 + px^3 + qx + 5$ where p, q are constants.
The remainder when $f(x)$ is divided by $(x + 1)$ is 7, and the remainder when $f(x)$ is divided by $(x - 2)$ is 1. Find the value of p and the value of q.

2. [Maximum mark: 7]

Let $y = \sin^2\theta,\ 0 \le \theta \le \pi$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}\theta}$. [2]

(b) Hence find the values of θ for which $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 2y$. [5]

3. [Maximum mark: 5]

Two unbiased tetrahedral (four-sided) dice with faces labelled $1, 2, 3, 4$ are thrown and the scores recorded. Let the random variable T be the maximum of these two scores. The probability distribution of T is given in the following table.

t	1	2	3	4
$\mathrm{P}(T=t)$	$\frac{1}{16}$	a	b	$\frac{7}{16}$

(a) Find the value of a and the value of b. [3]

(b) Find the expected value of T. [2]

4. [Maximum mark: 6]

Given that $\int_{-2}^{2} f(x)\mathrm{d}x = 10$ and $\int_{0}^{2} f(x)\mathrm{d}x = 12$, find

(a) $\int_{-2}^{0} \left(f(x)+2\right)\mathrm{d}x$; [4]

(b) $\int_{-2}^{0} f(x+2)\,\mathrm{d}x$. [2]

5. [Maximum mark: 6]

Solve $(\ln x)^2 - (\ln 2)(\ln x) < 2(\ln 2)^2$.

6. [Maximum mark: 7]

Use the principle of mathematical induction to prove that

$$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^{2}+4\left(\frac{1}{2}\right)^{3}+\ldots+n\left(\frac{1}{2}\right)^{n-1}=4-\frac{n+2}{2^{n-1}}, \text{ where } n\in\mathbb{Z}^{+}.$$

7. [Maximum mark: 9]

Let $y = \arccos\left(\frac{x}{2}\right)$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(b) Find $\int_0^1 \arccos\left(\frac{x}{2}\right)\mathrm{d}x$. [7]

8. [Maximum mark: 5]

Let $a = \sin b,\ 0 < b < \frac{\pi}{2}$.

Find, in terms of b, the solutions of $\sin 2x = -a$, $0 \le x \le \pi$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

Let $f(x) = \frac{2 - 3x^5}{2x^3}, x \in \mathbb{R}, x \neq 0$.

(a) The graph of $y = f(x)$ has a local maximum at A. Find the coordinates of A. [5]

(b) (i) Show that there is exactly one point of inflexion, B, on the graph of $y = f(x)$.

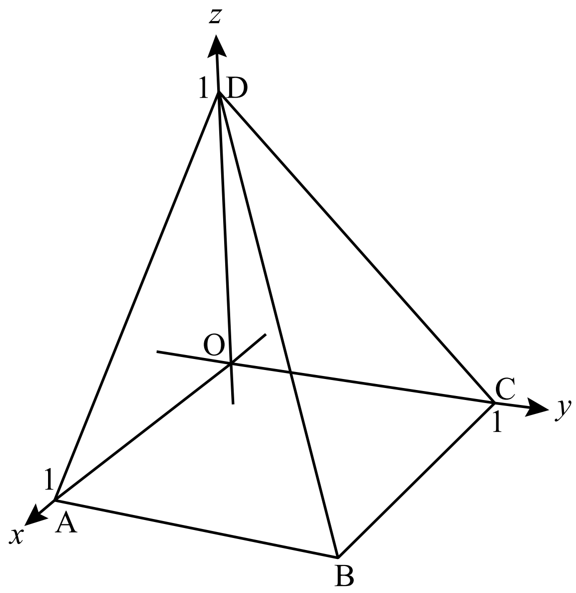
(ii) The coordinates of B can be expressed in the form $\mathrm{B}(2^a, b \times 2^{-3a})$, where $a, b \in \mathbb{Q}$. Find the value of a and the value of b. [8]

(c) Sketch the graph of $y = f(x)$ showing clearly the position of the points A and B. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 19]

The following figure shows a square based pyramid with vertices at $\mathrm{O}(0, 0, 0)$, $\mathrm{A}(1, 0, 0)$, $\mathrm{B}(1, 1, 0)$, $\mathrm{C}(0, 1, 0)$ and $\mathrm{D}(0, 0, 1)$.



(a) Find the Cartesian equation of the plane Π_1, passing through the points A, B and D. [3]

The Cartesian equation of the plane Π_2, passing through the points B, C and D, is $y + z = 1$.

(b) Find the angle between the faces ABD and BCD. [4]

The plane Π_3 passes through O and is normal to the line BD.

(c) Find the Cartesian equation of Π_3. [3]

Π_3 cuts AD and BD at the points P and Q respectively.

(d) Show that P is the midpoint of AD. [4]

(e) Find the area of the triangle OPQ. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 14]

Consider $w = 2\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$.

(a) (i) Express w^2 and w^3 in modulus-argument form.

(ii) Sketch on an Argand diagram the points represented by w^0, w^1, w^2 and w^3. [5]

These four points form the vertices of a quadrilateral, Q.

(b) Show that the area of the quadrilateral Q is $\frac{21\sqrt{3}}{2}$. [3]

Let $z = 2\left(\cos\frac{\pi}{n} + \mathrm{i}\sin\frac{\pi}{n}\right)$, $n \in \mathbb{Z}^+$. The points represented on an Argand diagram by $z^0, z^1, z^2, \ldots, z^n$ form the vertices of a polygon P_n.

(c) Show that the area of the polygon P_n can be expressed in the form $a\left(b^n - 1\right)\sin\frac{\pi}{n}$, where $a, b \in \mathbb{R}$. [6]