Markscheme

May 2018

Mathematics

Higher level

Paper 1

17 pages

Section A

1. $\cos\theta = \dfrac{(3\boldsymbol{i}-4\boldsymbol{j}-5\boldsymbol{k})\bullet(5\boldsymbol{i}-4\boldsymbol{j}+3\boldsymbol{k})}{|3\boldsymbol{i}-4\boldsymbol{j}-5\boldsymbol{k}||5\boldsymbol{i}-4\boldsymbol{j}+3\boldsymbol{k}|}$ ***(M1)***

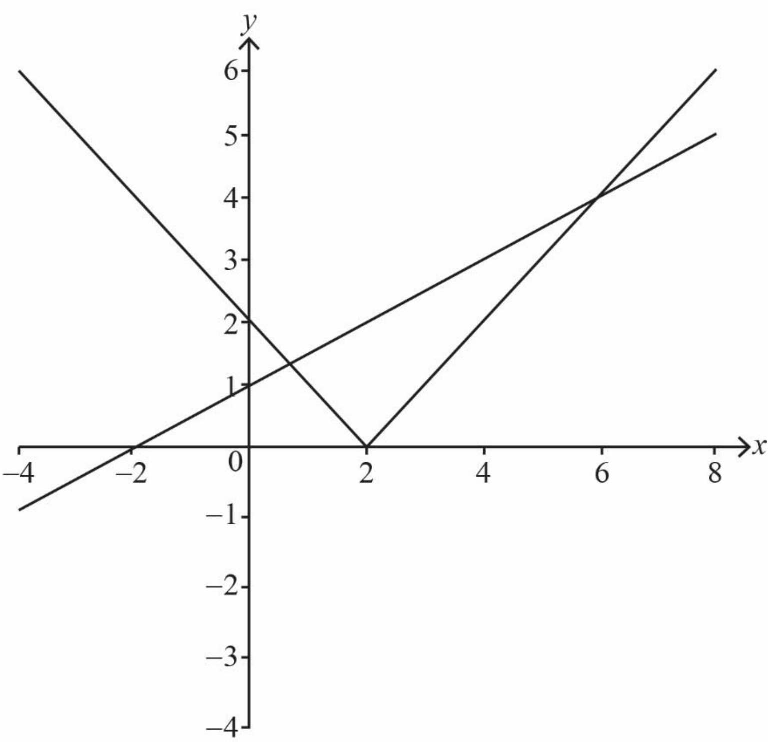
$= \dfrac{16}{\sqrt{50}\sqrt{50}}$ ***A1A1***

Note: ***A1*** for correct numerator and ***A1*** for correct denominator.

$= \dfrac{8}{25}\left(=\dfrac{16}{50}=0.32\right)$ ***A1***

[4 marks]

2. (a)



straight line graph with correct axis intercepts ***A1***
modulus graph: V shape in upper half plane ***A1***
modulus graph having correct vertex and y-intercept ***A1***

[3 marks]

continued…

Question 2 continued

(b) **METHOD 1**

attempt to solve $\frac{x}{2} + 1 = x - 2$ ***(M1)***

$x = 6$ ***A1***

Note: Accept $x = 6$ using the graph.

attempt to solve (algebraically) $\frac{x}{2} + 1 = 2 - x$ ***M1***

$x = \frac{2}{3}$ ***A1***

[4 marks]

METHOD 2

$\left(\frac{x}{2} + 1\right)^2 = (x - 2)^2$ ***M1***

$\frac{x^2}{4} + x + 1 = x^2 - 4x + 4$

$0 = \frac{3x^2}{4} - 5x + 3$

$3x^2 - 20x + 12 = 0$

attempt to factorise (or equivalent) ***M1***

$(3x-2)(x-6) = 0$

$x = \frac{2}{3}$ ***A1***

$x = 6$ ***A1***

[4 marks]

Total [7 marks]

3. (a) equating sum of probabilities to 1 ($p + 0.5 - p + 0.25 + 0.125 + p^3 = 1$) ***M1***

$p^3 = 0.125 = \frac{1}{8}$

$p = 0.5$ ***A1***

[2 marks]

(b) (i) $\mu = 0 \times 0.5 + 1 \times 0 + 2 \times 0.25 + 3 \times 0.125 + 4 \times 0.125$ ***M1***

$= 1.375 \left(= \frac{11}{8}\right)$ ***A1***

continued…

Question 3 continued

(ii) $P(X > \mu) = P(X = 2) + P(X = 3) + P(X = 4)$ ***(M1)***

$= 0.5$ ***A1***

Note: Do not award follow through ***A*** marks in (b)(i) from an incorrect value of p.

Note: Award ***M*** marks in both (b)(i) and (b)(ii) provided no negative probabilities, and provided a numerical value for μ has been found.

[4 marks]

Total [6 marks]

4. valid attempt to find $\frac{dy}{dx}$ ***M1***

$\frac{dy}{dx} = \frac{1}{(1-x)^2} - \frac{4}{(x-4)^2}$ ***A1A1***

attempt to solve $\frac{dy}{dx} = 0$ ***M1***

$x = 2,\ x = -2$ ***A1A1***

[6 marks]

5. (a) **METHOD 1**

state that $u_n = u_1 r^{n-1}$ (or equivalent) ***A1***

attempt to consider a_n and use of at least one log rule ***M1***

$\log_2 |u_n| = \log_2 |u_1| + (n-1)\log_2 |r|$ ***A1***

(which is an AP) with $d = \log_2 |r|$ (and 1st term $\log_2 |u_1|$) ***A1***

so A is an arithmetic sequence ***AG***

Note: Condone absence of modulus signs.

Note: The final ***A*** mark may be awarded independently.

Note: Consideration of the first two or three terms only will score ***M0***.

[4 marks]

continued…

Question 5 continued

METHOD 2

consideration of $(d =)\, a_{n+1} - a_n$ ***M1***

$(d) = \log_2 |u_{n+1}| - \log_2 |u_n|$

$(d) = \log_2 \left| \dfrac{u_{n+1}}{u_n} \right|$ ***M1***

$(d) = \log_2 |r|$ ***A1***

which is constant ***R1***

Note: Condone absence of modulus signs.

Note: the final ***A*** mark may be awarded independently.

Note: Consideration of the first two or three terms only will score ***M0***.

(b) attempting to solve $\dfrac{3}{1-r} = 4$ ***M1***

$r = \dfrac{1}{4}$ ***A1***

$d = -2$ ***A1***

[3 marks]

Total [7 marks]

6. (a) (i) attempt at product rule ***M1***

$f'(x) = -\mathrm{e}^{-x}\sin x + \mathrm{e}^{-x}\cos x$ ***A1***

(ii) $g'(x) = -\mathrm{e}^{-x}\cos x - \mathrm{e}^{-x}\sin x$ ***A1***

[3 marks]

(b) **METHOD 1**

Attempt to add $f'(x)$ and $g'(x)$ ***(M1)***

$f'(x) + g'(x) = -2\mathrm{e}^{-x}\sin x$ ***A1***

$\int_0^{\pi} \mathrm{e}^{-x}\sin x\,\mathrm{d}x = \left[-\dfrac{\mathrm{e}^{-x}}{2}(\sin x + \cos x) \right]_0^{\pi}$ (or equivalent) ***A1***

Note: Condone absence of limits.

$= \dfrac{1}{2}\left(1 + \mathrm{e}^{-\pi}\right)$ ***A1***

continued…

Question 6 continued

METHOD 2

$I = \int e^{-x} \sin x dx$

$= -e^{-x} \cos x - \int e^{-x} \cos x dx$ **OR** $= -e^{-x} \sin x + \int e^{-x} \cos x dx$ ***M1A1***

$= -e^{-x} \sin x - e^{-x} \cos x - \int e^{-x} \sin x dx$

$I = -\frac{1}{2} e^{-x} (\sin x + \cos x)$ ***A1***

$\int_0^{\pi} e^{-x} \sin x dx = \frac{1}{2}\left(1 + e^{-\pi}\right)$ ***A1***

[4 marks]

Total [7 marks]

7. (a) $\frac{z + w}{z - w} = \frac{(a + c) + i(b + d)}{(a - c) + i(b - d)}$

$= \frac{(a + c) + i(b + d)}{(a - c) + i(b - d)} \times \frac{(a - c) - i(b - d)}{(a - c) - i(b - d)}$ ***M1A1***

real part $= \frac{(a+c)(a-c) + (b+d)(b-d)}{(a - c)^2 + (b - d)^2} \left(= \frac{a^2 - c^2 + b^2 - d^2}{(a - c)^2 + (b - d)^2} \right)$ ***A1A1***

Note: Award ***A1*** for numerator, ***A1*** for denominator.

[4 marks]

(b) $|z| = |w| \Rightarrow a^2 + b^2 = c^2 + d^2$ ***R1***

hence real part $= 0$ ***A1***

Note: Do not award ***R0A1***.

[2 marks]

Total [6 marks]

8. (a) $\frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$ (accept $du = \frac{1}{2} x^{-\frac{1}{2}} dx$ or equivalent) ***A1***

substitution, leading to an integrand in terms of u ***M1***

$\int \frac{2u du}{u^3 + u}$ or equivalent ***A1***

$= 2\arctan(\sqrt{x})(+c)$ ***A1***

[4 marks]

continued…

Question 8 continued

(b) $\frac{1}{2}\int_1^9 \frac{\mathrm{d}x}{x^{\frac{3}{2}} + x^{\frac{1}{2}}} = \arctan 3 - \arctan 1$ ***A1***

$\tan(\arctan 3 - \arctan 1) = \frac{3 - 1}{1 + 3 \times 1}$ ***(M1)***

$\tan(\arctan 3 - \arctan 1) = \frac{1}{2}$

$\arctan 3 - \arctan 1 = \arctan \frac{1}{2}$ ***A1***

[3 marks]

Total [7 marks]

Section B

9. (a) (i) a pair of opposite sides have equal length and are parallel ***R1***
hence ABCD is a parallelogram ***AG***

(ii) attempt to rewrite the given information in vector form ***M1***
$\boldsymbol{b}-\boldsymbol{a}=\boldsymbol{c}-\boldsymbol{d}$ ***A1***
rearranging $\boldsymbol{d}-\boldsymbol{a}=\boldsymbol{c}-\boldsymbol{b}$ ***M1***
hence $\overrightarrow{AD}=\overrightarrow{BC}$ ***AG***

> **Note:** Candidates may correctly answer part i) by answering part ii) correctly and then deducing there are two pairs of parallel sides.

[4 marks]

(b) **EITHER**

use of $\overrightarrow{AB}=\overrightarrow{DC}$ ***(M1)***

$$\begin{pmatrix}2\\-3\\p+3\end{pmatrix}=\begin{pmatrix}q+1\\1-r\\4\end{pmatrix}$$ ***A1A1***

OR

use of $\overrightarrow{AD}=\overrightarrow{BC}$ ***(M1)***

$$\begin{pmatrix}-2\\r-2\\1\end{pmatrix}=\begin{pmatrix}q-3\\2\\2-p\end{pmatrix}$$ ***A1A1***

THEN

attempt to compare coefficients of $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ in their equation or statement to that effect ***M1***
clear demonstration that the given values satisfy their equation ***A1***
$p=1,\ q=1,\ r=4$ ***AG***

[5 marks]

(c) attempt at computing $\overrightarrow{AB}\times\overrightarrow{AD}$ (or equivalent) ***M1***

$$\begin{pmatrix}-11\\-10\\-2\end{pmatrix}$$ ***A1***

area $=\left|\overrightarrow{AB}\times\overrightarrow{AD}\right|\left(=\sqrt{225}\right)$ ***(M1)***

$=15$ ***A1***

[4 marks]

continued…

Question 9 continued

(d) valid attempt to find $\overrightarrow{\text{OM}}\left(=\frac{1}{2}(\boldsymbol{a}+\boldsymbol{c})\right)$ ***(M1)***

$\begin{pmatrix} 1 \\ \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix}$ ***A1***

the equation is

$\boldsymbol{r}=\begin{pmatrix} 1 \\ \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix}+t\begin{pmatrix} 11 \\ 10 \\ 2 \end{pmatrix}$ or equivalent ***M1A1***

Note: Award maximum ***M1A0*** if $'\mathbf{r}=\ldots'$ (or equivalent) is not seen.

[4 marks]

(e) attempt to obtain the equation of the plane in the form $ax+by+cz=d$ ***M1***

$11x+10y+2z=25$ ***A1A1***

Note: ***A1*** for right hand side, ***A1*** for left hand side.

[3 marks]

(f) (i) putting two coordinates equal to zero ***(M1)***

$\text{X}\left(\frac{25}{11},\,0,\,0\right),\ \text{Y}\left(0,\,\frac{5}{2},\,0\right),\ \text{Z}\left(0,\,0,\,\frac{25}{2}\right)$ ***A1***

(ii) $\text{YZ}=\sqrt{\left(\frac{5}{2}\right)^2+\left(\frac{25}{2}\right)^2}$ ***M1***

$=\sqrt{\frac{325}{2}}\left(=\frac{5\sqrt{104}}{4}=\frac{5\sqrt{26}}{2}\right)$ ***A1***

[4 marks]

Total [24 marks]

10. (a) attempt to make x the subject of $y = \frac{ax+b}{cx+d}$ ***M1***

$y(cx+d) = ax + b$ ***A1***

$x = \frac{dy-b}{a-cy}$ ***A1***

$f^{-1}(x) = \frac{dx-b}{a-cx}$, ***A1***

Note: Do not allow $y =$ in place of $f^{-1}(x)$.

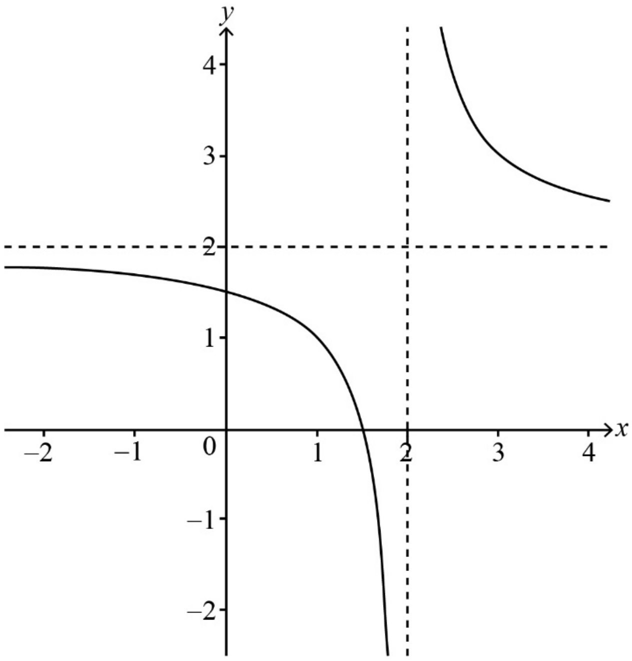
$x \neq \frac{a}{c}$, $(x \in \mathbb{R})$ ***A1***

Note: The final ***A*** mark is independent.

[5 marks]

(b) (i) $g(x) = 2 + \frac{1}{x-2}$ ***A1A1***

(ii)



hyperbola shape, with single curves in second and fourth quadrants and third quadrant blank, including vertical asymptote $x = 2$ ***A1***

horizontal asymptote $y = 2$ ***A1***

intercepts $\left(\frac{3}{2}, 0\right), \left(0, \frac{3}{2}\right)$ ***A1***

[5 marks]

continued…

Question 10 continued

(c) the domain of $h \circ g$ is $x \le \frac{3}{2},\ x > 2$ ***A1A1***

the range of $h \circ g$ is $y \ge 0,\ y \ne \sqrt{2}$ ***A1A1***

[4 marks]

Total [14 marks]

11. (a) **METHOD 1**

$\log_{r^2} x = \frac{\log_r x}{\log_r r^2} \left(= \frac{\log_r x}{2\log_r r} \right)$ ***M1A1***

$= \frac{\log_r x}{2}$ ***AG***

[2 marks]

METHOD 2

$\log_{r^2} x = \frac{1}{\log_x r^2}$ ***M1***

$= \frac{1}{2\log_x r}$ ***A1***

$= \frac{\log_r x}{2}$ ***AG***

[2 marks]

(b) **METHOD 1**

$\log_2 y + \log_4 x + \log_4 2x = 0$

$\log_2 y + \log_4 2x^2 = 0$ ***M1***

$\log_2 y + \frac{1}{2}\log_2 2x^2 = 0$ ***M1***

$\log_2 y = -\frac{1}{2}\log_2 2x^2$

$\log_2 y = \log_2 \left(\frac{1}{\sqrt{2}x} \right)$ ***M1A1***

$y = \frac{1}{\sqrt{2}} x^{-1}$ ***A1***

Note: For the final ***A*** mark, y must be expressed in the form px^q.

[5 marks]

continued…

Question 11 continued

METHOD 2

$\log_2 y + \log_4 x + \log_4 2x = 0$

$\log_2 y + \frac{1}{2}\log_2 x + \frac{1}{2}\log_2 2x = 0$ ***M1***

$\log_2 y + \log_2 x^{\frac{1}{2}} + \log_2 (2x)^{\frac{1}{2}} = 0$ ***M1***

$\log_2\left(\sqrt{2}xy\right) = 0$ ***M1***

$\sqrt{2}xy = 1$ ***A1***

$y = \frac{1}{\sqrt{2}}x^{-1}$ ***A1***

Note: For the final ***A*** mark, y must be expressed in the form px^q.

[5 marks]

(c) the area of R is $\int_1^{\alpha} \frac{1}{\sqrt{2}}x^{-1}\mathrm{d}x$ ***M1***

$= \left[\frac{1}{\sqrt{2}}\ln x\right]_1^{\alpha}$ ***A1***

$= \frac{1}{\sqrt{2}}\ln\alpha$ ***A1***

$\frac{1}{\sqrt{2}}\ln\alpha = \sqrt{2}$ ***M1***

$\alpha = \mathrm{e}^2$ ***A1***

Note: Only follow through from part (b) if y is in the form $y = px^q$.

[5 marks]

Total [12 marks]