Markscheme

May 2018

Mathematics

Higher level

Paper 2

14 pages

Section A

1. (a) $u_1 + 2d = 1407$, $u_1 + 9d = 1183$ ***(M1)(A1)***

$u_1 = 1471$, $d = -32$ ***A1A1***

[4 marks]

(b) $1471 + (n-1)(-32) > 0$ ***(M1)***

$\Rightarrow n < \frac{1471}{32} + 1$

$n < 46.96\ldots$ ***(A1)***

so 46 positive terms ***A1***

[3 marks]

Total [7 marks]

2. **METHOD 1**

$\alpha + \beta = 5$, $\alpha\beta = -7$ ***(M1)(A1)***

Note: Award ***M1A0*** if only one equation obtained.

$(\alpha + 1) + (\beta + 1) = 5 + 2 = 7$ ***A1***

$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$ ***(M1)***

$= -7 + 5 + 1 = -1$

$p = -7$, $q = -1$ ***A1A1***

METHOD 2

$\alpha = \frac{5 + \sqrt{53}}{2} = 6.1\ldots$; $\beta = \frac{5 - \sqrt{53}}{2} = -1.1\ldots$ ***(M1)(A1)***

$\alpha + 1 = \frac{7 + \sqrt{53}}{2} = 7.1\ldots$; $\beta + 1 = \frac{7 - \sqrt{53}}{2} = -0.1\ldots$ ***A1***

$(x - 7.14\ldots)(x + 0.14\ldots) = x^2 - 7x - 1$ ***(M1)***

$p = -7$, $q = -1$ ***A1A1***

Note: Exact answers only.

[6 marks]

3. $\tan(x + \pi) = \tan x \left(= \frac{\sin x}{\cos x} \right)$ ***(M1)A1***

$\cos\left(x - \frac{\pi}{2} \right) = \sin x$ ***(M1)A1***

Note: The two ***M1***'s can be awarded for observation or for expanding.

$\tan(x + \pi)\cos\left(x - \frac{\pi}{2} \right) = \frac{\sin^2 x}{\cos x}$ ***A1***

[5 marks]

4. (a) $P(L \geq 5) = 0.910$ ***(M1)A1***

[2 marks]

(b) X is the number of wolves found to be at least 5 years old
recognising binomial distribution ***M1***
$X \sim B(8,\, 0.910\ldots)$
$P(X > 6) = 1 - P(X \leq 6)$ ***(M1)***
$= 0.843$ ***A1***

Note: Award ***M1A0*** for finding $P(X \geq 6)$.

[3 marks]

Total [5 marks]

5. (a) $2x^3 - 3x + 1 = Ax\left(x^2+1\right) + Bx + C$
$A = 2,\ C = 1,$ ***A1***
$A + B = -3 \Rightarrow B = -5$ ***A1***

[2 marks]

(b) $\int \frac{2x^3-3x+1}{x^2+1}\,\mathrm{d}x = \int\left(2x - \frac{5x}{x^2+1} + \frac{1}{x^2+1}\right)\mathrm{d}x$ ***M1M1***

Note: Award ***M1*** for dividing by (x^2+1) to get $2x$, ***M1*** for separating the $5x$ and 1.

$= x^2 - \frac{5}{2}\ln\left(x^2+1\right) + \arctan x\,(+c)$ ***(M1)A1A1***

Note: Award ***(M1)A1*** for integrating $\frac{5x}{x^2+1}$, ***A1*** for the other two terms.

[5 marks]

Total [7 marks]

6. X is number of squirrels in reserve
$X \sim \mathrm{Po}(179.2)$ ***A1***

Note: Award ***A1*** if 179.2 or 56×3.2 seen or implicit in future calculations.

recognising conditional probability ***M1***
$P\left(X > 190 \mid X \geq 168\right)$
$= \frac{P(X > 190)}{P(X \geq 168)}\left(= \frac{0.19827\ldots}{0.80817\cdots}\right)$ ***(A1)(A1)***
$= 0.245$ ***A1***

[5 marks]

7. (a) **EITHER**

2019: $2500 \times 0.93 + 250 = 2575$ ***(M1)A1***

2020: $2575 \times 0.93 + 250$ ***M1***

OR

2020: $2500 \times 0.93^2 + 250(0.93 + 1)$ ***M1M1A1***

Note: Award ***M1*** for starting with 2500, ***M1*** for multiplying by 0.93 and adding 250 twice. ***A1*** for correct expression. Can be shown in recursive form.

THEN

$(= 2644.75) = 2645$ ***AG***

[3 marks]

(b) 2020: $2500 \times 0.93^2 + 250(0.93 + 1)$

2042: $2500 \times 0.93^{24} + 250\left(0.93^{23} + 0.93^{22} + \ldots + 1\right)$ ***(M1)(A1)***

$= 2500 \times 0.93^{24} + 250\dfrac{\left(0.93^{24} - 1\right)}{(0.93 - 1)}$ ***(M1)(A1)***

$= 3384$ ***A1***

Note: If recursive formula used, award ***M1*** for $u_n = 0.93\, u_{n-1} + 250$ **and** u_0 or u_1 seen (can be awarded if seen in part (a)). Then award ***M1A1*** for attempt to find u_{24} or u_{25} respectively (different term if other than 2500 used) (***M1A0*** if incorrect term is being found) and ***A2*** for correct answer.

Note: Accept all answers that round to 3380.

[5 marks]

Total [8 marks]

8. **METHOD 1**

let p have no pets, q have one pet and r have two pets ***(M1)***

$p + q + r + 2 = 25$ ***(A1)***

$0p + 1q + 2r + 6 = 18$ ***A1***

Note: Accept a statement that there are a total of 12 pets.

attempt to use variance equation, or evidence of trial and error ***(M1)***

$\dfrac{0p + 1q + 4r + 18}{25} - \left(\dfrac{18}{25}\right)^2 = \left(\dfrac{24}{25}\right)^2$ ***(A1)***

attempt to solve a system of linear equations ***(M1)***

$p = 14$ ***A1***

continued…

Question 8 continued

METHOD 2

x	0	1	2	3
$\mathrm{P}(X = x)$	p	q	r	$\frac{2}{25}$

(M1)

$p + q + r + \frac{2}{25} = 1$ ***(A1)***

$q + 2r + \frac{6}{25} = \frac{18}{25} \left(\Rightarrow q + 2r = \frac{12}{25} \right)$ ***A1***

$q + 4r + \frac{18}{25} - \left(\frac{18}{25} \right)^2 = \frac{576}{625} \left(\Rightarrow q + 4r = \frac{18}{25} \right)$ ***(M1)(A1)***

$q = \frac{6}{25}$, $r = \frac{3}{25}$ ***(M1)***

$p = \frac{14}{25}$ ***A1***

so 14 have no pets

[7 marks]

Section B

9. (a) differentiating implicitly: ***M1***

$2xy + x^2 \frac{\mathrm{d}y}{\mathrm{d}x} = -4y^3 \frac{\mathrm{d}y}{\mathrm{d}x}$ ***A1A1***

Note: Award ***A1*** for each side.

if $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ then either $x = 0$ or $y = 0$ ***M1A1***

$x = 0 \Rightarrow$ two solutions for $y\left(y = \pm\sqrt[4]{5}\right)$ ***R1***

$y = 0$ not possible (as $0 \neq 5$) ***R1***

hence exactly two points ***AG***

Note: For a solution that only refers to the graph giving two solutions at $x = 0$ and no solutions for $y = 0$ award ***R1*** only.

[7 marks]

(b) at $(2, 1)$ $4 + 4\frac{\mathrm{d}y}{\mathrm{d}x} = -4\frac{\mathrm{d}y}{\mathrm{d}x}$ ***M1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}$ ***(A1)***

gradient of normal is 2 ***M1***

$1 = 4 + c$ ***(M1)***

equation of normal is $y = 2x - 3$ ***A1***

[5 marks]

(c) substituting ***(M1)***

$x^2(2x - 3) = 5 - (2x - 3)^4$ or $\left(\frac{y+3}{2}\right)^2 y = 5 - y^4$ ***(A1)***

$x = 0.724$ ***A1***

[3 marks]

continued…

Question 9 continued

(d) recognition of two volumes ***(M1)***

volume 1 $= \pi\int_1^{\sqrt[4]{5}} \frac{5-y^4}{y}\,\mathrm{d}y (= 1.01\pi = 3.178\ldots)$ ***M1A1A1***

Note: Award ***M1*** for attempt to use $\pi\int x^2\mathrm{d}y$, ***A1*** for limits, ***A1*** for $\frac{5-y^4}{y}$. Condone omission of π at this stage.

volume 2

EITHER

$= \frac{1}{3}\pi \times 2^2 \times 4 (= 16.75\ldots)$ ***(M1)(A1)***

OR

$= \pi\int_{-3}^{1}\left(\frac{y+3}{2}\right)^2 \mathrm{d}y \left(= \frac{16\pi}{3} = 16.75...\right)$ ***(M1)(A1)***

THEN

total volume $= 19.9$ ***A1***

[7 marks]

Total [22 marks]

10. (a) $a\left[\int_0^{0.5} 3x\,\mathrm{d}x + \int_{0.5}^{2}(2-x)\mathrm{d}x\right] = 1$ ***M1***

Note: Award the ***M1*** for the total integral equalling 1, or equivalent.

$a\left(\frac{3}{2}\right) = 1$ ***(M1)A1***

$a = \frac{2}{3}$ ***AG***

[3 marks]

continued…

Question 10 continued

(b) **EITHER**

$\int_0^{0.5} 2x \, \mathrm{d}x + \frac{2}{3}\int_{0.5}^{1} (2 - x) \, \mathrm{d}x$ ***(M1)(A1)***

$= \frac{2}{3}$ ***A1***

OR

$\frac{2}{3}\int_1^2 (2 - x) \, \mathrm{d}x = \frac{1}{3}$ ***(M1)***

so $\mathrm{P}(X < 1) = \frac{2}{3}$ ***(M1)A1***

[3 marks]

(c) $\mathrm{P}(s < X < 0.8) = \int_s^{0.5} 2x \, \mathrm{d}x + \frac{2}{3}\int_{0.5}^{0.8} (2 - x) \, \mathrm{d}x$ ***M1A1***

$= \left[x^2\right]_s^{0.5} + 0.27$

$0.25 - s^2 + 0.27$ ***(A1)***

$\mathrm{P}(2s < X < 0.8) = \frac{2}{3}\int_{2s}^{0.8} (2 - x) \, \mathrm{d}x$ ***A1***

$= \frac{2}{3}\left[2x - \frac{x^2}{2}\right]_{2s}^{0.8}$

$\frac{2}{3}\left(1.28 - (4s - 2s^2)\right)$

equating

$0.25 - s^2 + 0.27 = \frac{4}{3}\left(1.28 - (4s - 2s^2)\right)$ ***(A1)***

attempt to solve for s ***(M1)***

$s = 0.274$ ***A1***

[7 marks]

Total [13 marks]

11. (a) $\boldsymbol{r}_A = \boldsymbol{r}_B$ ***(M1)***

$2 - t = -0.5t \Rightarrow t = 4$ ***A1***

checking $t = 4$ satisfies $4 + t = 3.2 + 1.2t$ and $-1 - 0.15t = -2 + 0.1t$ ***R1***

$\mathrm{P}(-2, 8, -1.6)$ ***A1***

Note: Do not award final ***A1*** if answer given as column vector.

[4 marks]

(b) (i) $0.9 \times \begin{pmatrix} -0.5 \\ 1.2 \\ 0.1 \end{pmatrix} = \begin{pmatrix} -0.45 \\ 1.08 \\ 0.09 \end{pmatrix}$ ***A1***

Note: Accept use of cross product equalling zero.

hence in the same direction ***AG***

(ii) $\begin{pmatrix} -0.45t \\ 3.2 + 1.08t \\ -2 + 0.09t \end{pmatrix} = \begin{pmatrix} -2 \\ 8 \\ -1.6 \end{pmatrix}$ ***M1***

Note: The ***M1*** can be awarded for any one of the resultant equations.

$\Rightarrow t = \frac{40}{9} = 4.44\ldots$ ***A1***

[3 marks]

(c) (i) $\boldsymbol{r}_A - \boldsymbol{r}_B = \begin{pmatrix} 2 - t \\ 4 + t \\ -1 - 0.15t \end{pmatrix} - \begin{pmatrix} -0.45t \\ 3.2 + 1.08t \\ -2 + 0.09t \end{pmatrix}$ ***(M1)(A1)***

$= \begin{pmatrix} 2 - 0.55t \\ 0.8 - 0.08t \\ 1 - 0.24t \end{pmatrix}$ ***(A1)***

Note: Accept $\boldsymbol{r}_B - \boldsymbol{r}_A$.

distance $D = \sqrt{(2 - 0.55t)^2 + (0.8 - 0.08t)^2 + (1 - 0.24t)^2}$ ***M1A1***

$(= \sqrt{8.64 - 2.688t + 0.317t^2})$

(ii) minimum when $\frac{\mathrm{d}D}{\mathrm{d}t} = 0$ ***(M1)***

$t = 3.83$ ***A1***

(iii) 0.511 (km) ***A1***

[8 marks]

Total [15 marks]