1. [Maximum points: 6]

Consider line L_1 described by the equation

$$\frac{x-3}{2} = \frac{y+1}{4} = z + 2$$

(a) Find the vector equation of the line. [2]

Line L_2 is described by the equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

(b) Find the equation of the plane which is parallel to both L_1 and L_2 and passes through the point (1,3,2). Write your answer in Cartesian form. [4]

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2. [Maximum points: 10]

Consider the system of equations given by the equations

$$ax + y + z = 1$$

$$2x + y + z = 3$$

$$x - 2y - z = 4$$

where $a \in \mathbb{R}$.

(a) Find the solution to the system in terms of a. [7]

(b) Write down the value of a for which the system has no solutions. [1]

(c) For the value of a found in part (b) explain the geometrical interpretation of the system. [2]

3. [Maximum points: 6]

Calculate the acute angle between the line

$$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

and the plane

$$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

4. [Maximum points: 6]

Three planes are described by the equations

$$4x - y - z = -1$$

$$x + 2y - 3z = -6$$

$$2x + ky - z = 1$$

Find the restrictions on k if there are no points common to all three planes.

5. [Maximum points: 4]

Vectors **a** and **b** are perpendicular, $|\mathbf{a}| = m$ and $|\mathbf{b}| = n$. Find the conditions on m and n if $(\mathbf{a} - 4\mathbf{b}) \cdot (\mathbf{a} + \mathbf{b})$ is perpendicular.

6. [Maximum points: 6]

Determine the area of the triangle with vertices $(1,0,4)$, $(3,-2,1)$ and $(2,1,3)$.

7. [Maximum points: 7]

Determine the angle between the plane $2x - y + 3z = 1$ and the line $x - 1 = y + 3 = -2z$.

8. [Maximum points: 9]

Determine the exact distance from the point (0,3,1) to the plane $2x - y + 3z = 2$.

9. [Maximum points: 13]

Consider line L_1 with equation $\frac{x-3}{4} = \frac{4-y}{2} = z - 1$ and the plane $x + y - 2z = 1$.

(a) Find the vector equation of the line. [2]

(b) Show that the line is parallel to the plane. [3]

Line L_2 passes through the point (3,4,1) and is perpendicular to the plane.

(c) Write down the equation of line L_2. [2]

(d) Find the coordinates of the point of intersection of lines L_2 and the plane. [4]

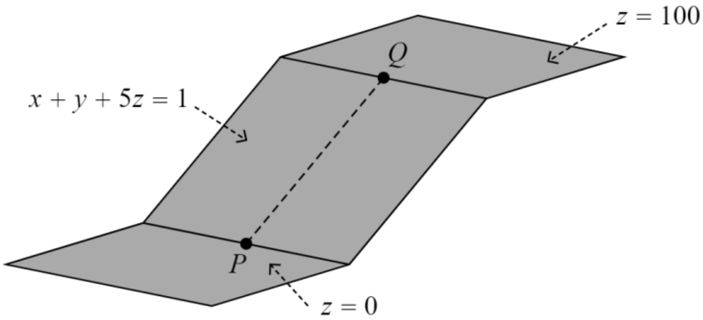
(e) Find the distance between line L_1 and the plane. [2]

10. [Maximum points: 19]

The sloped part of a hill, which starts at sea level, can be described by the equation $x + y + 5z = 1$ where the z-coordinate represents the height above sea level and units of coordinates are metres.

The flat ground (at sea level) at the bottom of the hill can be described by the plane $z = 0$, and the flat ground at the top of the hill can be described by the plane $z = 100$.

A hiker walks directly up the hill along the line PQ. This is shown in the diagram below.



(a) Show that the equation of the line of intersection between the plane $x + y + 5z = 1$ and the plane $z = 0$ is [4]

$$\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

(b) Write down a vector perpendicular to the plane [2]

(i) $z = 0$

(ii) $x + y + 5z = 1$

(c) Show that the acute angle between the plane $x + y + 5z = 1$ and the plane $z = 0$ is equal to 15.8° to 3 significant figures. [3]

Point P has coordinates (1,0,0). Line PQ is such that the angle between line PQ and the plane $z = 0$ is equal to the angle in part (c).

(d) Write down the z-coordinate of point Q. [1]

Let $\overrightarrow{PQ} = \mathbf{d}$.

(e) Write down two different vectors both of which are perpendicular to $\mathbf{d}$. [2]

(f) Hence find [5]

(i) vector **d**

(ii) the coordinates of point Q

11. [Maximum points: 20]

A plane has Cartesian equation $5x - 3y - 4z = 7$. The vector equation of the plane can be written in the form $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{i} + 3\mathbf{j} - \mathbf{k}) + \mu\mathbf{b}$ where $|\mathbf{b}| = 3$.

Let $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$.

(a) Show the following are true [2]

$$3b_3 + b_2 = 5$$

$$-b_1 - b_3 = -3$$

$$b_2 - 3b_1 = -4$$

(b) Write the following in terms of b_1 [4]

(i) b_2

(ii) b_3

(c) Show that the two possible values of b_1 are 8/11 and 2. [4]

(d) Find the possible values of [4]

(i) b_2

(ii) b_3.

(e) Find the smallest possible value of $|\mathbf{a}|$. [6]