1. (a) $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}$ A1A1

(b) Find a vector normal to the plane e.g.

$$\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \\ -4 \\ -6 \end{pmatrix}$$ M1A1

The equation is therefore

$$11x - 4y - 6z = 11 - 12 - 12$$ M1

Which gives

$$11x - 4y - 6z = -13$$ A1

2. (a) We have

$$\left[\begin{array}{ccc|c} a & 1 & 1 & 1 \\ 2 & 1 & 1 & 3 \\ 1 & -2 & -1 & 4 \end{array}\right]$$

Eliminate one variable from two equations e.g. row 1 + row 3, row 2 + row 3 M1

$$\left[\begin{array}{ccc|c} a & 1 & 1 & 1 \\ 1+a & -1 & 0 & 5 \\ 3 & -1 & 0 & 7 \end{array}\right]$$ A1

Eliminate one variable from one equation e.g. row 3 − row 2 M1

$$\left[\begin{array}{ccc|c} a & 1 & 1 & 1 \\ 1+a & -1 & 0 & 5 \\ 2-a & 0 & 0 & 2 \end{array}\right]$$ A1

Therefore

$$x = \frac{2}{2-a}$$ A1

$$y = \frac{7a-8}{2-a}$$ A1

$$z = \frac{10(1-a)}{2-a}$$ A1

(b) 2 A1

(c) Two parallel planes and one plane which is not parallel. A1A1

3. A vector normal to the plane is

$$\begin{pmatrix}3\\2\\1\end{pmatrix} \times \begin{pmatrix}-1\\2\\1\end{pmatrix} = \begin{pmatrix}0\\-4\\8\end{pmatrix} \quad \text{M1A1}$$

The angle θ between the line and the normal satisfies

$$-8 + 16 = 3\sqrt{80} \cdot \cos\theta \quad \text{M1}$$

So

$$\theta = 72.654° \quad \text{A1}$$

The angle between the line and the plane is therefore

$$90 - 72.654 = 17.3° \quad \text{M1A1}$$

4. Eliminate one variable from two rows e.g. M1

$$\left[\begin{array}{ccc|c}4 & -1 & -1 & -1\\1 & 2 & -3 & -6\\2 & k & -1 & 1\end{array}\right]$$

Row 3 - Row 1, 3 × Row 1 - Row 2

$$\left[\begin{array}{ccc|c}4 & -1 & 1 & -3\\-2 & k+1 & 0 & 2\\11 & -5 & 0 & 3\end{array}\right] \quad \text{A1}$$

Eliminate one variable from one row e.g. 2 × Row 3 + 11 × Row 2 M1

$$\left[\begin{array}{ccc|c}4 & -1 & 1 & -3\\-2 & k+1 & 0 & 2\\0 & 11k+1 & 0 & 28\end{array}\right] \quad \text{A1}$$

The system has a solution if $(11k + 1)y = 28$. M1

So there are no solutions if $k = -1/11$. A1

5. We have

$$(\mathbf{a} - 4\mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = |\mathbf{a}|^2 - 3\mathbf{a} \cdot \mathbf{b} - 4|\mathbf{b}|^2 = |\mathbf{a}|^2 - 4|\mathbf{b}|^2 \quad \text{A1A1}$$

Therefore

$$m^2 - 4n^2 = 0 \quad \text{M1}$$

So

$$m = 2n \quad \text{A1}$$

6. Determine the two vectors from any point to the two other points. M1

e.g. $2\mathbf{i} - 2\boldsymbol{j} - 3\boldsymbol{k}$ and $\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ A1

Determine the cross product of these vectors. M1

$$\begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ 4 \end{pmatrix}$$ A1

The area is then half of the magnitude of this vector. M1

This is equal to $\frac{\sqrt{42}}{2}$. A1

7. Determine the vector equation of the line. M1

$$x - 1 = \lambda$$

$$y + 3 = \lambda$$

$$-2z = \lambda$$

So

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1/2 \end{pmatrix}$$ A1

A vector normal to the plane is

$$\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$ A1

Determine the angle between the line and this normal vector. M1

$$2 - 1 - 3/2 = \sqrt{2.25} \cdot \sqrt{14} \cos \theta$$

So $\theta = 95.11°$. A1

The angle between the line and the plane is therefore

$$95.1 - 90 = 5.11°$$ M1A1

8. The vector normal to the plane is $2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$. A1

The line through the point and perpendicular to the plane is

$$\boldsymbol{r} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$ A1

At the point of intersection of this line and the plane we have

$$x = 2\lambda$$

$$y = 3 - \lambda$$

$$z = 1 + 3\lambda$$ A1

Substitute these into the equation of the plane and solve. M1

$$4\lambda - (3 - \lambda) + 3 + 9\lambda = 2$$

So $\lambda = \frac{1}{7}$. A1

Substitute this value into the expressions for x, y and z to determine the coordinate of the point of intersection. M1

$$x = \frac{2}{7}$$

$$y = 3 - \frac{1}{7} = \frac{20}{7}$$

$$z = 1 + \frac{3}{7} = \frac{10}{7}$$ A1

Use the distance formula to determine the distance between the two points. M1

$$\sqrt{\left(0 - \frac{2}{7}\right)^2 + \left(3 - \frac{20}{7}\right)^2 + \left(1 - \frac{10}{7}\right)^2}$$

This is equal to $\sqrt{\frac{2}{7}}$. A1

9. (a) Write in parametric form — M1

$$x = 3 + 4\lambda$$

$$y = 4 - 2\lambda$$

$$z = 1 + \lambda$$

So the vector equation is

$$\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$ A1

(b) We have

$$\begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = 4 - 2 - 2 = 0$$ M1A1

Since the line is perpendicular to the normal it must be parallel to the plane. R1

(c) $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ A1A1

(d) We have

$$3 + \mu + 4 + \mu - 2(1 - 2\mu) = 1$$ M1

So

$$\mu = -\frac{2}{3}$$ A1

The coordinates are therefore

$$(3 - 2/3, 4 - 2/3, 1 + 4/3) = (7/3, 10/3, 7/3)$$ M1A1

(e) The distance is

$$\sqrt{(3 - 7/3)^2 + (4 - 10/3)^2 + (1 - 7/3)^2} = \sqrt{\frac{8}{3}} = \frac{2\sqrt{6}}{3}$$ M1A1

10. (a) Replace z with 0 M1

$$x + y + 5 \cdot 0 = 1 \quad \text{A1}$$

So

$$y = 1 - x \quad \text{A1}$$

Let $x = \lambda$ so in vector form this is

$$\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad \text{A1}$$

(b)

(i) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (or a scalar multiple of this vector) A1

(ii) $\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$ A1

(c) Find the angle between the vectors in part (b) M1

$$5 = \sqrt{27}\cos\theta \quad \text{A1}$$

So

$$0 = \arccos\left(\frac{5}{\sqrt{27}}\right) = 15.8° \quad \text{A1}$$

(d) 100 A1

(e) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$ A1A1

(f) Calculate the vector product using the two vectors in part (d) M1

$$\begin{pmatrix} -1 \cdot 5 - 0 \cdot 1 \\ 0 \cdot 1 - 1 \cdot 5 \\ 1 \cdot 1 - (-1) \cdot 1 \end{pmatrix} = \begin{pmatrix} -5 \\ -5 \\ 2 \end{pmatrix} \quad \text{A1}$$

We therefore have

$$c \times \begin{pmatrix} -5 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 100 \end{pmatrix} \quad \text{M1}$$

This gives $c = 50$ so A1

$$\overrightarrow{PQ} = \begin{pmatrix} -250 \\ -250 \\ 100 \end{pmatrix} \quad \text{A1}$$

(g) The position vector is

$$\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}-250\\-250\\100\end{pmatrix} = \begin{pmatrix}-249\\-250\\100\end{pmatrix}$$ M1A1

11. (a) Since

$$\begin{pmatrix}1\\3\\-1\end{pmatrix} \times \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} = \begin{pmatrix}5\\-3\\-4\end{pmatrix}$$ M1

we have

$$3b_3 + b_2 = 5$$

$$-b_1 - b_3 = -3$$

$$b_2 - 3b_1 = -4$$ A1

(b)

(i) $b_2 = 3b_1 - 4$ A1A1

(ii) $b_3 = 3 - b_1$. A1A1

(c) We have

$$(b_1)^2 + (3b_1 - 4)^2 + (3 - b_1)^2 = 9$$ M1

So

$$11(b_1)^2 - 30b_1 + 16 = 0$$ A1

Solve using any method e.g. factorisation

$$(11b_1 - 8)(b_1 - 2) = 0$$ M1

So

$$b_1 = \frac{8}{11} \text{ or } 2$$ A1

(d)

(i) $b_2 = -\frac{20}{11}$ or 2 A1A1

(ii) $b_3 = \frac{25}{11}$ or 1 A1A1

(e) This occurs when position vector **a** represents the closest point on the plane to the origin. R1

The line perpendicular to the plane through the origin has equation

$$\mathbf{r} = \lambda\begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix}$$ A1

So we have

$$25\lambda + 9\lambda + 16\lambda = 7$$ M1

So

$$\lambda = \frac{7}{50}$$ A1

Giving

$$|\mathbf{a}| = \sqrt{\left(\frac{7}{10}\right)^2 + \left(\frac{-21}{50}\right)^2 + \left(\frac{14}{25}\right)^2} = 0.990$$ M1A1