$$\cos\theta = \frac{\mathbf{a.b}}{|\mathbf{a}||\mathbf{b}|}$$ (M1)

$$= \frac{-4+14}{\sqrt{20}\sqrt{50}}$$ (A1)

$$= \frac{10}{10\sqrt{10}}$$

$$= \frac{1}{\sqrt{10}} \ (= 0.3162)$$ (A1)

$\theta = 72°$ (to the nearest degree) (A1) (C4)

***Note:** Award (C2) for a radian answer between 1.2 and 1.25.*

[4]

$$\begin{pmatrix}1\\2\end{pmatrix}\cdot\begin{pmatrix}6\\-8\end{pmatrix} = 6 - 16 = -10$$ (A1)

$$\left|\begin{pmatrix}1\\2\end{pmatrix}\right| = \sqrt{1^2+2^2} = \sqrt{5}, \left|\begin{pmatrix}6\\-8\end{pmatrix}\right| = \sqrt{6^2+8^2} = \sqrt{100} = 10$$ (A1)

$$\begin{pmatrix}1\\2\end{pmatrix}\cdot\begin{pmatrix}6\\-8\end{pmatrix} = \left|\begin{pmatrix}1\\2\end{pmatrix}\right|\left|\begin{pmatrix}6\\-8\end{pmatrix}\right|\cos\theta$$

$$-10 = \sqrt{5} \times 10\cos\theta \Rightarrow \cos\theta = \frac{-10}{10\sqrt{5}} = -\frac{1}{\sqrt{5}} \Rightarrow \theta = \arccos\frac{-1}{\sqrt{5}}$$ (M1)

$\theta \approx 117°$ (A1)

[4]

(a) $\begin{pmatrix}2x\\x-3\end{pmatrix}\bullet\begin{pmatrix}x+1\\5\end{pmatrix} = 0$ (M1)(M1)

$\Rightarrow 2x(x+1) + (x-3)(5) = 0$ (A1)

$\Rightarrow 2x^2 + 7x - 15 = 0$ (C3)

(b) **METHOD 1**

$2x^2 + 7x - 15 = (2x-3)(x+5) = 0$

$\Rightarrow x = \frac{3}{2}$ or $x = -5$ (A1) (C1)

METHOD 2

$$x = \frac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)}$$

$\Rightarrow x = \frac{3}{2}$ or $x = -5$ (A1) (C1)

[4]

(a) $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$ (A1) (C1)

(b) $\overrightarrow{OA} = \frac{1}{2}\overrightarrow{CD}$

$= \frac{1}{2}(\overrightarrow{OD} - \overrightarrow{OC})$ (A1) (C1)

(c) $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \overrightarrow{OD} - \frac{1}{2}(\overrightarrow{OD} - \overrightarrow{OC})$ (A1)

$= \frac{1}{2}\overrightarrow{OD} + \frac{1}{2}\overrightarrow{OC}$ (A1) (C2)

Note: *Deduct [1 mark] (once only) if appropriate vector notation is omitted.*

[4]

(a) $\overrightarrow{OB} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ (A1) (C1)

$\overrightarrow{AC} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ (A1) (C1)

(b) $\overrightarrow{OB} \cdot \overrightarrow{AC} = (10 \times (-3)) + (5 \times 6) = 0$ (M1)

Angle = 90° (A1) (C2)

[4]

(a) (i) evidence of combining vectors (M1)

e.g. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$ A1 N2

(ii) $\overrightarrow{AC} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ A1 N1

(b) (i) $\overrightarrow{AB} \bullet \overrightarrow{AC} = (-2)(3) + (-3)(-2) = 0$ A1 N1

(ii) scalar product 0 = $\Rightarrow$ perpendicular, $\theta = 90°$ (R1)

$\sin \theta = 1$ A1 N2

[6]

(a) evidence of appropriate approach (M1)

e.g. $\overline{\text{AC}} - \overline{\text{AB}}, \begin{pmatrix} 4 & -3 \\ 4 & -1 \end{pmatrix}$

$\overline{\text{BC}} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ A1 N2 2

(b) **METHOD 1**

$\overline{\text{AD}} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (A1)

correct approach A1

e.g. $\overline{\text{AD}} - \overline{\text{AB}}, \begin{pmatrix} 1 & -3 \\ 3 & -1 \end{pmatrix}$

$\overline{\text{BD}} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ AG N0 2

METHOD 2

Recognizing $\overline{\text{CD}} = \overline{\text{BA}}$ (A1)

correct approach A1

e.g. $\overline{\text{BC}} + \overline{\text{CD}}, \begin{pmatrix} 1 & -3 \\ 3 & -1 \end{pmatrix}$

$\overline{\text{BD}} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ AG N0 2

(c) **METHOD 1**

evidence of scalar product (M1)

e.g. $\overline{\text{BD}} \bullet \overline{\text{AC}}, \begin{pmatrix} -2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

correct substitution A1

e.g. $(-2)(4) + (2)(4), -8 + 8$

$\overline{\text{BD}} \bullet \overline{\text{AC}} = 0$ A1

therefore vectors $\overline{\text{BD}}$ and $\overline{\text{AC}}$ are perpendicular AG N0 3

METHOD 2

attempt to find angle between two vectors (M1)

e.g. $\frac{\mathbf{a} \bullet \mathbf{b}}{\mathbf{ab}}$

correct substitution A1

e.g. $\frac{(-2)(4) + (2)(4)}{\sqrt{8}\sqrt{32}}, \cos\theta = 0$

$\theta = 90°$ A1

therefore vectors $\overline{\text{BD}}$ and $\overline{\text{AC}}$ are perpendicular AG N0

[7]

(a) Finding **correct** vectors, $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ A1A1

Substituting correctly in the scalar product $\overrightarrow{AB} \cdot \overrightarrow{AC} = 4(-3) + 3(1)$ A1

$= -9$ AG N0

(b) $\left|\overrightarrow{AB}\right| = 5$ $\left|\overrightarrow{AC}\right| = \sqrt{10}$ (A1)(A1)

Evidence of using scalar product formula M1

e.g. $\cos B\hat{A}C = \frac{-9}{5\sqrt{10}} = -0.569$ (3 s.f.)

$B\hat{A}C = 2.47$ (radians), $125°$ A1 N3

[7]

(a) (i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}-3\\-1\end{pmatrix} - \begin{pmatrix}2\\-2\end{pmatrix}$ (M1)

$= \begin{pmatrix}-5\\1\end{pmatrix}$ (A1) (N2) 2

(ii) $\left|\overrightarrow{AB}\right| = \sqrt{25+1}$ (M1)

$= \sqrt{26}$ (= 5.10 to 3 sf) (A1) (N2) 2

Note: *An answer of 5.1 is subject to* ***AP****.*

(b) $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \begin{pmatrix}d\\23\end{pmatrix} - \begin{pmatrix}2\\-2\end{pmatrix}$

$= \begin{pmatrix}d-2\\25\end{pmatrix}$ (A1)(A1) 2

(c) (i) **EITHER**

$B\hat{A}D = 90° \Rightarrow \overrightarrow{AB} \bullet \overrightarrow{AD} = 0$ or mention of scalar (dot) product. (M1)

$\Rightarrow \begin{pmatrix}-5\\1\end{pmatrix} \bullet \begin{pmatrix}d-2\\25\end{pmatrix} = 0$

$-5d + 10 + 25 = 0$ (A1)

$d = 7$ (AG)

OR

$\left.\begin{array}{l}\text{Gradient of AB} = -\frac{1}{5}\\ \text{Gradient of AD} = \frac{25}{d-2}\end{array}\right\}$ (A1)

$\left(\frac{25}{d-2}\right) \times \left(-\frac{1}{5}\right) = -1$ (A1)

$\Rightarrow d = 7$ (AG)

(ii) $\overrightarrow{OD} = \begin{pmatrix}7\\23\end{pmatrix}$ (correct answer only) (A1) 3

(d) $\overrightarrow{AD} = \overrightarrow{BC}$ (M1)

$\overrightarrow{BC} = \begin{pmatrix} 5 \\ 25 \end{pmatrix}$ (A1)

$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$ (M1)

$\overrightarrow{OC} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 5 \\ 25 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 24 \end{pmatrix}$ (A1) (N3) 4

Note: *Many other methods, including scale drawing, are acceptable.*

(e) $\left|\overrightarrow{AD}\right| \left(\text{or } \left|\overrightarrow{BC}\right|\right) = \sqrt{5^2 + 25^2} = \sqrt{650}$ (A1)

Area = $\sqrt{26} \times \sqrt{650}$ =(5.099 × 25.5)

= 130 (A1) 2

[15]