33 Consider the function $f(x) = \dfrac{x+2}{x-1}$.

a Find the domain and range of f.

b Write down the equations of the asymptotes of $y = f(x)$.

c Find the axes intercepts.

d Draw a sign diagram for $f(x)$.

e Describe the behaviour of the function near the asymptotes.

f Sketch the function.

34 Consider the graph of $y = f(x)$ where $f(x) = 2 + \dfrac{4}{x+1}$.

a Find the axes intercepts.

b Calculate $f(-2)$.

c Determine the equation of the:

i horizontal asymptote

ii vertical asymptote.

d Sketch the graph of $y = 2 + \dfrac{4}{x+1}$. Label the axes intercepts and asymptotes clearly.

35 For each of the following functions:

i Find the equations of the asymptotes.

ii Find the axes intercepts.

iii Draw a sign diagram of the function.

iv Hence discuss the behaviour of the function near the asymptotes.

v Sketch the graph of the function.

a $y = \dfrac{2-x}{x^2+4x-21}$

b $y = \dfrac{5x-2}{2x^2+9x+9}$

36 Consider the function $f(x) = \dfrac{x+2}{x^2+bx+3}$ where b is a constant.

a Find the axes intercepts.

b Find the possible value(s) of b such that $f(x)$ has:

i no vertical asymptotes

ii one vertical asymptote

iii two vertical asymptotes.

37 For each of the following functions:

i Find the equation of the vertical asymptote.

ii Find the axes intercepts.

iii Find the oblique asymptote.

iv Draw a sign diagram of the function.

v Hence discuss the behaviour of the function near its asymptotes.

vi Sketch the graph of the function.

a $y = \dfrac{x^2-2x-8}{x-3}$

b $y = \dfrac{6x^2-7x-5}{3x+2}$

70 Fully describe the transformations which map $y = f(x)$ onto:

a $y = f(2(x-1))+3$

b $y = 5 - 2f(\frac{1}{4}x)$

c $y = 6f(\frac{1}{3}x - 2) + 4$

71 The function $y = f(x)$ is illustrated. Sketch the graphs of:

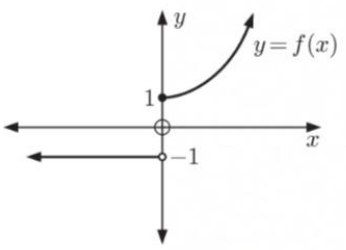
a $y = -f(x)$

b $y = f(-x)$

c $y = f(x-2)$

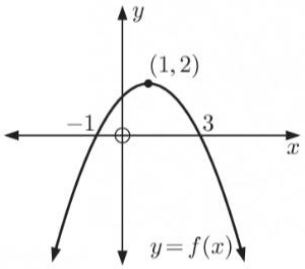
d $y = 2f(x)$

e $y = \dfrac{1}{f(x)}$

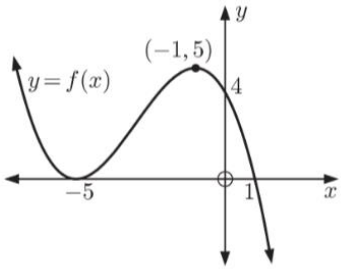


72 Copy the following graphs for $y = f(x)$ and on the same axes graph $y = \dfrac{1}{f(x)}$:

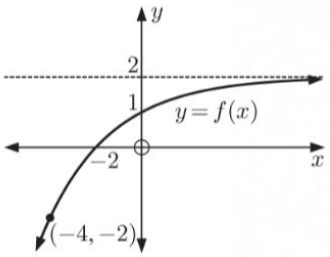
a



b



73 Copy the graph alongside, and on the same set of axes, sketch the graph of $y = [f(x)]^2 - 1$.



74 Consider the function $f(x) = \dfrac{6 - 2x}{x + 3}$.

a Find the axes intercepts and asymptotes of the function.

b Hence find the axes intercepts and asymptotes of $y = [f(x)]^2$.

c Which points are invariant when $y = f(x)$ is transformed to $y = [f(x)]^2$?

d Sketch $y = f(x)$ and $y = [f(x)]^2$ on the same set of axes.

75 **a** Sketch the graph of $f(x) = x^2 - 2x$, $x \in \mathbb{R}$, showing clearly the x-intercepts and vertex.

b Hence sketch the graph of:

i $y = f(|x|)$ **ii** $y = |f(x)|$

77 Solve for x:

a $|3 - 2x| = 5$ **b** $\left|\dfrac{2x + 5}{3 - x}\right| = 2$ **c** $|2 - x| = 3|x + 4|$

82 Suppose $f(x) = (5x - 2)\left(\dfrac{a}{x} + 3\right)$, $a \in \mathbb{R}$ is an odd function. Find the value of a.

83 **a** Can an even function have an inverse? Explain your answer.

b What domain restriction could be placed on $y = x^4 + x^2$ so that the new function obtained has an inverse?