Investigation of mathematics in barcodes

A: Clear structure.

Introduction

Barcodes are everywhere. There is a barcode label on almost every product. The mobile phone can be used as a barcode reader. By scanning the barcode, consumers can access information about the product easily. Information includes product description, price, the location of retailers, and product review.[1] Moreover, I have recently read the website about "Barcoded Medication Administration" (BCMA) which is the "system that uses barcodes to prevent human errors in the distribution of prescription medications at hospitals"[2].

C: Some personal engagement.

My interest in the barcodes arose after work experience over the summer in the fashion company, I worked in several departments including the shop assistant which I check out consumer's product. I started thinking about how barcodes work, how many different types there were, how to express information, and how many different arrangements of the black and white strips were possible and became interested to know the mathematics behind the barcodes. I read some articles and found the word "Symbology" to express the purpose of barcode. This is a way of mapping information into numbers and strips of black and white and it seemed similar to a function that we learn in SL course.

C: Student has carried out their own research.

This investigation aims to find out the number of combinations to express a number in the barcode. I will find this using EAN-8, EAN-13 and Code 128 and use tree diagram and combination formula to find out this solution. Barcodes contain check digit to avoid errors and I will explain how this works. I will also try to find out how to calculate the number of different possible barcodes using my own simplified set of rules.

A: Aim given.

B: Definition given.

What is EAN barcode?

EAN (European Article number) is the barcodes that are used worldwide for the consumer goods that are sold at retail point of sale. The numbers in the barcode include a large quantity

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[1] (No Date) Available at: http://apsoutheast1.accounts.ivvy.com.s3.amazonaws.com/account14/events/50/files/4dafc65ed4b71.pdf (Accessed: 9 November 2016).
[2] (No Date) Available at: http://searchhealthit.techtarget.com/definition/Bar-Coded-Medication-Administration (Accessed: 24 November 2016).

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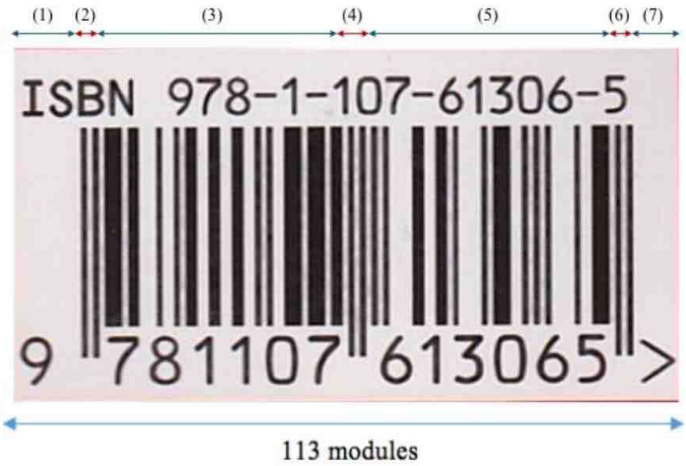
of information. There are two types of EAN; EAN-13 and EAN-8. EAU 8 is used for the smaller packages that have only limited space available for a barcode. (e.g. medicine). Firstly, I will be going to investigate EAN-13 barcode. I used the barcode which was on the back of the math book as an example (figure 1). EAN-13 barcode is commonly used than EAN-8.

B: Definitions given.

Figure 1: EAN-13[3] (this is a footnote and not the power of 3)

C: Shows personal engagement.



Module: Unit for width for composing barcode

(1) Left margin: 9 modules
(2) Left guard bar: 3 modules
(3) Left data character: 6 character 42 modules
(4) Centre guard: 5 modules
(5) Right data character: 6 character 42 modules
(6) Right guard bar: 3 module
(7) Right margin: 9 modules

B: Definitions.

9 781107 61306 5

↙ ↓ ↓ ↘

Country code Manufacturer code Product code Check digit

e.g. 978: ISBN (International Standard Book Number)
50: UK

→ Make sure the barcode does not have any errors

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[3] Paul Fennon, Vesna Kadelburg, Ben Woolley and Stephen Ward, Mathematics standard level for the IB diploma, Cambridge university press, 2012

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A: Helpful presentation.

Characteristics:

- Number consists of 0-9.
- It has 13 digits
- EAN barcode is separated into 4 sections 1) The country code, 2) The manufacturer code, 3) The product code, and 4) The check digit
- Centre guard pattern is white, black, white, black, white
- Left guard and right guard patterns are: black, white, black
- Right data character and left data character consists of 4 bars. Pattern are made from of 2 white and 2 black bars.
- Each bar varies in width, but the total width of the 4 bars is equal to 7

I will be looking at EAN 8 barcode. This has same characteristics with EAN 13 barcode but it has 8 digits instead of 13.

Figure 2: EAN-8 barcode.[4] (this is a footnote and not the power of 4)

(1) Left margin: 7 modules

(2) Left guard bar; 3 modules

(3) Left data character: 4 character 28 modules

(4) Centre bar; 5 modules

(5) Right data character; 4 character 28 module

(6) Right guard bar:3 module

[4] EAN-13, EAN-8 and UPC bar code image services, and bar code label printing and EAN specification (no date) Available at: http://www.terrapin.co.uk/services/bcspecean13.html (Accessed: 13 February 2017).

(7) Right margin; 7 module

Meaning of the numbers

50 1843 8 5

↙ ↓ ↓ ↘

Country code Manufacturer code Product code Check digit

→ Make sure the barcode does not have any errors

A: Well-organized structure.

<u>Ways of expressing a number in EAN 13 and EAN 8</u>

A number consist of 42÷6=7 module and consist of white, black, white and black stripe pattern. For example, the number 3 consists of 1 white, 4 black, 1 white and 1 black. (Shown in figure 3). The number 4 consists of 1 white, 1 black, 3 white and 2 black. If we express black and white pattern with a,b,c,d, a formula will be $a+b+c+d=7$ module. Which means each pattern consist of minimum 1 and maximum 4 modules ($1 \leq a,b,c,d \leq 4$). How many ways are there to make a number with 7 modules? There are 2 ways of calculating this.

B: A bit confusing.

Figure 3: The number 3

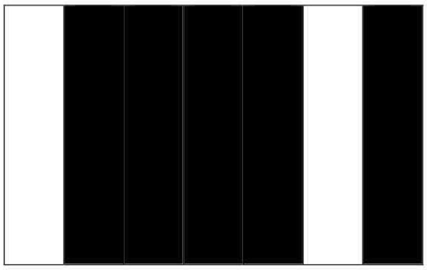
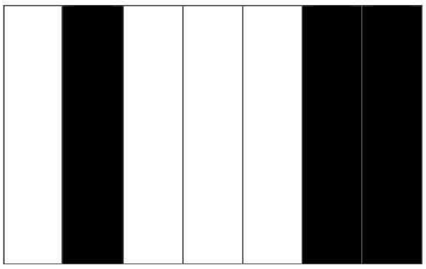


Figure 4: The number 4



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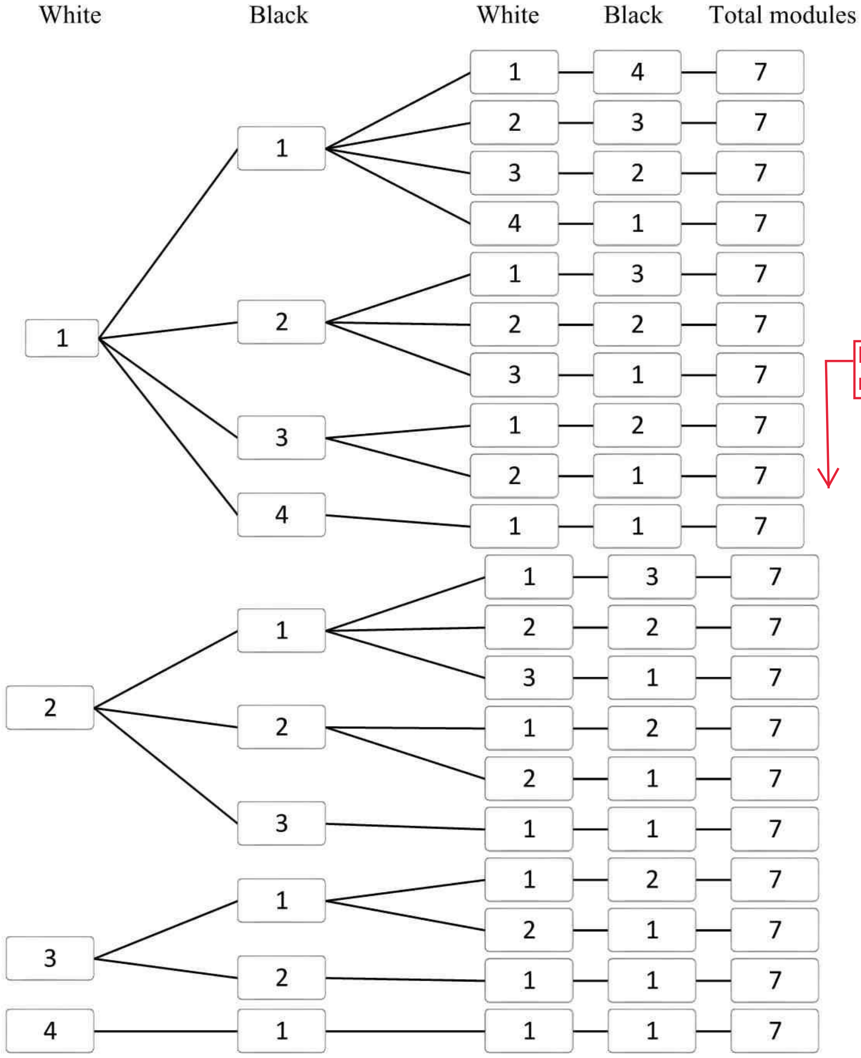
1. Figure 5: Tree diagram showing the number of combinations of expressing number



B: Useful representation.

There are 20 ways of expressing the number

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2. Combination formula

The minimum module is 1 and the maximum module is 4. The formula for this will be 1≤ a,b,c,d≤4

B: This would be clearer on one line.

If you subtract a minimum of 1 module from a,b,c,d, then it will be $7-(1+1+1+1)=3$. Thus, we need to find the number of ways of 3 module from a,b,c,d. We use combination formula to solve this question.

$$_{n+r-1}C_r = \frac{(n+r-1)!}{(n-1)!\,r!}$$

B: n and r are not defined.

When you applied the number into this formula, it will be this this

$$_{4+3-1}C_3 = {}_6\,C_3 = \frac{6!}{(6-3)!\,3!} = 20$$

E: Use of combinations.

There are 20 ways to expressing a number using 7 modules. However, there is both white, black, white, black and also opposite of black, white, black, white. (Odd parity and even parity)

If the number of black modules is odd number, then we call this "odd parity"
If the number of black modules is even number, then we call this "even parity"

B: Definitions given.

Figure 6

Number	Ratio	Odd parity	Even parity
0	3:2:1:1 / 1:1:2:3		

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1	2:2:2:1/ 1:2:2:2		
2	2:1:2:2/ 2:2:1:2		
3	1:4:1:1 /1:1:4:1		
4	1:1:3:2 / 2:3:1:1		
5	1:2:3:2 / 1:3:2:1		
6	1:1:1:4 /4:1:1:1		
7	1:3:1:2 /2:1:3:1		
8	1:2:1:3 / 3:1:2:1		
9	3:1:1:2/ 2:1:1:3		

e.g. number 「0」

Either 「3:2:1:1」、「1:1:2:3」

B: It is not clear what this means.

Figure 7:

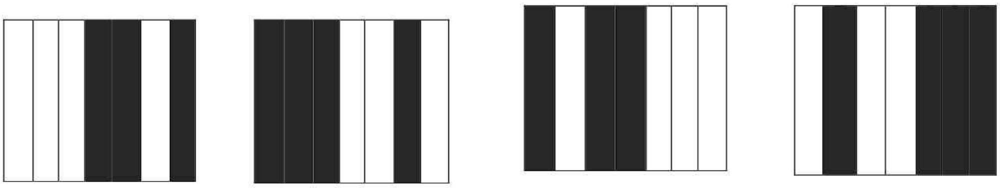
0

↙ ↘

3:2:1:1 1:1:2:3

↙ ↘ ↙ ↘

odd parity even parity odd parity even parity



If we include both odd and even parity, 20×2=40.

Thus, as the result, there are 40 ways of expressing a number with 7 modules.

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Calculate the check digit

Barcode could easily be misread so check digit is very important way to detect errors and it helps to increase the accuracy in the barcode. The check digit is a last digit of a barcode number and it is calculated by using all of the other numbers in the barcode.

You can calculate this by…

1. Add the digits in the even numbered position from left e.g. 2,4,6,8---
2. Multiply this result by 3
3. Add the digits in the odd number positions e.g. position 1,3,5,7---
4. Sum the result of step 2 and 3
5. Calculate the difference between that number and the next multiple of 10

Example : 978110761306

E: Example given.

1. 7+1+0+6+3+6=23
2. 23×3=69
3. 9+8+1+7+1+0=26
4. 69+26=95
5. The next multiple of 10 that is greater than 95 is 100
6. 100−95=5
7. Therefore, check digit is 5 and barcode number in full is 9781107613065

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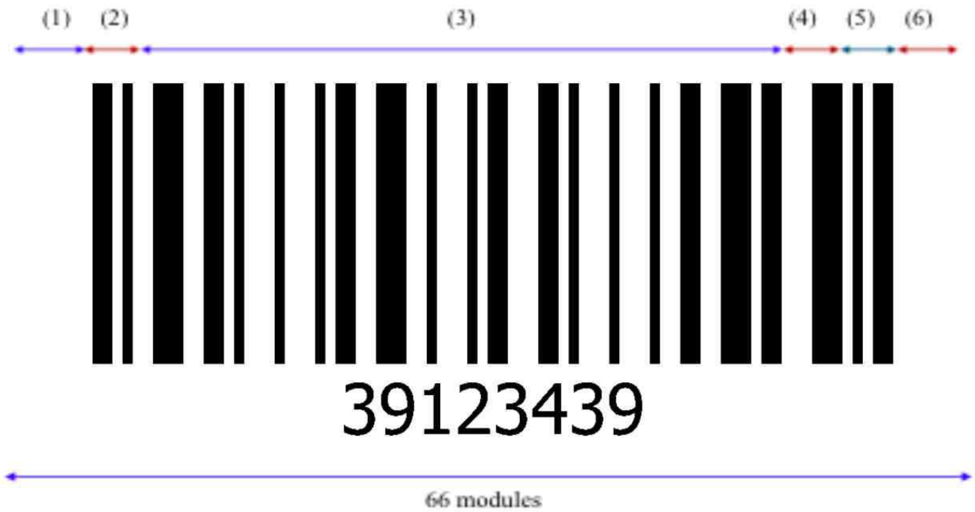
Next, I will be looking at code 128. This is a very high density symbology which permits the encoding of alphanumeric and all-numeric data.

Figure 8: Code 128[5] (It is a footnote and not the power of 5)



(1) Quiet zone: 10 modules
(2) Start character: 11 modules
(3) Data: 1 character 11 modules
(4) Check character: 11 modules
(5) Stop character: 13 modules
(6) Quiet zone: 10 modules

<u>Characteristics;</u>

- It consists number 0-99
- It consists alphabetical letter A-Z (lower and upper)
- Each character consists of 3 black and 3 white bars

[5] Code 39 & 128 Barcodes (2016) Available at: http://worldbarcodes.com/code-128-code-39/ (Accessed: 13 December 2017).

Figure 9: Number 1

Figure 10: Letter A

For example, the number 1 is made of 2 black, 2 white, 2 black, 1 white, 2 black and 2 white. And letter A is made of 1 black, 1 white, 1 black, 3 white, 2 black and 3 white. (Shown in figures 9 and10).

In code 128, 1 character consists of 11 modules. It is very difficult and time consuming write a tree diagram. Therefore, I will just calculate this using combination formula.

D: Limited reflection.

In the case of 11 modules,

$(a + b + c + d + e + f) = 11$

Which means each pattern consists of minimum 1 and maximum 6 modules.

If you subtract a minimum of 1 module from $(a + b + c + d + e + f)$ then it will be $11 - (1 + 1 + 1 + 1 + 1 + 1) = 5$. Thus we need to find the probability of 5 module from a,b,c,d,e,f.

When you applied the number into combination formula, it will be this this;

$$C(n, r) = {}_{10}C_5 = {}_{10}C_5 \binom{n}{r} = \frac{10!}{(10-5)!\,5!} = 252$$

E: Shows some knowledge and understanding.

And if you include both odd and even parity, there are $252 \times 2 = 504$ ways of expressing a number with 11 modules.

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Calculate the check digit

Example 1: 3912343 I will show check digit is 9

1. See the character set table to obtain the value of the start character and all data characters [6]

2. Using the table, multiply each value by its position number

e.g. $1 \times 19, 2 \times 25, 3 \times 17, 4 \times 18, 5 \times 19, 6 \times 20, 7 \times 19$

A: Even-odd placed digit pattern?

3. Add the value from step 2 with the start code value

$103 + 19 + 50 + 51 + 72 + 95 + 120 + 133 = 643$

4. Divide the value by 103 and obtain the remainder

$643 \div 103 = 6.243$ Remainder=25

5. Use the table, find the character with this value

25→9

Therefore, check digit is 9 and full barcode number is 39123439

Figure 11

Barcode	Start A	3	9	1	2	3	4	3
Character position		1	2	3	4	5	6	7
Character value	103	19	25	17	18	19	20	19
Calculation		1×19	2×25	3×17	4×18	5×19	6×20	7×19
Totals	103	19	50	51	72	95	120	133

B: Use of a table to show results.

[6] Code 128 Barcode FAQ & Tutorial (2017) Available at: http://www.idautomation.com/barcode-faq/code-128/ (Accessed: 1 February 2017).

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There is a lack of coherence here.

Calculation of how many possible barcodes are there?

In my shop in Japan, I asked this question several times. Millions, billions, trillions, were some of the answers. How to work this out? One method is to consider that each strip can be either B= Black or W= White. Then if there are 2 choices at each stage. Then if
x= Width of barcode which is the same as the total number of strips and
Y= Number of different barcodes with width x
The formula would be $Y = 2^x$
And using my first example with 113 strips there could be 2^{113} different barcodes of this width. This is 10^{34} on GDC
Would they all be possible choices?
In reality large number of W or B strips all together would not be good; there could be a restriction so that the colour would have to change frequently between B and W.
Starting and ending with B strip might make it easier to read the code. I will set this as my condition and also limit the number of strips of B or W to a maximum of 2
With a width of 5, the possible barcodes would be
BWBWB; BWWBB; BBWWB; BBWBB

B: Use of notation.

If x is a small number such as 5, writing out all the possibilities can be done as above but as x increases that will become impossible. Is there some function method that could be used?
As I was considering this, I recalled how Pascal's triangle numbers can be built up by adding numbers from the line above. Could such a method help?

To sum up my system as follows:
In every barcode, start with 1 black strip and also end with 1 black strip and in between, there are alternate of black and white. Each black section could be 1 or 2 strips and each white section 1 or 2 strips.

Y(x) will be discrete function. Domain is set of positive integers.
If there is just 1 strip, (x = 1) B is the only possibility as it starts and finishes with B
$\therefore Y(1)=1$

For 2 strips, (x = 2) BB is the only possibility as it starts and finishes with B
$\therefore Y(2) = 1$

For 3 strips, BWB, is the only possibility, as we can't have BBB (only 2B allowed)
$\therefore Y(3) = 1$

For 4 strips, can get BBWB, BWBB, BWWB, (we can't have BBBB)
$\therefore Y(4) = 3$

D: The student reflects on the results.

I already have from above $Y(5) = 4$ and looking at these choices, I thought they could all be made by adding some of the previous information as in the Pascal triangle method.

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A strip of 5 could start BW and then strip of 3 and $Y(3) = 1$
Or it could start BBW and strip of 2 and $Y(2) = 1$
Or it could start BWW and again strip of 2 and $Y(2) = 1$
Or it could start BBWW and strip of 1 and $Y(1)$
Since there can't be 3W's, these are the only possibilities.
$Y(5) = Y(3) + 2Y(2) + Y(1) = 4$
If this works then
$Y(6) = Y(4) + 2Y(3) + Y(2) = 6$
BW BBWB; BW BWBB; BW BWWB; BBW BWB; BWW BWB; BBWW BB
I put gaps to show how the addition works.

This suggests the formula,
$Y(x) = Y(x-2) + 2Y(x-3) + Y(x-4)$ using this, work out the next values as follows,

$Y(7) = Y(5) + 2Y(4) + Y(3) = 11$

$Y(8) = Y(6) + 2Y(5) + Y(4) = 17$

B: A table of values would be useful here.

$Y(8) = Y(6) + 2Y(5) + Y(4) = 27$

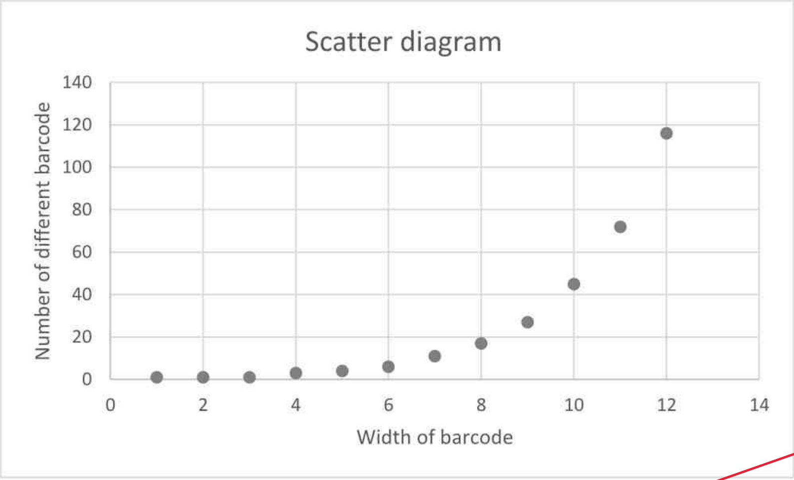
$Y(10) = Y(8) + 2Y(7) + Y(6) = 45$

$Y(11) = Y(9) + 2Y(8) + Y(7) = 72$

$Y(12) = Y(10) + 2Y(9) + Y(8) = 116$

I calculated some other values in the same way, (see appendix).

This is much quicker than writing out all the possibilities but it would still take time to reach a value for $Y(131)$
However, we can see the values are increasing quite rapidly. Does it follow an equation? Using GDC, I created a scatter diagram for x values of 1 to 12 as my domain and Y values have a range greater than 0.



D: There is no reflection on whether this is an exact match.

The graph looks like an exponential function
It gives the equation of $y = 0.409(1.59)^x$ using exponential regression on GDC.

Will this work for future values?
If I substitute $X = 13$ as an example and $Y = 0.409(1.59)^x = 170$
The answer from the formula is 189
$Y(13) = Y(11) + 2Y(10) + Y(9) = 189$
Therefore, this equation does not seem to work for future values as my barcode function is increasing at much faster rate.
The scatter diagram looks like standard exponential function and I tried some variations of this on my GDC and looked at the table values but could not find any function that gave similar values.

C: Tests some values.

<u>Conclusion:</u>
In this exploration, I have learned that there are 2 mathematical approaches to find the number of combinations to express a number in barcodes. When expressing a character with 7 modules, I came up with the idea of drawing a tree diagram, which demonstrates all the possible combinations; whereas the combination formula. ($C(n, r) = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$) gives the total number of ways. Both of these approaches worked fine. However, for the barcode 128 which have 11 modules for each number, it is too difficult and time consuming to draw a tree diagram, because it has 504 possible combinations. Thus, I have only used the combination formula.
I have also investigated into calculation of number of different possible barcodes based on a simplified system with constraints. $y = 0.409(1.59)^x$ using exponential regression seems to work for first 12 values, however, it could not be used for additional values. This shows the

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limitation on using regression to predict a value outside of the given domain. The value increased much more rapidly as shown in the table in the appendix, Y(31) would be over a million. EAN-13 barcode which I used at the beginning had 113 modules. By using my equation, Y(113) is likely to be billions or trillions. Since barcode is universally used for many situations, it is necessary to have a large number of different possibilities.

D: Reflects on the results.

In this investigation, it is proven that in EAN barcode there are 40 ways to express number with 7 modules and code 128 has 504 ways with 11 modules. The check digit, is a way of preventing errors and is easy to calculate as I showed.

D: Reflects on the limitations of the exploration.

Evaluation

This investigation has deepened my understanding of barcode however my exploration was limited because I have only explored EAN and Code 128. There are more barcodes that are used in BCMA therefore if have a chance explore more on this topic, I would like to look at other barcodes such as JAN codes of code 39. This investigation could be extended by looking at the relationship between error rate and length of the barcode. From this, it is may be possible to find the optimum length of the barcode and minimize the possibility of error.

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Appendix

$Y(14) = Y(12) + 2Y(11) + Y(10) = 305$

$Y(15) = Y(13) + 2Y(12) + Y(11) = 724$

$Y(16) = Y(14) + 2Y(13) + Y(12) = 799$

$Y(17) = Y(15) + 2Y(14) + Y(13) = 1523$

$Y(18) = Y(16) + 2Y(15) + Y(14) = 2552$

$Y(19) = Y(17) + 2Y(16) + Y(15) = 3845$

$Y(20) = Y(18) + 2Y(17) + Y(16) = 6397$

$Y(21) = Y(19) + 2Y(18) + Y(17) = 10472$

$Y(22) = Y(20) + 2Y(19) + Y(18) = 16639$

$Y(23) = Y(21) + 2Y(20) + Y(19) = 27111$

$Y(24) = Y(22) + 2Y(21) + Y(20) = 43980$

$Y(25) = Y(23) + 2Y(22) + Y(21) = 70861$

$Y(26) = Y(24) + 2Y(23) + Y(22) = 114841$

$Y(27) = Y(25) + 2Y(24) + Y(23) = 185932$

$Y(28) = Y(26) + 2Y(25) + Y(24) = 300543$

$Y(29) = Y(27) + 2Y(26) + Y(25) = 486475$

$Y(30) = Y(28) + 2Y(27) + Y(26) = 789248$

$Y(31) = Y(29) + 2Y(28) + Y(27) = 1273493$

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