7.4 THE ALGEBRA OF LOGARITHMS

The following logarithmic laws are a direct consequence of the definition of a logarithm and the index laws already established.

First law: **The logarithm of a product** $\log_a(x \times y) = \log_a x + \log_a y,\ x > 0,\ y > 0$

Proof: Let $M = \log_a x$ and $N = \log_a y$ so that $x = a^M$ and $y = a^N$.

Then, $x \times y = a^M \times a^N$

$$\Leftrightarrow x \times y = a^{M+N}$$

$$\Leftrightarrow \log_a(x \times y) = M + N$$

$$\Leftrightarrow \log_a(x \times y) = \log_a x + \log_a y$$

EXAMPLE 7.14

Simplify the expression

(a) $\log_3 x + \log_3(4x)$ (b) $\log_2 x + \log_2(4x)$

Solution

(a) $\log_3 x + \log_3(4x) = \log_3(x \times 4x)$

$$= \log_3 4x^2$$

(b) This time we note that because the base is '2' and there is a '4' in one of the logarithmic expressions, we could first try to 'remove the '4'.

$$\log_2 x + \log_2(4x) = \log_2 x + (\log_2 4 + \log_2 x)$$

$$= \log_2 x + 2 + \log_2 x$$

$$= 2\log_2 x + 2$$

EXAMPLE 7.15

Given that $\log_a p = 0.70$ and $\log_a q = 2$, evaluate the following

(a) $\log_a p^2$ (b) $\log_a(p^2 q)$ (c) $\log_a(apq)$

Solution

(a) $\log_a p^2 = \log_a(p \times p) = \log_a p + \log_a p$

$$= 2\log_a p$$

$$= 2 \times 0.70$$

$$= 1.40$$

(b) $\log_a(p^2 q) = \log_a p^2 + \log_a q$

$$= 2\log_a p + \log_a q$$

$$= 1.40 + 2$$

$$= 3.40$$

(c) $\log_a(apq) = \log_a a + \log_a p + \log_a q$

$= 1 + 0.70 + 2$

$= 3.70$

EXAMPLE 7.16

Find $\{x \mid \log_2 x + \log_2(x+2) = 3\}$.

Solution

$$\log_2 x + \log_2(x+2) = 3 \Leftrightarrow \log_2[x \times (x+2)] = 3$$
$$\Leftrightarrow x(x+2) = 2^3$$
$$\Leftrightarrow x^2 + 2x = 8$$
$$\Leftrightarrow x^2 + 2x - 8 = 0$$
$$\Leftrightarrow (x+4)(x-2) = 0$$
$$\Leftrightarrow x = -4 \text{ or } x = 2$$

Next, we must check our solutions.

When $x = -4$, substituting into the **original equation**, we have:

L.H.S = $\log_2(-4) + \log_2(-4+2)$ – which cannot be evaluated (as the logarithm of a negative number does not exist).

Therefore, $x = -4$, is not a possible solution.

When $x = 2$, substituting into the **original equation**, we have:

L.H.S $= \log_2(2) + \log_2(2+2)$

$= \log_2 8$

$= 3$

$=$ R.H.S

Therefore, $\{x \mid \log_2 x + \log_2(x+2) = 3\} = \{2\}$.

Second law: The logarithm of a quotient $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y,\ x > 0,\ y > 0$

Proof: Let $M = \log_a x$ and $N = \log_a y$ so that $x = a^M$ and $y = a^N$.

Then, $\frac{x}{y} = \frac{a^M}{a^N}$

$\Leftrightarrow \frac{x}{y} = a^{M-N}$

$\Leftrightarrow \log_a\left(\frac{x}{y}\right) = M - N$

$\Leftrightarrow \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

EXAMPLE 7.17

Simplify

(a) $\log_{10}100x - \log_{10}xy$ (b) $\log_2 8x^3 - \log_2 x^2 + \log_2\left(\frac{y}{x}\right)$

Solution

(a) $$\log_{10}100x - \log_{10}xy = \log_{10}\left(\frac{100x}{xy}\right)$$
$$= \log_{10}\left(\frac{100}{y}\right)$$

Note: We could then express $\log_{10}\left(\frac{100}{y}\right)$ as $\log_{10}100 - \log_{10}y = 2 - \log_{10}y$.

(b) $$\log_2 8x^3 - \log_2 x^2 + \log_2\left(\frac{y}{x}\right) = \log_2\left(\frac{8x^3}{x^2}\right) + \log_2\left(\frac{y}{x}\right)$$
$$= \log_2 8x + \log_2\left(\frac{y}{x}\right)$$
$$= \log_2\left(8x \times \frac{y}{x}\right)$$
$$= \log_2 8y$$

Note: We could then express $\log_2 8y$ as $\log_2 8 + \log_2 y = 3 + \log_2 y$.

EXAMPLE 7.18

Find $\{x|\ \log_{10}(x+2) - \log_{10}(x-1) = 1\}$

Solution

$$\log_{10}(x+2) - \log_{10}(x-1) = 1 \Leftrightarrow \log_{10}\left(\frac{x+2}{x-1}\right) = 1$$
$$\Leftrightarrow \left(\frac{x+2}{x-1}\right) = 10^1$$
$$\Leftrightarrow x + 2 = 10x - 10$$
$$\Leftrightarrow 12 = 9x$$
$$\Leftrightarrow x = \frac{4}{3}$$

Next, we check our answer. Substituting into the original equation, we have:

$$\text{L.H.S} = \log_{10}\left(\frac{4}{3} + 2\right) - \log_{10}\left(\frac{4}{3} - 1\right) = \log_{10}\frac{10}{3} - \log_{10}\frac{1}{3} = \log_{10}\left(\frac{10}{3} \div \frac{1}{3}\right)$$
$$= \log_{10}10$$
$$= 1 = \text{R.H.S}$$

Therefore, $\{x|\ \log_{10}(x+2) - \log_{10}(x-1) = 1\} = \left\{\frac{4}{3}\right\}$

Third law: The logarithm of a power $\log_a x^n = n\log_a x,\ x > 0$

Proof: This follows from repeated use of the First Law or it can be shown as follows:

Let $M = \log_a x \Leftrightarrow a^M = x$

$\Leftrightarrow (a^M)^n = x^n$ [Raising both sides to the power of n]

$\Leftrightarrow a^{nM} = x^n$ [Using the index laws]

$\Leftrightarrow nM = \log_a x^n$ [Converting from exponential to log form]

$\Leftrightarrow n\log_a x = \log_a x^n$

EXAMPLE 7.19

Given that $\log_a x = 0.2$ and $\log_a y = 0.5$, evaluate

(a) $\log_a x^3 y^2$ (b) $\log_a \sqrt{\frac{x}{y^4}}$

Solution

(a) $\log_a x^3 y^2 = \log_a x^3 + \log_a y^2$

$= 3\log_a x + 2\log_a y$

$= 3 \times 0.2 + 2 \times 0.5$

$= 1.6$

(b) $\log_a \sqrt{\frac{x}{y^4}} = \log_a \left(\frac{x}{y^4}\right)^{1/2} = \frac{1}{2}\log_a \left(\frac{x}{y^4}\right)$

$= \frac{1}{2}[\log_a (x) - \log_a y^4]$

$= \frac{1}{2}[\log_a x - 4\log_a y]$

$= \frac{1}{2}[0.2 - 4 \times 0.5]$

$= -0.9$

Fourth law: Change of base $\log_a b = \frac{\log_k b}{\log_k a},\ a, k \in \mathbb{R}^+ \backslash \{1\}$

Proof: Let $\log_a b = N$ so that $a^N = b$

Taking the logarithms to base k of both sides of the equation we have:

$$\log_k (a^N) = \log_k b \Leftrightarrow N\log_k a = \log_k b$$

$$\Leftrightarrow N = \frac{\log_k b}{\log_k a}$$

However, we have that $\log_a b = N$, therefore, $\log_a b = \frac{\log_k b}{\log_k a}$.

Other observations include:

1. $\log_a a = 1$
2. $\log_a 1 = 0$
3. $\log_a x^{-1} = -\log_a x, x > 0$
4. $\log_{\frac{1}{a}} x = -\log_a x$
5. $a^{\log_a x} = x, x > 0$

MISCELLANEOUS EXAMPLES

EXAMPLE 7.20

Express y in terms of x if (a) $2 + \log_{10} x = 4\log_{10} y$

(b) $\log x = \log(a - by) - \log a$

Solution

(a) Given that $2 + \log_{10} x = 4\log_{10} y$ then $2 = 4\log_{10} y - \log_{10} x$

$$\Leftrightarrow 2 = \log_{10} y^4 - \log_{10} x$$

$$\Leftrightarrow 2 = \log_{10}\left(\frac{y^4}{x}\right)$$

$$\Leftrightarrow 10^2 = \frac{y^4}{x}$$

$$\Leftrightarrow y^4 = 100x$$

$$\Leftrightarrow y = \sqrt[4]{100x} \quad (\text{ as } y > 0)$$

(b) Given that $\log x = \log(a - by) - \log a$ then $\log x = \log\frac{a - by}{a}$

$$\Leftrightarrow x = \frac{a - by}{a}$$

$$\Leftrightarrow ax = a - by$$

$$\Leftrightarrow by = a - ax$$

$$\Leftrightarrow y = \frac{a}{b}(1 - x)$$

EXAMPLE 7.21

Find x if (a) $\log_x 64 = 3$ (b) $\log_{10} x - \log_{10}(x - 2) = 1$

Solution

(a) $\log_x 64 = 3 \Leftrightarrow x^3 = 64$

$$\Leftrightarrow x^3 = 4^3$$

$$\Leftrightarrow x = 4$$

(b) $\log_{10} x - \log_{10}(x - 2) = 1 \Leftrightarrow \log_{10}\left(\frac{x}{x - 2}\right) = 1$

$$\Leftrightarrow \frac{x}{x-2} = 10^1$$
$$\Leftrightarrow x = 10x - 20$$
$$\Leftrightarrow -9x = -20$$
$$\Leftrightarrow x = \frac{20}{9}$$

We still need to check our answer: substituting $x = \frac{20}{9}$ into the original equation we have:

$$\text{L.H.S} = \log_{10}\frac{20}{9} - \log_{10}\left(\frac{20}{9} - 2\right) = \log_{10}\frac{20}{9} - \log_{10}\left(\frac{2}{9}\right) = \log_{10}\left(\frac{20}{9} \times \frac{9}{2}\right)$$
$$= \log_{10}10$$
$$= 1$$
$$= \text{R.H.S}$$

Therefore. solution is $x = \frac{20}{9}$.

EXAMPLE 7.22

Find $\{x \mid 5^x = 2^{x+1}\}$. Give both an exact answer and one to 2 d.p.

Solution

Taking the logarithm of base 10 of both sides $5^x = 2^{x+1}$ gives:

$$5^x = 2^{x+1} \Leftrightarrow \log_{10}5^x = \log_{10}2^{x+1}$$
$$\Leftrightarrow x\log_{10}5 = (x+1)\log_{10}2$$
$$\Leftrightarrow x\log_{10}5 - x\log_{10}2 = \log_{10}2$$
$$\Leftrightarrow x(\log_{10}5 - \log_{10}2) = \log_{10}2$$
$$\Leftrightarrow x = \frac{\log_{10}2}{\log_{10}5 - \log_{10}2}$$

And so, $x = 0.75647\ldots = 0.76$ (to 2 d.p).

Exact answer = $\left\{\frac{\log_{10}2}{\log_{10}5 - \log_{10}2}\right\}$, answer to 2 d.p = $\{0.76\}$

EXAMPLE 7.23

Find x, where $6e^{2x} - 17 \times e^x + 12 = 0$

Solution

We first note that $6e^{2x} - 17 \times e^x + 12$ can be written as $6 \times e^{2x} - 17 \times e^x + 12$.

Which in turn can be expressed as $6 \times (e^x)^2 - 17 \times e^x + 12$.

Therefore, making the substitution $y = e^x$, we have that

$6 \times (e^x)^2 - 17 \times e^x + 12 = 6y^2 - 17y + 12$ (i.e., we have a 'hidden' quadratic)

Solving for y, we have:

$$6y^2 - 17y + 12 = 0 \Leftrightarrow (2y-3)(3y-4) = 0$$

So that $y = \frac{3}{2}$ or $y = \frac{4}{3}$

However, we wish to solve for x, and so, we need to substitute back:

$$e^x = \frac{3}{2} \text{ or } e^x = \frac{4}{3}$$

$$\Leftrightarrow x = \ln\frac{3}{2} \text{ or } x = \ln\frac{4}{3}$$

EXAMPLE 7.24

Solve for x, where $8^{2x+1} = 4^{5-x}$

Solution

Taking logs of both sides of the equation $8^{2x+1} = 4^{5-x}$, we have

$$\log 8^{2x+1} = \log 4^{5-x} \Leftrightarrow (2x+1)\log 8 = (5-x)\log 4$$

$$\Leftrightarrow (2x+1)\log 2^3 = (5-x)\log 2^2$$

$$\Leftrightarrow 3(2x+1)\log 2 = 2(5-x)\log 2$$

Therefore, we have that $6x+3 = 10-2x \Leftrightarrow 8x = 7$

$$\therefore x = \frac{7}{8}$$

EXERCISES 7.4

1. Without using a calculator, evaluate the following.

(a) $\log_2 8 + \log_2 4$ (b) $\log_6 18 + \log_6 2$ (c) $\log_5 2 + \log_5 12.5$

(d) $\log_3 18 - \log_3 6$ (e) $\log_2 20 - \log_2 5$ (f) $\log_2 10 - \log_2 5$

2. Write down an expresion for $\log a$ in terms of $\log b$ and $\log c$ for the following.

(a) $a = bc$ (b) $a = b^2c$ (c) $a = \frac{1}{c^2}$

(d) $a = b\sqrt{c}$ (e) $a = b^3c^4$ (f) $a = \frac{b^2}{\sqrt{c}}$

3. Given that $\log_a x = 0.09$, find

(a) $\log_a x^2$ (b) $\log_a \sqrt{x}$ (c) $\log_a\left(\frac{1}{x}\right)$

4. Express each of the following as an equation that does not involve a logarithm.

(a) $\log_2 x = \log_2 y + \log_2 z$
(b) $\log_{10} y = 2\log_{10} x$
(c) $\log_2(x+1) = \log_2 y + \log_2 x$
(d) $\log_2 x = y + 1$
(e) $\log_2 y = \frac{1}{2}\log_2 x$
(f) $3\log_2(x+1) = 2\log_2 y$

5. Solve the following equations

(a) $\log_2(x+1) - \log_2 x = \log_2 3$
(b) $\log_{10}(x+1) - \log_{10} x = \log_{10} 3$
(c) $\log_2(x+1) - \log_2(x-1) = 4$
(d) $\log_{10}(x+3) - \log_{10} x = \log_{10} x + \log_{10} 2$
(e) $\log_{10}(x^2+1) - 2\log_{10} x = 1$
(f) $\log_2(3x^2+28) - \log_2(3x-2) = 1$
(g) $\log_{10}(x^2+1) = 1 + \log_{10}(x-2)$
(h) $\log_2(x+3) = 1 - \log_2(x-2)$
(i) $\log_6(x+5) + \log_6 x = 2$
(j) $\log_3(x-2) + \log_3(x-4) = 2$
(k) $\log_2 x - \log_2(x-1) = 3\log_2 4$
(l) $\log_{10}(x+2) - \log_{10} x = 2\log_{10} 4$

6. Simplify the following

(a) $\log_3(2x) + \log_3 w$
(b) $\log_4 x - \log_4(7y)$
(c) $2\log_a x + 3\log_a(x+1)$
(d) $5\log_a x - \frac{1}{2}\log_a(2x-3) + 3\log_a(x+1)$
(e) $\log_{10} x^3 + \frac{1}{3}\log x^3 y^6 - 5\log_{10} x$
(f) $2\log_2 x - 4\log_2\left(\frac{1}{y}\right) - 3\log_2 xy$

7. Solve the following

(a) $\log_2(x+7) + \log_2 x = 3$
(b) $\log_3(x+3) + \log_3(x+5) = 1$
(c) $\log_{10}(x+7) + \log_{10}(x-2) = 1$
(d) $\log_3 x + \log_3(x-8) = 2$
(e) $\log_2 x + \log_2 x^3 = 4$
(f) $\log_3\sqrt{x} + 3\log_3 x = 7$

8. Solve for x.

(a) $\log_2 x^2 = (\log_2 x)^2$
(b) $\log_3 x^3 = (\log_3 x)^3$
(c) $\log_4 x^4 = (\log_4 x)^4$
(d) $\log_5 x^5 = (\log_5 x)^5$

Investigate the solution to $\log_n x^n = (\log_n x)^n$

9. Solve the following, giving an exact answer and an answer to 2 d.p.

(a) $2^x = 14$ (b) $10^x = 8$ (c) $3^x = 125$

(d) $\dfrac{1}{1-2^x} = 12$ (e) $3^{4x+1} = 10$ (f) $0.8^{x-1} = 0.4$

(g) $10^{-2x} = 2$ (h) $2.7^{0.3x} = 9$ (i) $0.2^{-2x} = 20$

(j) $\dfrac{2}{1+0.4^x} = 5$ (k) $\dfrac{2^x}{1-2^x} = 3$ (l) $\dfrac{3^x}{3^x+3} = \dfrac{1}{3}$

10. Solve for x

(a) $(\log_2 x)^2 - \log_2 x - 2 = 0$ (b) $\log_2(2^{x+1} - 8) = x$

(c) $\log_{10}(x^2 - 3x + 6) = 1$ (d) $(\log_{10} x)^2 - 11\log_{10} x + 10 = 0$

(e) $\log_x(3x^2 + 10x) = 3$ (f) $\log_{x+2}(3x^2 + 4x - 14) = 2$

11. Solve the following simultaneous equations

(a) $x^y = 5x - 9$, $\log_x 11 = y$ (b) $\log_{10} x - \log_{10} y = 1$, $x + y^2 = 200$ (c) $xy = 2$, $2\log_2 x - \log_2 y = 2$

12. Express each of the following as an equation that does not involve a logarithm.

(a) $\log_e x = \log_e y - \log_e z$ (b) $3\log_e x = \log_e y$ (c) $\ln x = y - 1$

13. Solve the following for x

(a) $\ln(x+1) - \ln x = 4$ (b) $\ln(x+1) - \ln x = \ln 4$

(c) $\log_e(x+1) + \log_e x = 0$ (d) $\log_e(x+1) - \log_e x = 0$

14. Solve the following for x

(a) $e^x = 21$ (b) $e^x - 2 = 8$ (c) $-5 + e^{-x} = 2$

(d) $200e^{-2x} = 50$ (e) $\dfrac{2}{1-e^{-x}} = 3$ (f) $70e^{-\frac{1}{2}x} + 15 = 60$

(g) $\ln x = 3$ (h) $2\ln(3x) = 4$ (i) $\ln(x^2) = 9$

(j) $\ln x - \ln(x+2) = 3$ (k) $\ln\sqrt{x+4} = 1$ (l) $\ln(x^3) = 9$

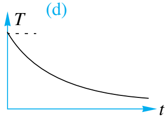
15. Solve the following for x

(a) $e^{2x} - 3e^x + 2 = 0$ (b) $e^{2x} - 4e^x - 5 = 0$

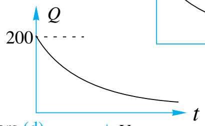
(c) $e^{2x} - 5e^x + 6 = 0$ (d) $e^{2x} - 2e^x + 1 = 0$

(e) $e^{2x} - 6e^x + 5 = 0$ (f) $e^{2x} - 9e^x - 10 = 0$

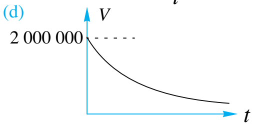
6. (a) 15 000°C (b) i. 11 900°C ii. 1500°C (c) 3.01 million yrs (d)



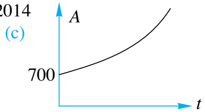
7. (a) 0.0151 (b) 12.50gm (c) 20 years (d)



8. (a) \$2 million (b) \$1.589 mil (c) 30.1 years (d)

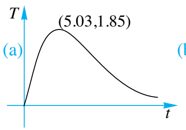


9. (b) 0.01761 (c) 199 230 (d) 22.6 years
10. (a) 20 cm^2 (b) 19.72 cm^2 (c) 100 days (d) 332 days
11. (a) 1 (b) i. 512170 ii. 517217 (c) 54.1 early 2014
12. (a) i. \$933.55 ii. \$935.50 (b) 11.95 years (c)



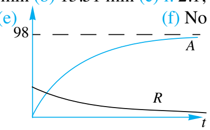
13. (a) 99 (b) $99 \times 2^{0.1394t}$ (c) 684

14. (a)

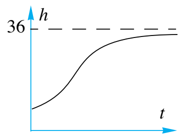


(b) 38.85°C at ~ midnight

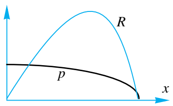
15. (a) 19 (b) 2.63 (c) 100
16. (a) 18 cm (b) 4 cm (c) 1.28 m (d) 36 m (e) i. 21.7yr ii. 27.6yr iii. 34.5yr (f) 36 (g)
17. (a) 5 mg/min (b) 13.51 min (c) i. 2.1, ii. 13.9 iii. 68min
(d) 19.6 mg (e)



(f) No



18. (a) i. \$499 ii. \$496 iii. \$467 (c) 15537 (d) i. \$499k ii. \$2.48mil iii. \$4.67mil (f) 12358
(g) \$5.14mil (b), (e)



EXERCISE 7.3

1. (a) 2 (b) 2 (c) 5 (d) 3 (e) –3 (f) –2 (g) 0 (h) 0 (i) –1 (j) –2 (k) 0.5 (l) –2 **2.** (a) $\log_{10}10000 = 4$ (b) $\log_{10}0.001 = -3$ (c) $\log_{10}(x+1) = y$ (d) $\log_{10}p = 7$ (e) $\log_2(x-1) = y$ (f) $\log_2(y-2) = 4x$ **3.** (a) $2^9 = x$ (b) $b^x = y$ (c) $b^{ax} = t$ (d) $10^{x^2} = z$ (e) $10^{1-x} = y$ (f) $2^y = ax - b$ **4.** (a) 16 (b) 2 (c) 2 (d) 9 (e) $\sqrt[4]{2}$ (f) 125 (g) 4 (h) 9 (i) $\sqrt[3]{\frac{1}{3}}$ (j) 21 (k) 3 (l) 13
5. (a) 54.5982 (b) 1.3863 (c) 1.6487 (d) 7.3891 (e) 1.6487 (f) 0.3679 (g) 52.5982 (h) 4.7183 (i) 0.6065

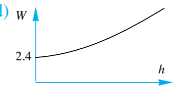
EXERCISE 7.4

1. (a) 5 (b) 2 (c) 2 (d) 1 (e) 2 (f) 1 **2.** (a) $\log a = \log b + \log c$ (b) $\log a = 2\log b + \log c$ (c) $\log a = -2\log c$ (d) $\log a = \log b + 0.5\log c$ (e) $\log a = 3\log b + 4\log c$ (f) $\log a = 2\log b - 0.5\log c$ **3.** (a) 0.18 (b) 0.045 (c) –0.09 **4.** (a) $x = yz$ (b) $y = x^2$ (c) $y = \frac{x+1}{x}$ (d) $x = 2^{y+1}$ (e) $y = \sqrt{x}$ (f) $y^2 = (x+1)^3$ **5.** (a) $\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{17}{15}$ (d) $\frac{3}{2}$ (e) $\frac{1}{3}$ (f) no real sol'n (g) 3,7 (h) $\frac{\sqrt{33}-1}{2}$ (i) 4 (j) $\sqrt{10}+3$ (k) $\frac{64}{63}$ (l) $\frac{2}{15}$ **6.** (a) $\log_3 2wx$ (b) $\log \frac{x}{47y}$ (c) $\log_a[x^2(x+1)^3]$ (d) $\log_a\left[\frac{(x^5)(x+1)^3}{\sqrt{2x-3}}\right]$ (e) $\log_{10}\left(\frac{y^2}{x}\right)$ (f) $\log_2\left(\frac{y}{x}\right)$ **7.** (a) 1 (b) –2 (c) 3 (d) 9 (e) 2 (f) 9 **8.** (a) 1,4 (b) $1, 3^{\pm\sqrt{3}}$ (c) $1, 4^{\sqrt[3]{4}}$ (d) $1, 5^{\pm\sqrt[4]{5}}$ **9.** (a) $\frac{\log 14}{\log 2} = 3.81$ (b) $\frac{\log 8}{\log 10} = 0.90$ (c) $\frac{\log 125}{\log 3} = 4.39$ (d) $\frac{1}{\log 2} \times \log\left(\frac{11}{3}\right) - 2 = -0.13$ (e) $\frac{\log 10 - \log 3}{4\log 3} = 0.27$ (f) 5.11 (g) $\frac{-\log 2}{2\log 10} = -0.15$ (h) 7.37 (i) 0.93 (j) no real solution (k) $\frac{\log 3}{\log 2} - 2 = -0.42$ (l) $\frac{\log 1.5}{\log 3} = 0.37$ **10.** (a) 0.5,4 (b) 3 (c) –1,4 (d) $10, 10^{10}$ (e) 5 (f) 3 **11.** (a) $(4, \log_4 11)$ (b) (100,10) (c) (2,1) **12.** (a) $y = xz$ (b) $y = x^3$ (c) $x = e^{y-1}$ **13.** (a) $\frac{1}{e^4-1}$ (b) $\frac{1}{3}$ (c) $\frac{\sqrt{5}-1}{2}$ (d) Ø **14.** (a) $\ln 21 = 3.0445$ (b) $\ln 10 = 2.3026$ (c) $-\ln 7 = -1.9459$ (d) $\ln 2 = 0.6931$ (e) $\ln 3 = 1.0986$ (f) $2\ln\left(\frac{14}{9}\right) = 0.8837$ (g) $e^3 = 20.0855$ (h) $\frac{1}{3}e^2 = 2.4630$ (i) $\pm\sqrt{e^9} = \pm 90.0171$ (j) Ø (k) $e^2 - 4 = 3.3891$ (l) $\sqrt[3]{e^9} = 20.0855$ **15.** (a) 0, $\ln 2$ (b) $\ln 5$ (c) $\ln 2$, $\ln 3$ (d) 0 (e) 0, $\ln 5$ (f) $\ln 10$ **16.** (a) 4.5222 (b) 0.2643 (c) 0,0.2619 (d) –1,0.3219 (e) –1.2925,0.6610 (f) 0,1.8928 (g) 0.25,2 (h) 1 (i) 121.5 (j) 2

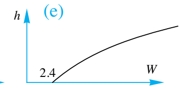
EXERCISE 7.5

1. (a) 10 (b) 30 (c) 40 **2.** (a) 31.64 kg (b) 1.65 (c) $W = 2.4 \times 10^{0.8h}$

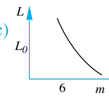
(d)



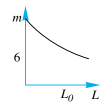
(e)



3. (a) 4.75 (b) $L = L_0 \times 10^{\left(\frac{6-m}{2.5}\right)}$ (c)



(d)



4. (a) [0,1[(b) i. 2.22 ii. 1.11 iii. 0.74 yrs (c) As c increases, reliability reduces.
(d) $x = 1 - 10^{-ct}$ (e)