Disguised quadratic equations

Introduction

Before you start make sure you are comfortable with solving quadratic equations using factorization, completing the square or quadratic formula and that you are able recognize when a quadratic equation has no solutions.

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This may look like a complicated equation, but in fact it can be easily reduced to a quadratic, which we can solve in few seconds.

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This can be easily solved using factorization:

$$\begin{aligned} (t-5)(t+2) &= 0 \\ t-5 = 0 \qquad &\text{or} \qquad t+2 = 0 \\ t = 5 \qquad &\text{or} \qquad t = -2 \end{aligned}$$

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$$\frac{1}{x+1} = 5 \quad \text{or} \quad \frac{1}{x+1} = -2$$
$$1 = 5x + 5 \quad \text{or} \quad 1 = -2x - 2$$
$$x = -\frac{4}{5} \quad \text{or} \quad x = -\frac{3}{2}$$

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$$1 = 5x + 5 \quad \text{or} \quad 1 = -2x - 2$$
$$x = -\frac{4}{5} \quad \text{or} \quad x = -\frac{3}{2}$$

And these are our final two solutions.

What we need to practice now is the ability to recognize when a seemingly complicated equation can be reduced to a quadratic by introducing a new variable.

Example 1

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$$x^6 - 10x^3 + 16 = 0$$

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Going back to x we get:

$$x^3 = 8 \quad \text{or} \quad x^3 = 2$$
$$x = 2 \quad \text{or} \quad x = \sqrt[3]{2}$$

And these are our solutions to the original equation.

Example 2

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$$(t-3)(t+2) = 0$$

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$$\begin{aligned} \sqrt{x} &= 3 \qquad \text{or} \qquad \sqrt{x} = -2 \\ x &= 9 \qquad \text{or} \qquad \text{no solution} \end{aligned}$$

So in the end we only have one solution $x = 9$.

Example 3

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$$2(x^2+1)^2-5(x^2+1)-3=0$$

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We factorize and get:

$$(2t+1)(t-3) = 0$$

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$$t = -\frac{1}{2} \qquad \text{or} \qquad t = 3$$

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$$\begin{aligned} x^2+1 &= -\frac{1}{2} \qquad \text{or} \qquad x^2+1=3 \\ x^2 &= -\frac{3}{2} \qquad \text{or} \qquad x^2=2 \\ \text{no real solutions} & \qquad\quad \text{or} \qquad x=\pm\sqrt{2} \end{aligned}$$

So we have two real solution $x=\sqrt{2}$ or $x=-\sqrt{2}$.

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$$\Delta = (-2)^2 - 4(1)(-4) = 20$$

Now we have:

$$t = \frac{-b \pm \sqrt{\Delta}}{2a}$$
$$t = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$$

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We have only one solution $x = 6+2\sqrt{5}$.

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Important point here. You may wonder why $\sqrt{x} = 1 - \sqrt{5}$ results in no solution. Why didn't we just square both side? It is because $1 - \sqrt{5}$ is a negative number (< 0). So there is no number whose square root is $1 - \sqrt{5}$.

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In the fist equation we're looking for a number whose square root is -3. Clearly there is no such number. Note that $(-3)^2 = 9$, but $\sqrt{9} \neq -3$, we have $\sqrt{9} = 3$.

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Now we go back to disguised quadratics. We will now try different examples.

Example 5

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$$t = 1 \qquad \text{or} \qquad t = 8$$

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$$2^x = 1 \quad \text{or} \quad 2^x = 8$$
$$x = 0 \quad \text{or} \quad x = 3$$

And these are our two solutions.

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$$t = \frac{1}{2} \qquad \text{or} \qquad t = 4$$

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$$2^x = \frac{1}{2} \qquad \text{or} \qquad 2^x = 4$$
$$x = -1 \qquad \text{or} \qquad x = 2$$

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We factorize and solve:

$$(3t+2)(t-2) = 0$$

$$3t + 2 = 0 \quad \text{or} \quad t - 2 = 0$$

$$t = -\frac{2}{3} \quad \text{or} \quad t = 2$$

Example 7

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$$3^x = -\frac{2}{3} \qquad \text{or} \qquad 3^x = 2$$
$$\text{no solution} \qquad \text{or} \qquad x = \log_3 2$$

So we only end up with one solution $x = \log_3 2$.

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We solve by factorization:

$$\begin{aligned} (t-5)(t+2) &= 0 \\ t-5 = 0 \qquad &\text{or} \qquad t+2 = 0 \\ t = 5 \qquad &\text{or} \qquad t = -2 \end{aligned}$$

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$$\begin{aligned} 3^x &= 5 \quad &\text{or}& \quad 3^x = -2 \\ x &= \log_3 5 \quad &\text{or}& \quad \text{no solution} \end{aligned}$$

We end up with one solution $x = \log_3 5$.

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We factorize and solve:

$$(3t - 1)(t + 4) = 0$$

$$3t - 1 = 0 \qquad \text{or} \qquad t + 4 = 0$$

$$t = \frac{1}{3} \qquad \text{or} \qquad t = -4$$

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$$3^x = \frac{1}{3} \qquad \text{or} \qquad 3^x = -4$$
$$x = -1 \qquad \text{or} \qquad \text{no solution}$$

We have one solution $x = -1$.

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$$(t - 3)(t + 5) = 0$$

$$t - 3 = 0 \quad \text{or} \quad t + 5 = 0$$

$$t = 3 \quad \text{or} \quad t = -5$$

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In the end we have two real solutions: $x = -3$ or $x = 1$.

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We factorize and get:

$$(2t-5)(t+2)=0$$

$$2t-5=0 \qquad \text{or} \qquad t+2=0$$

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Since we assumed that $x \neq 0$ we can multiply both sides of both equations $2x$ and x respectively and get:

$$2x^2 + 2 = 5x \qquad \text{or} \qquad x^2 + 1 = -2x$$
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We end up with three solution $x = \frac{1}{2}$ or $x = 2$ or $x = -1$.

The test will contain examples similar to the ones on the presentation.