8 For what values of a do the curves $y = ax^2 + 2x + 1$ and $y = -x^2 + ax - 1$:

a meet twice **b** touch **c** never meet?

REVIEW SET 14A

1 Use the vertex, axis of symmetry, and y-intercept to graph:

a $y = (x-2)^2 - 4$ **b** $y = -\frac{1}{2}(x+4)^2 + 6$

2 Find the points of intersection of $y = x^2 - 3x$ and $y = 3x^2 - 5x - 24$.

3 For what values of k does the graph of $y = -2x^2 + 5x + k$ *not* cut the x-axis?

4 Find the values of m for which $2x^2 - 3x + m = 0$ has:

a a repeated root **b** two distinct real roots **c** no real roots.

5 The sum of a number and its reciprocal is $2\frac{1}{30}$. Find the number.

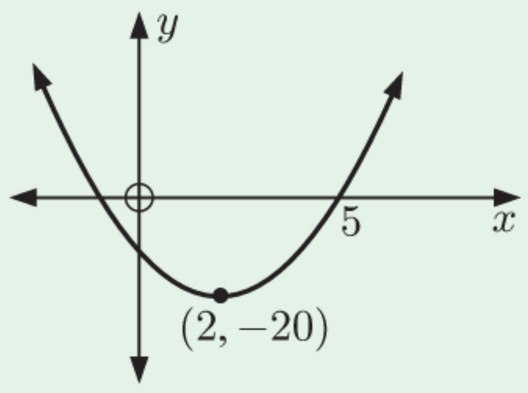
6 Show that no line with a y-intercept of 10 will ever be tangential to the curve with equation $y = 3x^2 + 7x - 2$.

7 **a** Write the quadratic $y = 2x^2 + 6x - 3$ in the form $y = a(x-h)^2 + k$.

b Hence sketch the graph of the quadratic.

8 Find the equation of the quadratic with graph:

a



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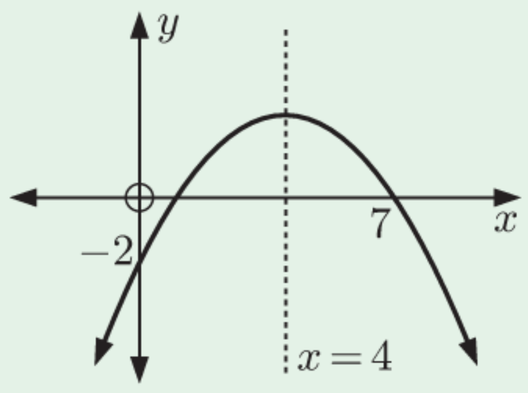
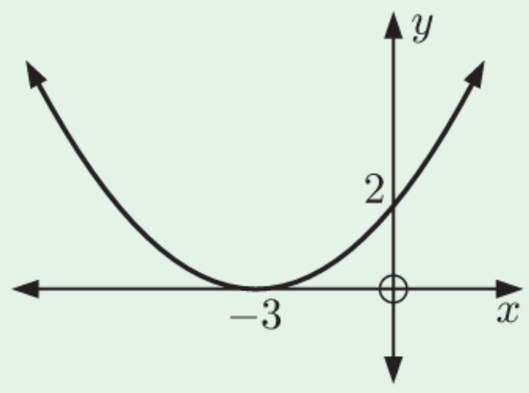


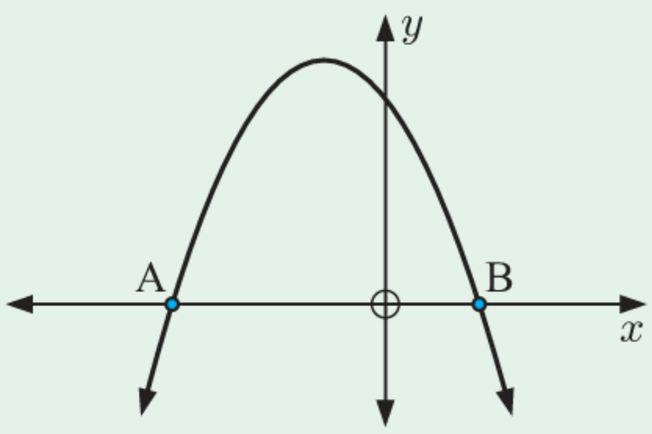
c



9 Draw the graph of $y = -x^2 + 2x$.

10 Find the y-intercept of the line with gradient -3 which is a tangent to the parabola $y = 2x^2 - 5x + 1$.

11 The graph shows the parabola $y = a(x+m)(x+n)$ where $m > n$.



a State the sign of:

i the discriminant Δ **ii** a.

b Find, in terms of m and n, the:

i coordinates of the x-intercepts A and B

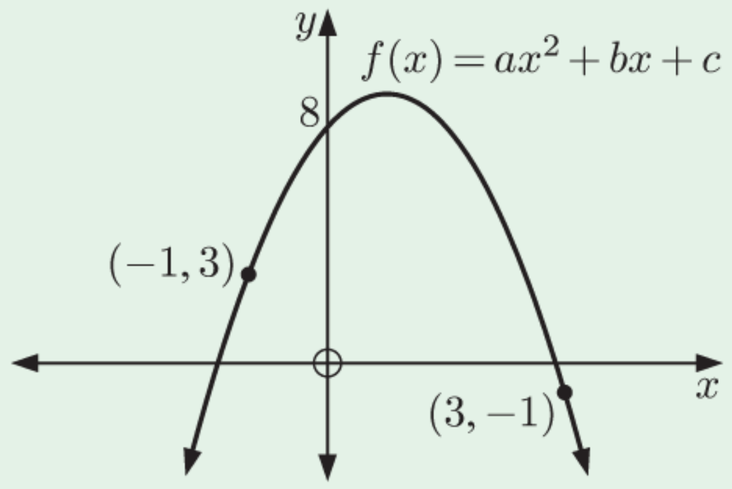
ii equation of the axis of symmetry.

12 For a quadratic function $y = ax^2 + bx + c$, suppose the constants a, b, and c are consecutive terms of a geometric sequence. Show that the function does not cut the x-axis.

13 Find the quadratic function which cuts the x-axis at 3 and -2 and which has y-intercept 24. Give your answer in the form $y = ax^2 + bx + c$.

14 Find the value of k for which the x-intercepts of $y = 3x^2 + 2kx + k - 1$ are closest together.

15 Consider the function $y = ax^2 + bx + c$ shown.



a State the value of c.

b Use the other information to write two equations involving a and b.

c Find a and b, and hence state the equation of the quadratic.

16 For what values of m are the lines $y = mx - 10$ tangents to the parabola $y = 3x^2 + 7x + 2$?

17 When Annie hits a softball, the height of the ball above the ground after t seconds is given by $h = -4.9t^2 + 19.6t + 1.4$ metres. Find the maximum height reached by the ball.

18 Draw a sign diagram for:

a $(3x + 2)(4 - x)$ **b** $-x^2 + 3x + 18$

19 Solve for x:

a $(3 - x)(x + 2) < 0$ **b** $x^2 - 4x - 5 \leqslant 0$ **c** $2x^2 + x > 10$

20 Find the values of k for which the function $f(x) = x^2 + kx + (3k - 4)$:

a cuts the x-axis twice **b** touches the x-axis **c** misses the x-axis.

REVIEW SET 14B

1 Consider the quadratic $y = \frac{1}{2}(x - 2)^2 - 4$.

a State the equation of the axis of symmetry.

b Find the coordinates of the vertex. **c** Find the y-intercept.

d Sketch the function.

2 Consider the quadratic $y = -3x^2 + 8x + 7$. Find the equation of the axis of symmetry, and the coordinates of the vertex.

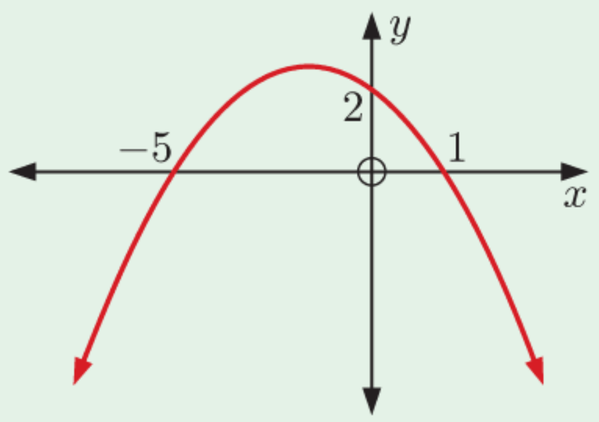
3 Use the discriminant only to find the relationship between the graph and the x-axis for:

a $y = 2x^2 + 3x - 7$ **b** $y = -3x^2 - 7x + 4$

4 Find the equation of the quadratic with vertex $(2, 25)$ and y-intercept 1.

5 **a** Find the equation of the quadratic illustrated.

b Hence find its vertex and axis of symmetry.



6 Consider the quadratic $y = 2x^2 + 4x - 1$.

a State the axis of symmetry.

b Find the coordinates of the vertex.

c Find the axes intercepts.

d Hence sketch the function.

7 Find, in the form $y = ax^2 + bx + c$, the quadratic function whose graph:

a touches the x-axis at 3 and passes through $(2, 2)$

b has x-intercepts 3 and -2, and y-intercept 3

c passes through $(-1, -9)$, $(1, 5)$, and $(2, 15)$

d has vertex $(3, 15)$ and passes through the point $(1, 7)$.

8 **a** For what values of c do the lines with equations $y = 3x + c$ intersect the parabola $y = x^2 + x - 5$ in two distinct points?

b Choose one such value of c and find the points of intersection in this case.

9 Find the maximum or minimum value of each quadratic, and the corresponding value of x:

a $y = 3x^2 + 4x + 7$

b $y = -2x^2 - 5x + 2$

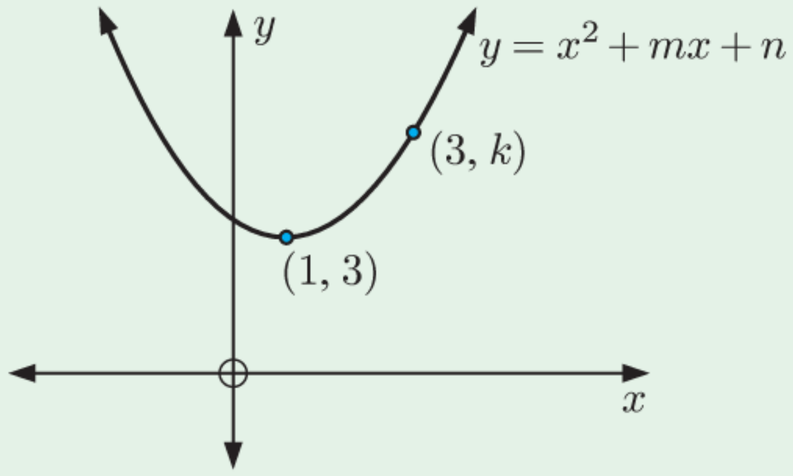
10 The graph of a quadratic function cuts the x-axis at -2 and 3, and passes through $(-3, 18)$.

a Find the equation of the function in the form $y = ax^2 + bx + c$.

b Write down the y-intercept of the function.

c Find the coordinates of the vertex.

11



Consider the graph of $y = x^2 + mx + n$.

a Determine the values of m and n.

b Hence find the value of k.

12 An open square-based box has capacity 120 mL. It is made from a square piece of tinplate with 4 cm squares cut from each of its corners. Find the dimensions of the original piece of tinplate.

13 Consider $y = -x^2 - 3x + 4$ and $y = x^2 + 5x + 4$.

a Solve for x: $-x^2 - 3x + 4 = x^2 + 5x + 4$.

b Sketch the curves on the same set of axes.

c Hence solve for x: $x^2 + 5x + 4 > -x^2 - 3x + 4$.

14 For each of the following quadratics:

i Write the quadratic in completed square form.

ii Write the quadratic in factored form.

iii Sketch the graph of the quadratic, identifying its axes intercepts, vertex, and axis of symmetry.

a $y = x^2 + 4x + 3$

b $y = x^2 + 2x - 3$

c $y = 2x^2 - 8x - 10$

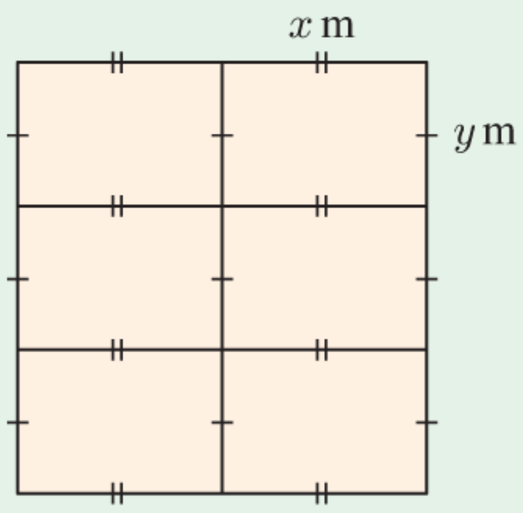
d $y = -x^2 + 6x + 7$

15 Two different quadratic functions of the form $y = 9x^2 - kx + 4$ both *touch* the x-axis.

a Find the two values of k.

b Find the point of intersection of the two quadratic functions.

16 600 m of fencing is used to construct 6 rectangular animal pens as shown.



a Show that the area A of each pen is

$$A = x\left(\frac{600 - 8x}{9}\right) \text{ m}^2.$$

b Find the dimensions of each pen so that it has the maximum possible area.

c What is the area of each pen in this case?

17 A retailer sells sunglasses for \$45, and has 50 customers per day. From market research, the retailer discovers that for every \$1.50 increase in the price of the sunglasses, he will lose a customer per day.

Let $\$x$ be the price increase of the sunglasses.

a Show that the revenue collected by the retailer each day is

$$R = (45 + x)\left(50 - \frac{x}{1.5}\right) \text{ dollars.}$$

b Find the price the retailer should set for his sunglasses in order to maximise his daily revenue. How much revenue is made per day at this price?

18 Draw a sign diagram for:

a $x^2 - 3x - 10$ **b** $-(x+3)^2$

19 Solve for x:

a $4x^2 - 3x < 0$ **b** $2x^2 - 3x - 5 \geqslant 0$ **c** $\frac{11}{3}x \leqslant 2x^2 + 1$

20 Find the values of m for which the function $y = mx^2 + 5x + (m + 12)$:

a cuts the x-axis twice **b** touches the x-axis **c** misses the x-axis.

14 **a** $y = -\frac{8}{9}x^2 + 8$

b No, as the tunnel is only 4.44 m high when it is the same width as the truck.

15 **a** $h = -5(t-2)^2 + 80$ **b** 75 m **c** 6 seconds

EXERCISE 14G

1 **a** min. -1, when $x = 1$ **b** max. 8, when $x = -1$

c max. $8\frac{1}{3}$, when $x = \frac{1}{3}$ **d** min. $-1\frac{1}{8}$, when $x = -\frac{1}{4}$

e min. $4\frac{15}{16}$, when $x = \frac{1}{8}$ **f** max. $6\frac{1}{8}$, when $x = \frac{7}{4}$

2 **a** 40 refrigerators **b** €4000

4 500 m by 250 m

5 **a** $41\frac{2}{3}$ m by $41\frac{2}{3}$ m **b** 50 m by $31\frac{1}{4}$ m

6 **b** $3\frac{1}{8}$ units **7** **a** $y = 6 - \frac{3}{4}x$ **b** 3 cm by 4 cm

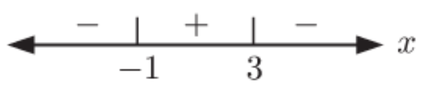
8 $m = \dfrac{\sum_{i=1}^{n} a_i b_i}{\sum_{i=1}^{n} a_i^{\,2}}$

9 $y = x^4 - 2(a^2+b^2)x^2 + (a^2-b^2)^2$

least value $= -4a^2b^2$

EXERCISE 14H.1

1 **a**



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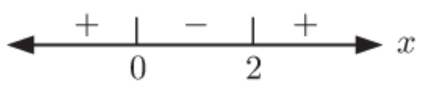

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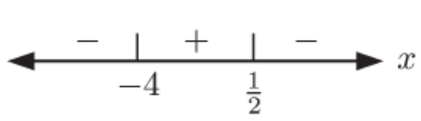

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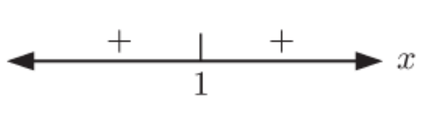

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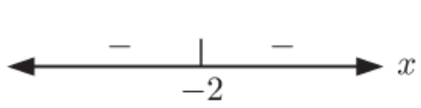

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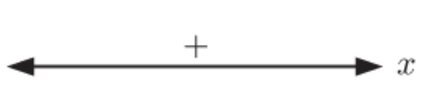
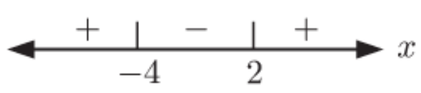


2 **a**



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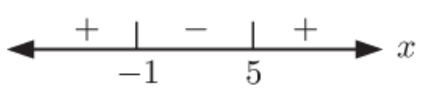

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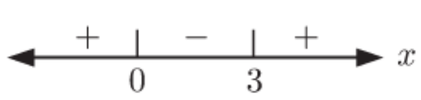

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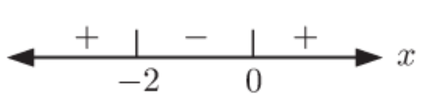

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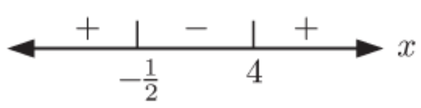

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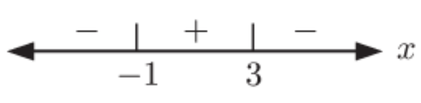

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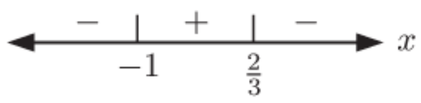

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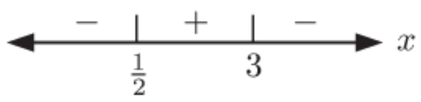

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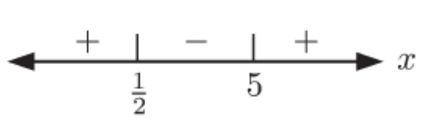

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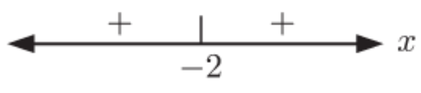

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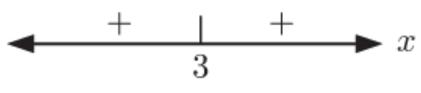

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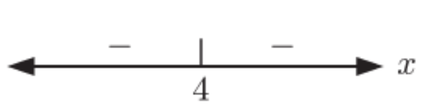

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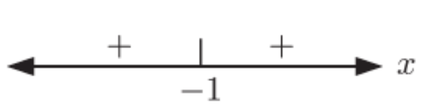

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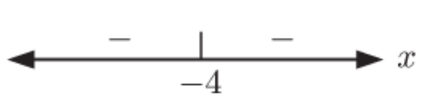

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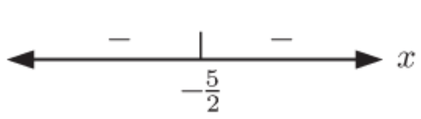
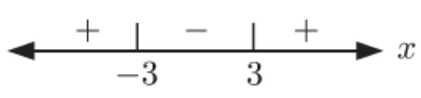


4 **a**



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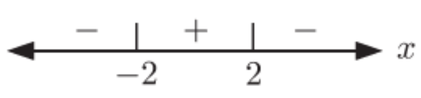

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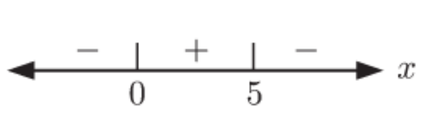

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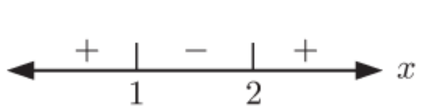

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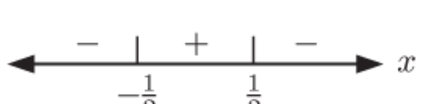

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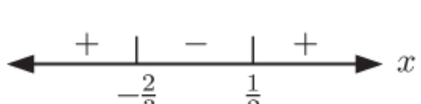

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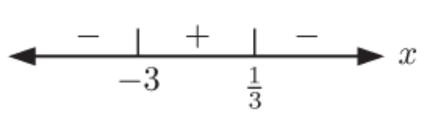

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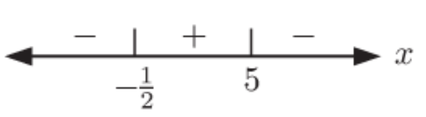

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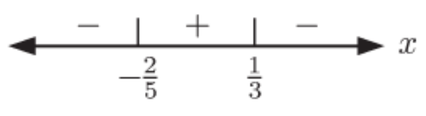
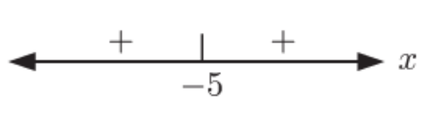


5 **a**



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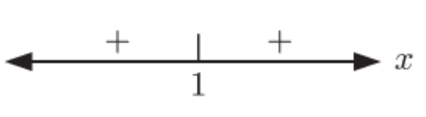

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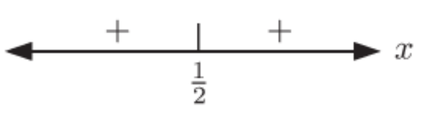
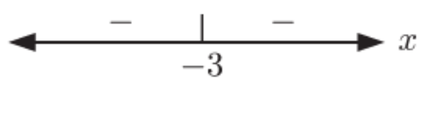
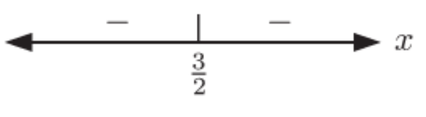


e



f



EXERCISE 14H.2

1 **a** $-5 \leqslant x \leqslant 2$ **b** $-3 \leqslant x \leqslant 2$ **c** no solutions

d all $x \in \mathbb{R}$ **e** $-\frac{1}{2} < x < 3$ **f** $-\frac{3}{2} < x < 4$

2 **a** $x \leqslant 0$ or $x \geqslant 1$ **b** $-\frac{2}{3} < x < 0$ **c** $x \neq -2$

d $-5 \leqslant x \leqslant 3$ **e** $x < -2$ or $x > 6$ **f** $-4 < x < 1$

3 **a** $x \leqslant 0$ or $x \geqslant 3$ **b** $-2 < x < 2$

c $x \leqslant -\sqrt{2}$ or $x \geqslant \sqrt{2}$ **d** $-3 \leqslant x \leqslant 7$

e $x < 5$ or $x > 6$ **f** $x < -6$ or $x > 7$

g $x \leqslant -1$ or $x \geqslant \frac{3}{2}$ **h** no solutions

i $-\frac{3}{2} < x < \frac{1}{3}$ **j** $x < -\frac{4}{3}$ or $x > 4$

k $x \neq 1$ **l** $\frac{1}{3} \leqslant x \leqslant \frac{1}{2}$ **m** $x < -\frac{1}{6}$ or $x > 1$

n $x \leqslant -\frac{1}{4}$ or $x \geqslant \frac{2}{3}$ **o** $x < \frac{3}{2}$ or $x > 3$

4 **a** **i** $k < -8$ or $k > 0$ **ii** $k = -8$ or 0

iii $-8 < k < 0$

b **i** $-1 < k < 1$, $k \neq 0$ **ii** $k = -1$ or 1

iii $k < -1$ or $k > 1$

c **i** $k < -6$ or $k > 2$ **ii** $k = -6$ or $k = 2$

iii $-6 < k < 2$

5 **a** **i** $k < -2$ or $k > 6$ **ii** $k = -2$ or $k = 6$

iii $-2 < k < 6$

b **i** $k < -\frac{13}{9}$ or $k > 3$ **ii** $k = -\frac{13}{9}$ or $k = 3$

iii $-\frac{13}{9} < k < 3$

c **i** $-\frac{4}{3} < k < 0$, $k \neq -1$ **ii** $k = -\frac{4}{3}$ or $k = 0$

iii $k < -\frac{4}{3}$ or $k > 0$

6 **a** $m > 3$ **b** $m < -1$

7 **a** $m < -1$ or $m > 7$ **b** $m = -1$ or $m = 7$

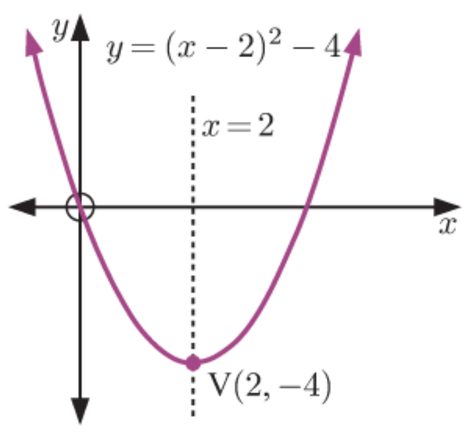
c $-1 < m < 7$

8 **a** $a < 6 - 2\sqrt{10}$ or $a > 6 + 2\sqrt{10}$ **b** $a = 6 \pm 2\sqrt{10}$

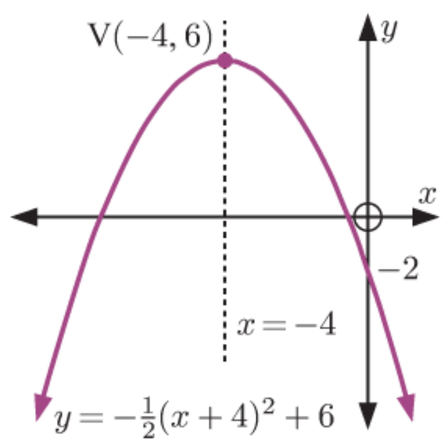
c $6 - 2\sqrt{10} < a < 6 + 2\sqrt{10}$

REVIEW SET 14A

1 **a**



b

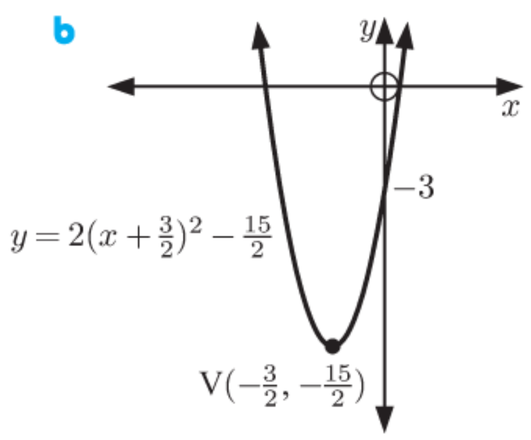


2 $(4, 4)$ and $(-3, 18)$ **3** $k < -3\frac{1}{8}$

4 **a** $m = \frac{9}{8}$ **b** $m < \frac{9}{8}$ **c** $m > \frac{9}{8}$ **5** $\frac{6}{5}$ or $\frac{5}{6}$

6 **Hint:** Let the line have equation $y = mx + 10$.

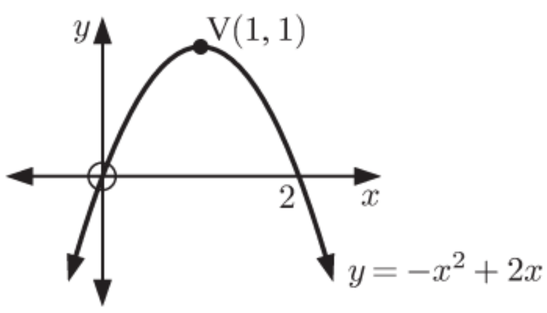
7 **a** $y = 2(x + \frac{3}{2})^2 - \frac{15}{2}$ **b**



8 **a** $y = \frac{20}{9}(x - 2)^2 - 20$ **b** $y = -\frac{2}{7}(x - 1)(x - 7)$

c $y = \frac{2}{9}(x + 3)^2$

9



10 $\frac{1}{2}$

11 **a** **i** $\Delta > 0$ **ii** $a < 0$

b **i** $A(-m, 0)$, $B(-n, 0)$ **ii** $x = \dfrac{-m - n}{2}$

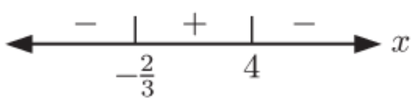
13 $y = -4x^2 + 4x + 24$ **14** $k = \frac{3}{2}$

15 **a** $c = 8$ **b** $3a + b = -3$, $a - b = -5$

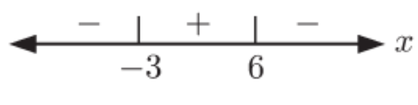
c $a = -2$, $b = 3$, $y = -2x^2 + 3x + 8$

16 $m = -5$ or 19 **17** 21 m

18 **a**



b



19 **a** $x < -2$ or $x > 3$ **b** $-1 \leqslant x \leqslant 5$

c $x < -\frac{5}{2}$ or $x > 2$

20 **a** $k < 6 - 2\sqrt{5}$ or $k > 6 + 2\sqrt{5}$ **b** $k = 6 \pm 2\sqrt{5}$

c $6 - 2\sqrt{5} < k < 6 + 2\sqrt{5}$

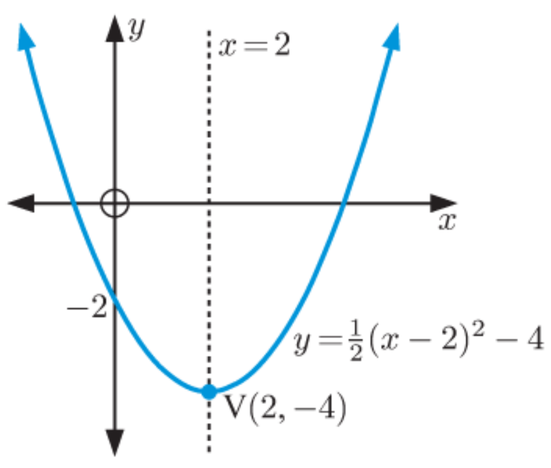
REVIEW SET 14B

1 **a** $x = 2$

b $(2, -4)$

c -2

d



2 $x = \frac{4}{3}$, $V(1\frac{1}{3}, 12\frac{1}{3})$

3 **a** $\Delta = 65$, the graph cuts the x-axis twice

b $\Delta = 97$, the graph cuts the x-axis twice

4 $y = -6(x - 2)^2 + 25$

5 **a** $y = -\frac{2}{5}(x + 5)(x - 1)$ **b** $(-2, 3\frac{3}{5})$, $x = -2$

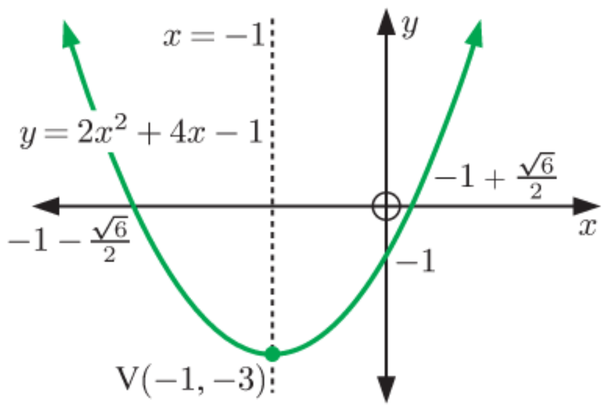
6 **a** $x = -1$

b $(-1, -3)$

c x-int. $-1 \pm \frac{\sqrt{6}}{2}$

y-intercept -1

d



7 **a** $y = 2x^2 - 12x + 18$ **b** $y = -\frac{1}{2}x^2 + \frac{1}{2}x + 3$

c $y = x^2 + 7x - 3$ **d** $y = -2x^2 + 12x - 3$

8 **a** $c > -6$

b For example, when $c = -2$, points of intersection are $(-1, -5)$ and $(3, 7)$.

9 **a** minimum is $5\frac{2}{3}$ when $x = -\frac{2}{3}$

b maximum is $5\frac{1}{8}$ when $x = -\frac{5}{4}$

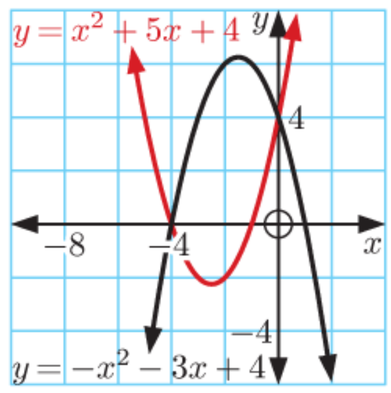
10 **a** $y = 3x^2 - 3x - 18$ **b** -18 **c** $(\frac{1}{2}, -18\frac{3}{4})$

11 **a** $m = -2$, $n = 4$ **b** $k = 7$ **12** ≈ 13.5 cm square

13 **a** $x = -4$ or 0

c $x < -4$ or $x > 0$

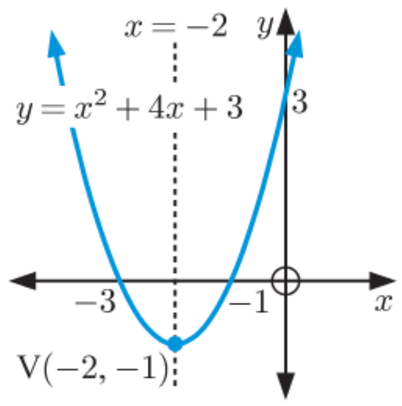
b



14 **a** **i** $y = (x + 2)^2 - 1$

ii $y = (x + 3)(x + 1)$

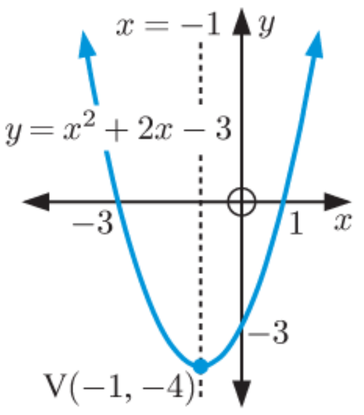
iii



b **i** $y = (x + 1)^2 - 4$

ii $y = (x + 3)(x - 1)$

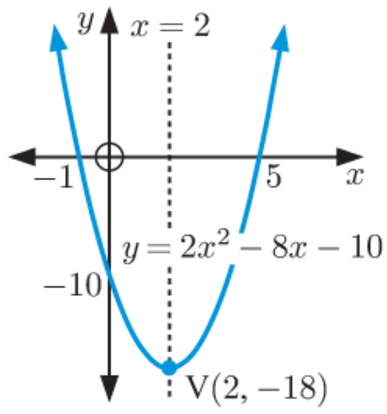
iii



c **i** $y = 2(x - 2)^2 - 18$

ii $y = 2(x - 5)(x + 1)$

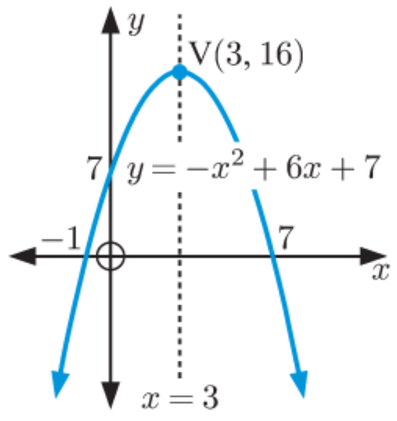
iii



d **i** $y = -(x - 3)^2 + 16$

ii $y = -(x - 7)(x + 1)$

iii

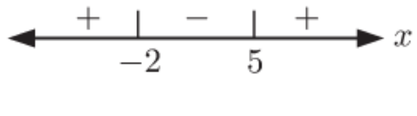


15 **a** $k = \pm 12$ **b** $(0, 4)$

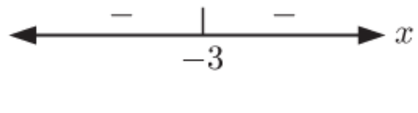
16 **b** $37\frac{1}{2}$ m by $33\frac{1}{3}$ m **c** 1250 m^2

17 **b** \$60, revenue is \$2400 per day

18 **a**



b



19 **a** $0 < x < \frac{3}{4}$ **b** $x \leqslant -1$ or $x \geqslant \frac{5}{2}$

c $x \leqslant \frac{1}{3}$ or $x \geqslant \frac{3}{2}$