2. [Maximum mark: 5]

The function f is defined as $f(x) = \frac{3x + 2}{x + 1}$, $x \in \mathbb{R}$, $x \neq -1$.

Sketch the graph of $y = f(x)$, clearly indicating and stating the equations of any asymptotes and the coordinates of any axes intercepts.

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

The function f is defined by $f(x) = \frac{3x}{x-2}$, $x \in \mathbb{R}$, $x \neq 2$.

(a) Sketch the graph of $y = f(x)$, indicating clearly any asymptotes and points of intersection with the x and y axes. [4]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Find all values of x for which $f(x) = f^{-1}(x)$. [3]

(d) Solve the inequality $|f(x)| < \frac{3}{2}$. [4]

(e) Solve the inequality $f(|x|) < \frac{3}{2}$. [2]

11. [Maximum mark: 16]

Consider the functions $f(x) = \tan x$, $0 \leq x < \frac{\pi}{2}$ and $g(x) = \frac{x+1}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$.

(a) Find an expression for $g \circ f(x)$, stating its domain. [2]

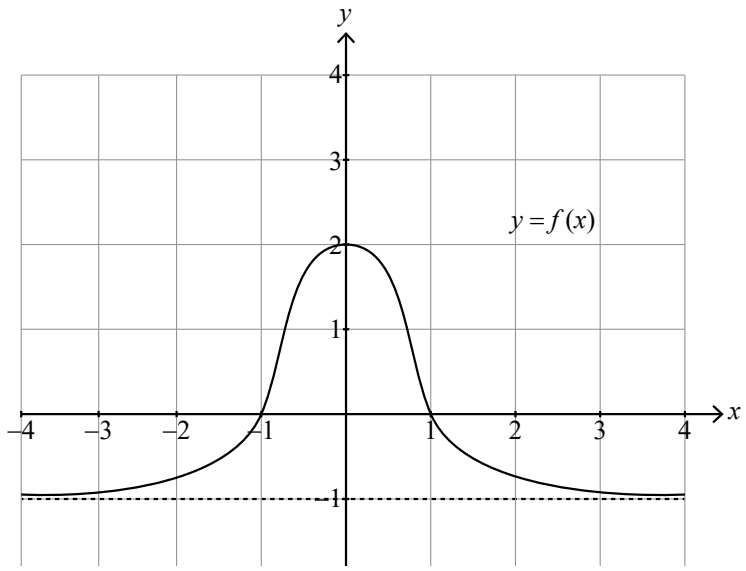
(b) Hence show that $g \circ f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$. [2]

(c) Let $y = g \circ f(x)$, find an exact value for $\frac{dy}{dx}$ at the point on the graph of $y = g \circ f(x)$ where $x = \frac{\pi}{6}$, expressing your answer in the form $a + b\sqrt{3}$, $a, b \in \mathbb{Z}$. [6]

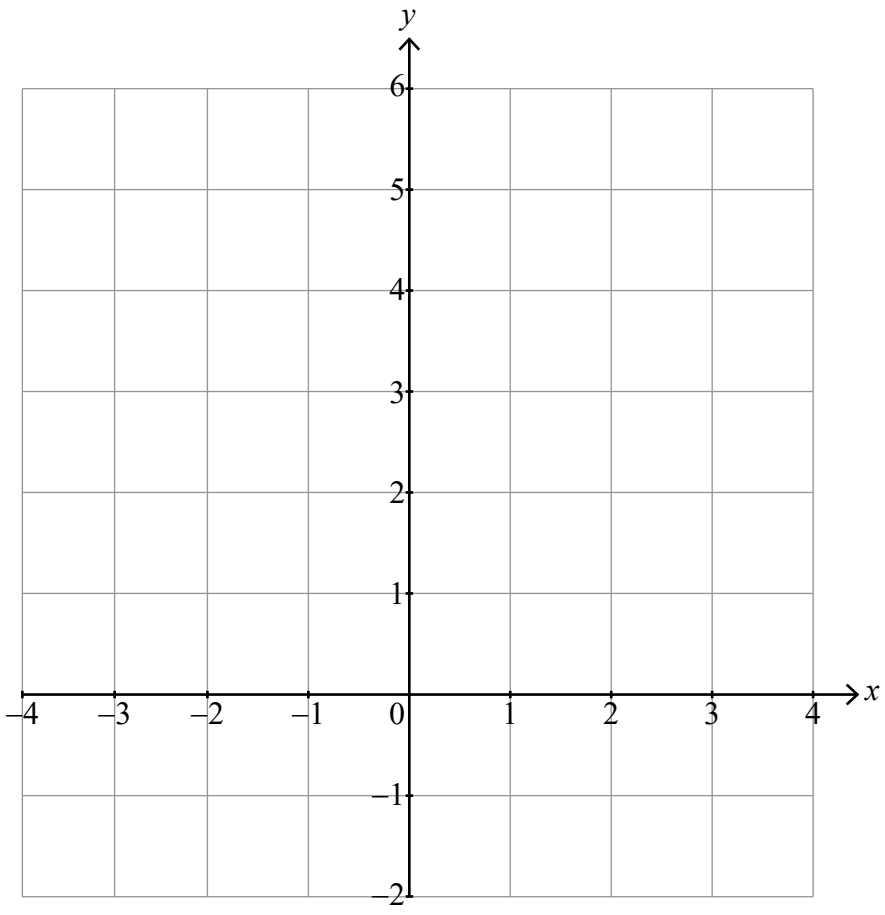
(d) Show that the area bounded by the graph of $y = g \circ f(x)$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{6}$ is $\ln\left(1 + \sqrt{3}\right)$. [6]

4. [Maximum mark: 5]

The following diagram shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = -1$. The graph crosses the x-axis at $x = -1$ and $x = 1$, and the y-axis at $y = 2$.



On the following set of axes, sketch the graph of $y = [f(x)]^2 + 1$, clearly showing any asymptotes with their equations and the coordinates of any local maxima or minima.



5. [Maximum mark: 6]

The functions f and g are defined by $f(x) = ax^2 + bx + c$, $x \in \mathbb{R}$ and $g(x) = p\sin x + qx + r$, $x \in \mathbb{R}$ where a, b, c, p, q, r are real constants.

(a) Given that f is an even function, show that $b = 0$. [2]

(b) Given that g is an odd function, find the value of r. [2]

The function h is both odd and even, with domain $\mathbb{R}$.

(c) Find $h(x)$. [2]

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7. [Maximum mark: 8]

(a) Sketch on the same axes the curve $y = \left| \frac{7}{x-4} \right|$ and the line $y = x + 2$, clearly indicating any axes intercepts and any asymptotes. [3]

(b) Find the exact solutions to the equation $x + 2 = \left| \frac{7}{x-4} \right|$. [5]

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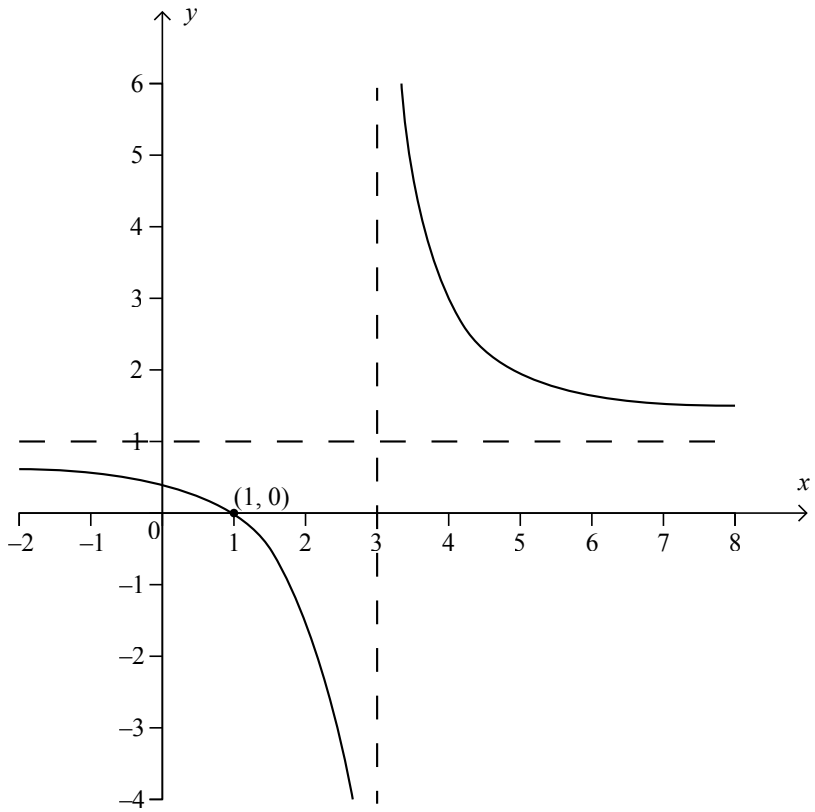
3. [Maximum mark: 4]

A rational function is defined by $f(x) = a + \frac{b}{x - c}$ where the parameters $a, b, c \in \mathbb{Z}$ and $x \in \mathbb{R} \setminus \{c\}$. The following diagram represents the graph of $y = f(x)$.



Using the information on the graph,

(a) state the value of a and the value of c; [2]

(b) find the value of b. [2]

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6. [Maximum mark: 5]

Consider the graphs of $y = |x|$ and $y = -|x| + b$, where $b \in \mathbb{Z}^+$.

(a) Sketch the graphs on the same set of axes. [2]

(b) Given that the graphs enclose a region of area 18 square units, find the value of b. [3]

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9. [Maximum mark: 17]

Consider the function f defined by $f(x) = x^2 - a^2, x \in \mathbb{R}$ where a is a positive constant.

(a) Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes.

(i) $y = f(x)$;

(ii) $y = \frac{1}{f(x)}$;

(iii) $y = \left|\frac{1}{f(x)}\right|$. [8]

11. [Maximum mark: 17]

(a) (i) Express $x^2 + 3x + 2$ in the form $(x + h)^2 + k$.

(ii) Factorize $x^2 + 3x + 2$. [2]

Consider the function $f(x) = \frac{1}{x^2 + 3x + 2}, x \in \mathbb{R}, x \neq -2, x \neq -1$.

(b) Sketch the graph of $f(x)$, indicating on it the equations of the asymptotes, the coordinates of the y-intercept and the local maximum. [5]

(e) Sketch the graph of $y = f(|x|)$. [2]