

1.

**METHOD 1**

$$5(2a + 9d) = 60 \quad (\text{or } 2a + 9d = 12) \quad \text{M1A1}$$

$$10(2a + 19d) = 320 \quad (\text{or } 2a + 19d = 32) \quad \text{A1}$$

solve simultaneously to obtain M1

$$a = -3, d = 2 \quad \text{A1}$$

$$\text{the 15}^{\text{th}} \text{ term is } -3 + 14 \times 2 = 25 \quad \text{A1}$$

**Note:** FT the final A1 on the values found in the penultimate line.

**METHOD 2**

with an AP the mean of an even number of consecutive terms equals  
the mean of the middle terms (M1)

$$\frac{a_{10} + a_{11}}{2} = 16 \quad (\text{or } a_{10} + a_{11} = 32) \quad \text{A1}$$

$$\frac{a_5 + a_6}{2} = 6 \quad (\text{or } a_5 + a_6 = 12) \quad \text{A1}$$

$$a_{10} - a_5 + a_{11} - a_6 = 20 \quad \text{M1}$$

$$5d + 5d = 20$$

$$d = 2 \text{ and } a = -3 \quad (\text{or } a_5 = 5 \text{ or } a_{10} = 15) \quad \text{A1}$$

$$\text{the 15}^{\text{th}} \text{ term is } -3 + 14 \times 2 = 25 \quad (\text{or } 5 + 10 \times 2 = 25 \text{ or } 15 + 5 \times 2 = 25) \quad \text{A1}$$

**Note:** FT the final A1 on the values found in the penultimate line.

[6]

2.

$$(a) \quad (i) \quad (g \circ f)(x) = \frac{1}{2x+3}, x \neq -\frac{3}{2} \quad (\text{or equivalent}) \quad \text{A1}$$

$$(ii) \quad (f \circ g)(x) = \frac{2}{x} + 3, x \neq 0 \quad (\text{or equivalent}) \quad \text{A1}$$

(b) **EITHER**

$$f(x) = (g^{-1} \circ f \circ g)(x) \Rightarrow (g \circ f)(x) = (f \circ g)(x) \quad \text{(M1)}$$

$$\frac{1}{2x+3} = \frac{2}{x} + 3 \quad \text{A1}$$

**OR**

$$(g^{-1} \circ f \circ g)(x) = \frac{1}{\frac{2}{x} + 3} \quad \text{A1}$$

$$2x + 3 = \frac{1}{\frac{2}{x} + 3} \quad \text{M1}$$

**THEN**

$$6x^2 + 12x + 6 = 0 \quad (\text{or equivalent}) \quad \text{A1}$$

$$x = -1, y = 1 \quad (\text{coordinates are } (-1, 1)) \quad \text{A1}$$

[6]

3.

(a) **METHOD 1**

$$f'(x) = q - 2x = 0 \quad \text{M1}$$

$$f'(3) = q - 6 = 0 \quad \text{A1}$$

$$q = 6 \quad \text{M1}$$

$$f(3) = p + 18 - 9 = 5 \quad \text{A1}$$

$$p = -4 \quad \text{A1}$$

**METHOD 2**

$$f(x) = -(x-3)^2 + 5 \quad \text{M1A1}$$

$$= -x^2 + 6x - 4$$

$$q = 6, p = -4 \quad \text{A1A1}$$

(b)  $g(x) = -4 + 6(x-3) - (x-3)^2 (= -31 + 12x - x^2)$  M1A1

**Note:** Accept any alternative form that is correct.  
Award M1A0 for a substitution of  $(x+3)$ .

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Method 1 uses calculus (we haven't covered it yet).

4.

$$g(x) = 0 \text{ or } 3 \quad \text{(M1)(A1)}$$

$$x = -1 \text{ or } 4 \text{ or } 1 \text{ or } 2 \quad \text{A1A1}$$

**Notes:** Award A1A1 for all four correct values,  
A1A0 for two or three correct values,  
A0A0 for less than two correct values.

Award M1 and corresponding A marks for correct attempt to find expressions for  $f$  and  $g$ .

[4]

5.

$$2^{2x-2} = 2^x + 8 \quad \text{(M1)}$$

$$\frac{1}{4} 2^{2x} = 2^x + 8 \quad \text{(A1)}$$

$$2^{2x} - 4 \times 2^x - 32 = 0 \quad \text{A1}$$

$$(2^x - 8)(2^x + 4) = 0 \quad \text{(M1)}$$

$$2^x = 8 \Rightarrow x = 3 \quad \text{A1}$$

**Notes:** Do not award final A1 if more than 1 solution is given.

[5]

6.

- (a) attempt at completing the square (M1)  
 $3x^2 - 6x + 5 = 3(x^2 - 2x) + 5 = 3(x - 1)^2 - 1 + 5$  (A1)  
 $= 3(x - 1)^2 + 2$  A1  
( $a = 3, b = -1, c = 2$ )
- (b) definition of suitable basic transformations:  
 $T_1 =$  stretch in  $y$  direction scale factor 3 A1  
 $T_2 =$  translation  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  A1  
 $T_3 =$  translation  $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$  A1

[6]

7.

$$g(x) = 0$$
$$\log_5 |2 \log_3 x| = 0 \quad (\text{M1})$$
$$|2 \log_3 x| = 1 \quad \text{A1}$$
$$\log_3 x = \pm \frac{1}{2} \quad (\text{A1})$$
$$x = 3^{\pm \frac{1}{2}} \quad \text{A1}$$

so the product of the zeros of  $g$  is  $3^{\frac{1}{2}} \times 3^{-\frac{1}{2}} = 1$  A1 N0

[5]

8.

- (a) **EITHER**  
graph of the cubic is shifted horizontally one unit to the right (M1)  
 $\Rightarrow x = -0.796$  A1
- OR**  
 $(x - 1) = -1.796$  (M1)  
 $x = -0.796$  A1
- (b) **EITHER**  
stretch factor of 0.5 in the  $x$ -direction (M1)  
 $\Rightarrow 2x = -1.796$  (M1)

9.

$$(a) \quad h(x) = g\left(\frac{4}{x+2}\right) \quad (\text{M1})$$

$$= \frac{4}{x+2} - 1 \quad \left( = \frac{2-x}{2+x} \right) \quad \text{A1}$$

(b) **METHOD 1**

$$x = \frac{4}{y+2} - 1 \quad (\text{interchanging } x \text{ and } y) \quad \text{M1}$$

Attempting to solve for  $y$  M1

$$(y+2)(x+1) = 4 \quad \left( y+2 = \frac{4}{x+1} \right) \quad (\text{A1})$$

$$h^{-1}(x) = \frac{4}{x+1} - 2 \quad (x \neq -1) \quad \text{A1} \quad \text{N1}$$

**METHOD 2**

$$x = \frac{2-y}{2+y} \quad (\text{interchanging } x \text{ and } y) \quad \text{M1}$$

Attempting to solve for  $y$  M1

$$xy + y = 2 - 2x \quad (y(x+1) = 2(1-x)) \quad (\text{A1})$$

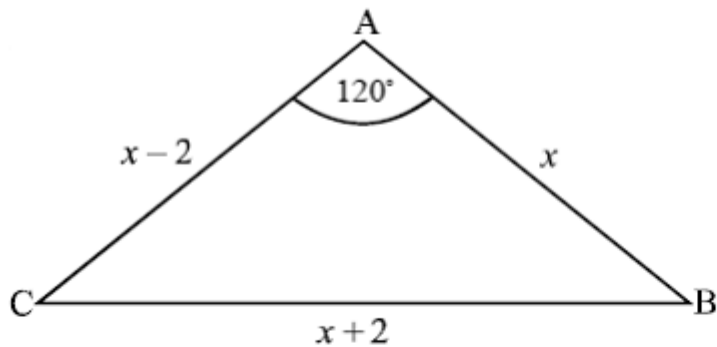
$$h^{-1}(x) = \frac{2(1-x)}{x+1} \quad (x \neq -1) \quad \text{A1} \quad \text{N1}$$

**Note:** In either **METHOD 1** or **METHOD 2** rearranging first and interchanging afterwards is equally acceptable.

[6]

10.

(a)



(M1)

$$(x+2)^2 = (x-2)^2 + x^2 - 2(x-2)x \cos 120^\circ$$

M1A1

$$x^2 + 4x + 4 = x^2 - 4x + 4 + x^2 + x^2 - 2x$$

(M1)

$$0 = 2x^2 - 10x$$

A1

$$0 = x(x-5)$$

$$x = 5$$

A1

(b)  $\text{Area} = \frac{1}{2} \times 5 \times 3 \times \sin 120^\circ$

M1A1

$$= \frac{1}{2} \times 15 \times \frac{\sqrt{3}}{2}$$

A1

$$= \frac{15\sqrt{3}}{4}$$

AG

(c)  $\sin A = \frac{\sqrt{3}}{2}$

$$\frac{15\sqrt{3}}{4} = \frac{1}{2} \times 5 \times 7 \times \sin B \Rightarrow \sin B = \frac{3\sqrt{3}}{14}$$

M1A1

Similarly  $\sin C = \frac{5\sqrt{3}}{14}$

A1

$$\sin A + \sin B + \sin C = \frac{15\sqrt{3}}{14}$$

A1

[13]

11.

(a)  $A^2 = \begin{pmatrix} 2a & -2 \\ -a & 2a+1 \end{pmatrix}$

(M1)A1

(b) **METHOD 1**

$$\det A^2 = 4a^2 + 2a - 2a = 4a^2$$

M1

$$a = \pm 2$$

A1A1 N2

**METHOD 2**

$$\det A = -2a$$

M1

$$\det A = \pm 4$$

$$a = \pm 2$$

A1A1 N2

[5]

12.

(a) Use of  $\bar{x} = \frac{\sum_{i=1}^4 x_i}{n}$  (M1)

$$\bar{x} = \frac{(k-2) + k + (k+1) + (k+4)}{4} \quad (\text{A1})$$

$$\bar{x} = \frac{4k+3}{4} \quad \left( = k + \frac{3}{4} \right) \quad \text{A1 N3}$$

(b) Either attempting to find the new mean or subtracting 3 from their  $\bar{x}$  (M1)

$$\bar{x} = \frac{4k+3}{4} - 3 \quad \left( = \frac{4k-9}{4}, k - \frac{9}{4} \right) \quad \text{A1 N2}$$

[5]

13.

**EITHER**

Using  $P(A | B) = \frac{P(A \cap B)}{P(B)}$  (M1)

$$0.6P(B) = P(A \cap B) \quad \text{A1}$$

Using  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  to obtain

$$0.8 = 0.6 + P(B) - P(A \cap B) \quad \text{A1}$$

Substituting  $0.6P(B) = P(A \cap B)$  into above equation (M1)

**OR**

As  $P(A | B) = P(A)$  then  $A$  and  $B$  are independent events (M1R1)

$$\text{Using } P(A \cup B) = P(A) + P(B) - P(A) \times P(B) \quad \text{A1}$$

$$\text{to obtain } 0.8 = 0.6 + P(B) - 0.6 \times P(B) \quad \text{A1}$$

**THEN**

$$0.8 = 0.6 + 0.4P(B) \quad \text{A1}$$

$$P(B) = 0.5 \quad \text{A1 N1}$$

[6]