1. The mean of the first ten terms of an arithmetic sequence is 6. The mean of the first twenty terms of the arithmetic sequence is 16. Find the value of the 15th term of the sequence.

(Total 6 marks)

2. Consider the functions given below.

$$f(x) = 2x + 3$$
$$g(x) = \frac{1}{x}, \; x \neq 0$$

   (a) (i) Find $(g \circ f)(x)$ and write down the domain of the function.

   (ii) Find $(f \circ g)(x)$ and write down the domain of the function.

   (2)

   (b) Find the coordinates of the point where the graph of $y = f(x)$ and the graph of $y = (g^{-1} \circ f \circ g)(x)$ intersect.
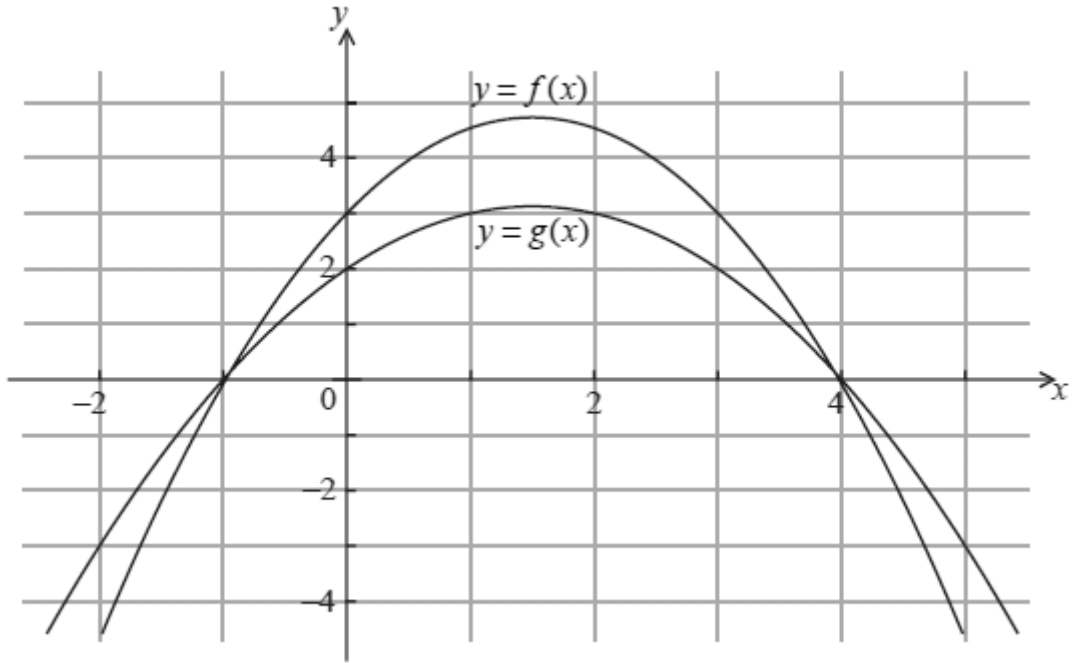
   (4)
   (Total 6 marks)

3. The quadratic function $f(x) = p + qx - x^2$ has a maximum value of 5 when $x = 3$.

   (a) Find the value of p and the value of q.

   (4)

   (b) The graph of $f(x)$ is translated 3 units in the positive direction parallel to the x-axis. Determine the equation of the new graph.

   (2)
   (Total 6 marks)

4. Shown below are the graphs of $y = f(x)$ and $y = g(x)$.

If $(f \circ g)(x) = 3$, find all possible values of x.

(Total 4 marks)

5. Solve the equation $4^{x-1} = 2^x + 8$.

(Total 5 marks)

6. (a) Express the quadratic $3x^2 - 6x + 5$ in the form $a(x + b)^2 + c$, where $a, b, c \in \mathbb{Z}$.

(3)

(b) Describe a sequence of transformations that transforms the graph of $y = x^2$ to the graph of $y = 3x^2 - 6x + 5$.

(3)
(Total 6 marks)

7. Let $g(x) = \log_5 \left| 2\log_3 x \right|$. Find the product of the zeros of g.

(Total 5 marks)

8. The real root of the equation $x^3 - x + 4 = 0$ is -1.796 to three decimal places. Determine the real root for each of the following.

(a) $(x-1)^3 - (x-1) + 4 = 0$

(2)

(b) $8x^3 - 2x + 4 = 0$

(3)
(Total 5 marks)

9. Let $f(x) = \dfrac{4}{x+2}, x \neq -2$ and $g(x) = x - 1$.

If $h = g \circ f$, find

(a) $h(x)$;

(2)

(b) $h^{-1}(x)$, where h^{-1} is the inverse of h.

(4)
(Total 6 marks)

10. The lengths of the sides of a triangle ABC are $x - 2$, x and $x + 2$. The largest angle is $120°$.

(a) Find the value of x.

(6)

(b) Show that the area of the triangle is $\dfrac{15\sqrt{3}}{4}$.

(3)

(c) Find $\sin A + \sin B + \sin C$ giving your answer in the form $\dfrac{p\sqrt{q}}{r}$ where $p, q, r \in \mathbb{Z}$.

(4)
(Total 13 marks)

11. Consider the matrix $A = \begin{pmatrix} 0 & 2 \\ a & -1 \end{pmatrix}$.

    (a)   Find the matrix A^2.

                               (2)

    (b)   If det $A^2 = 16$, determine the possible values of *a*.

                               (3)
                      (Total 5 marks)

12. Consider the data set $\{k - 2, k, k + 1, k + 4\}$, where $k \in \mathbb{R}$.

    (a)   Find the mean of this data set in terms of *k*.

                               (3)

Each number in the above data set is now **decreased** by 3.

    (b)   Find the mean of this **new** data set in terms of *k*.

                               (2)
                      (Total 5 marks)

13. Let *A* and *B* be events such that $P(A) = 0.6$, $P(A \cup B) = 0.8$ and $P(A \mid B) = 0.6$.

Find $P(B)$.

                               (Total 6 marks)