Name:

Mathematics IB HL diagnostics test

September 9, 2021

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.

- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.

- The maximum mark for this examination paper is [**72 marks**].

- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to **show all working**.

- Write your solutions in the space provided. <u>Underline</u> your answer to each part of the question.

1. [Maximum mark: 6]

In a group of 12 students, 4 can speak French, 2 can speak both French and German, 7 students **cannot** speak German.

(a) Find the number of students who can speak German but not French. [2]

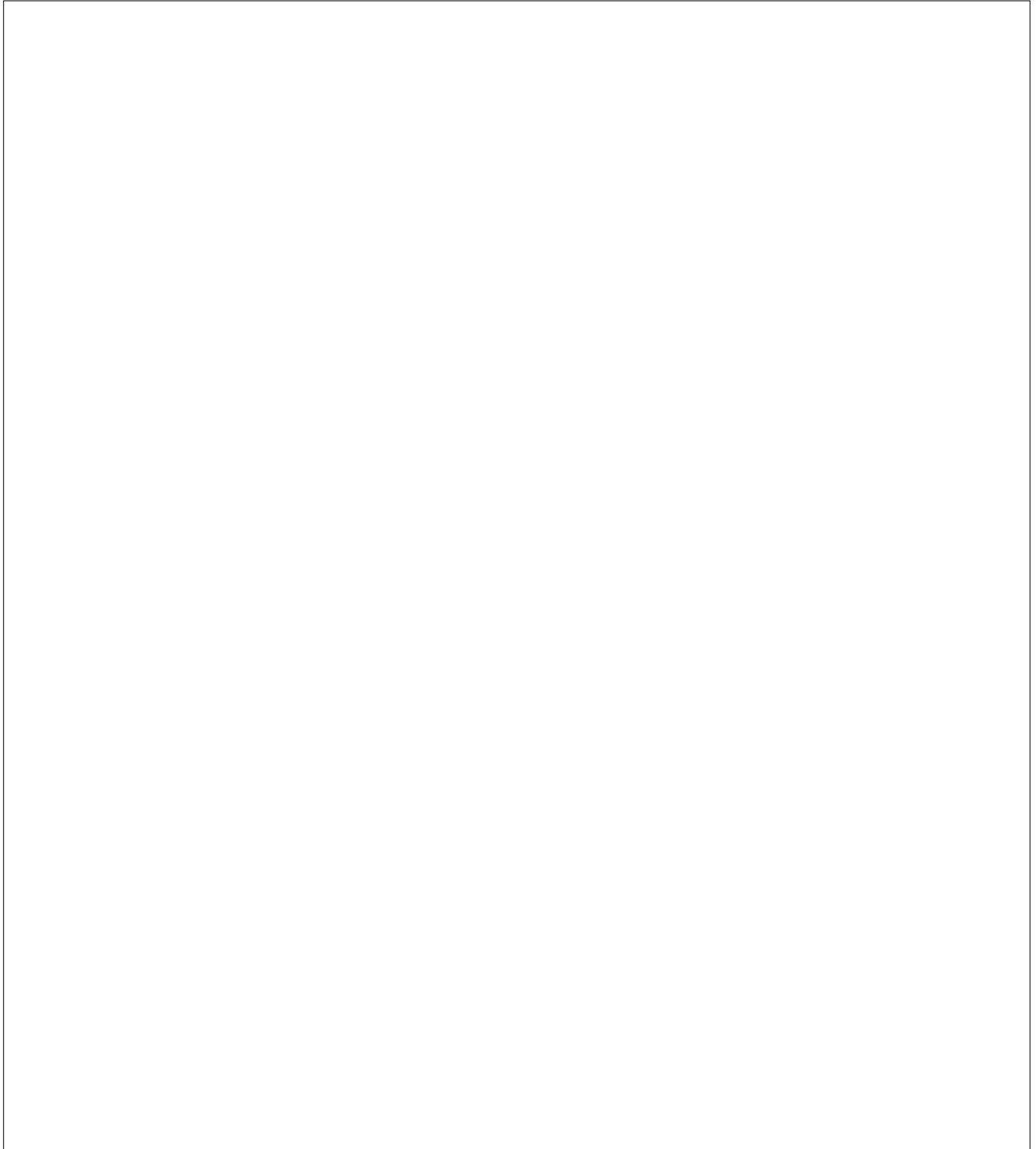Three students are selected at random.

(c) Find the probability that at least one of these students can speak both French and German. Write your answer as an irreducible fraction (in lowest terms). [4]

2. [Maximum mark: 7]

A box contain 3 blue and a certain number of green balls. Two balls are picked at random. The probability that both are blue is $\frac{1}{12}$.

(a) Show that there are 6 green balls in the box. [4]

(b) Two balls of the same colour were picked. Find the probability that they were both blue. [3]

3. [Maximum mark: 6]

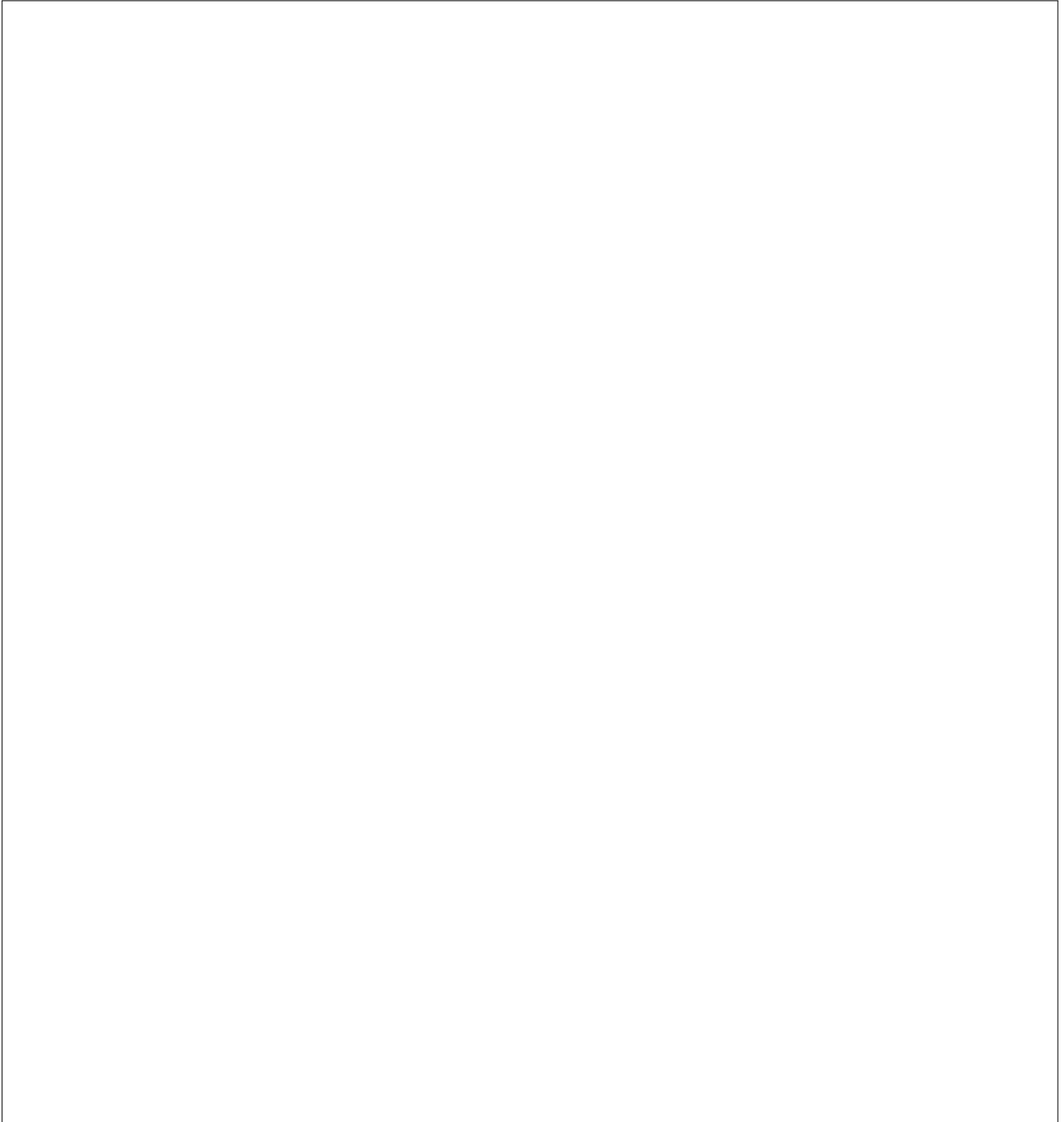Line l passes points $(2, -1)$ and $(6, 1)$

(a) Find the gradient of l. [2]

Line m is perpendicular to l and passes through $(2, 3)$, $(x, 1)$ and $(4, y)$
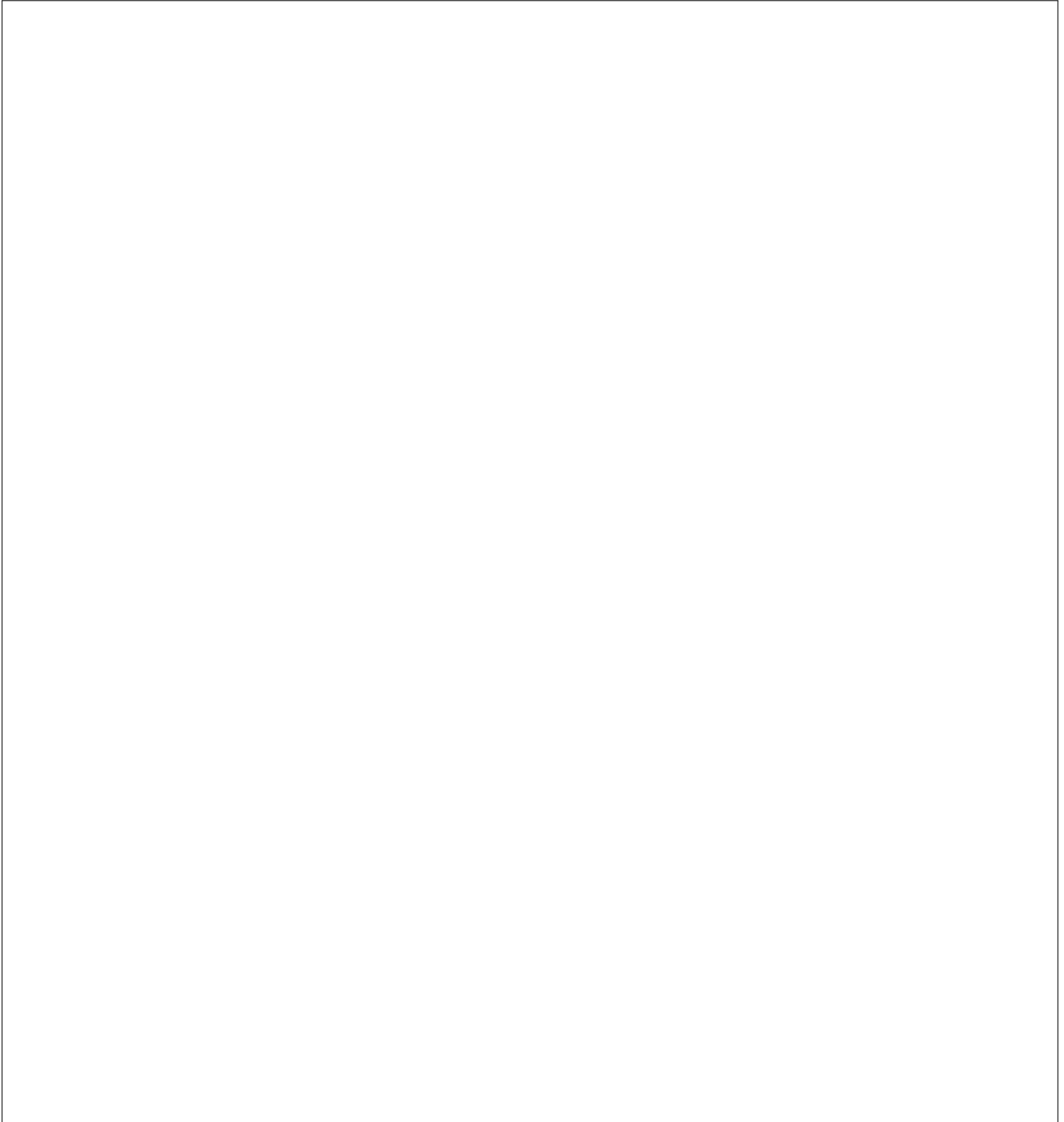
(b) Find x and y. [4]

4. [Maximum mark: 6]

Let $f(x) = \dfrac{2x - 1}{x + 3}$ and $g(x) = 5 - x$.

(a) Write down the largest possible domain of $f(x)$. [1]

(b) Find the coordinates of the points of intersection of the graphs of f and g. [3]

(d) Find $f^{-1}(x)$, the inverse of $f(x)$. [2]

5. [Maximum mark: 6]

Let $f(x) = \frac{1}{2}(x-1)^2 + 3$, where $x \in \mathbb{R}$.

(a) State the coordinates of the vertex of $f(x)$. [1]

(b) Sketch the graph of $y = f(x)$. [3]

(c) State the range of values of f. [1]

(d) State the maximum value of $g(x) = \frac{1}{f(x)}$. [1]

6. [Maximum mark: 6]

Solve the following equations:

(a)
$$\log_7(2x - 1) - \log_7(x + 3) = 0$$

[2]

(b)
$$3^{2x+1} = 4 \times 3^x + 1$$

[2]

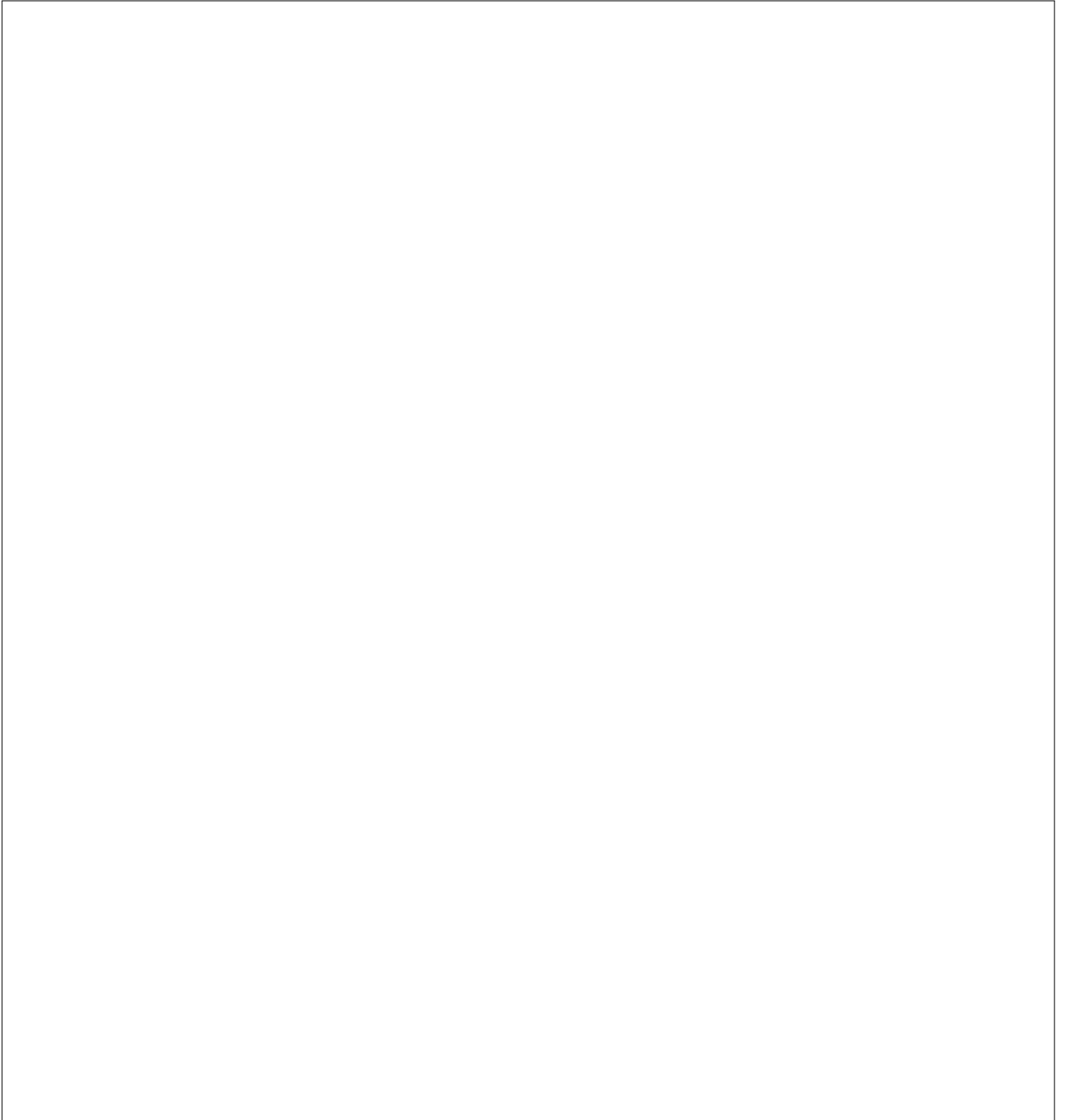(c)
$$|x + 1| = |2x - 3|$$

[2]

7. [Maximum mark: 5]

Consider the function $f(x) = 2x^2 + 6x + 1$.
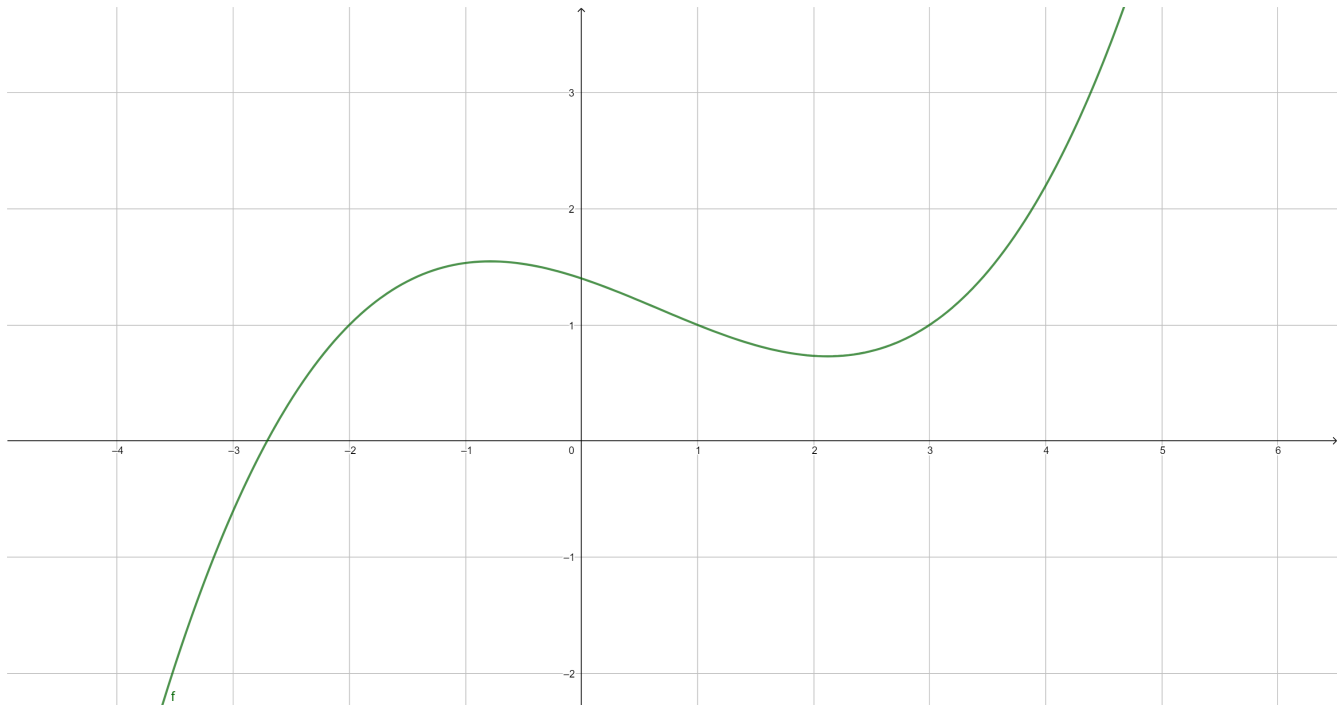
(a) Find the sequence of transformations that maps the graph of $y = x^2$ onto the graph of $y = f(x)$.   [3]

(b) The graph of $y = f(x)$ has been reflected in the y-axis and then translated by $\binom{1}{1}$ to form the graph of $y = g(x)$. Write down the coordinates of the vertex of $g(x)$.   [2]

8. [Maximum mark: 4]

The graph of the function $y = f(x)$ is shown below:

(a) Solve the equation $f(x) = 1$ [2]

(b) Let $g(x) = f(x-1) - 1$. State the zeroes of $g(x)$.   [2]
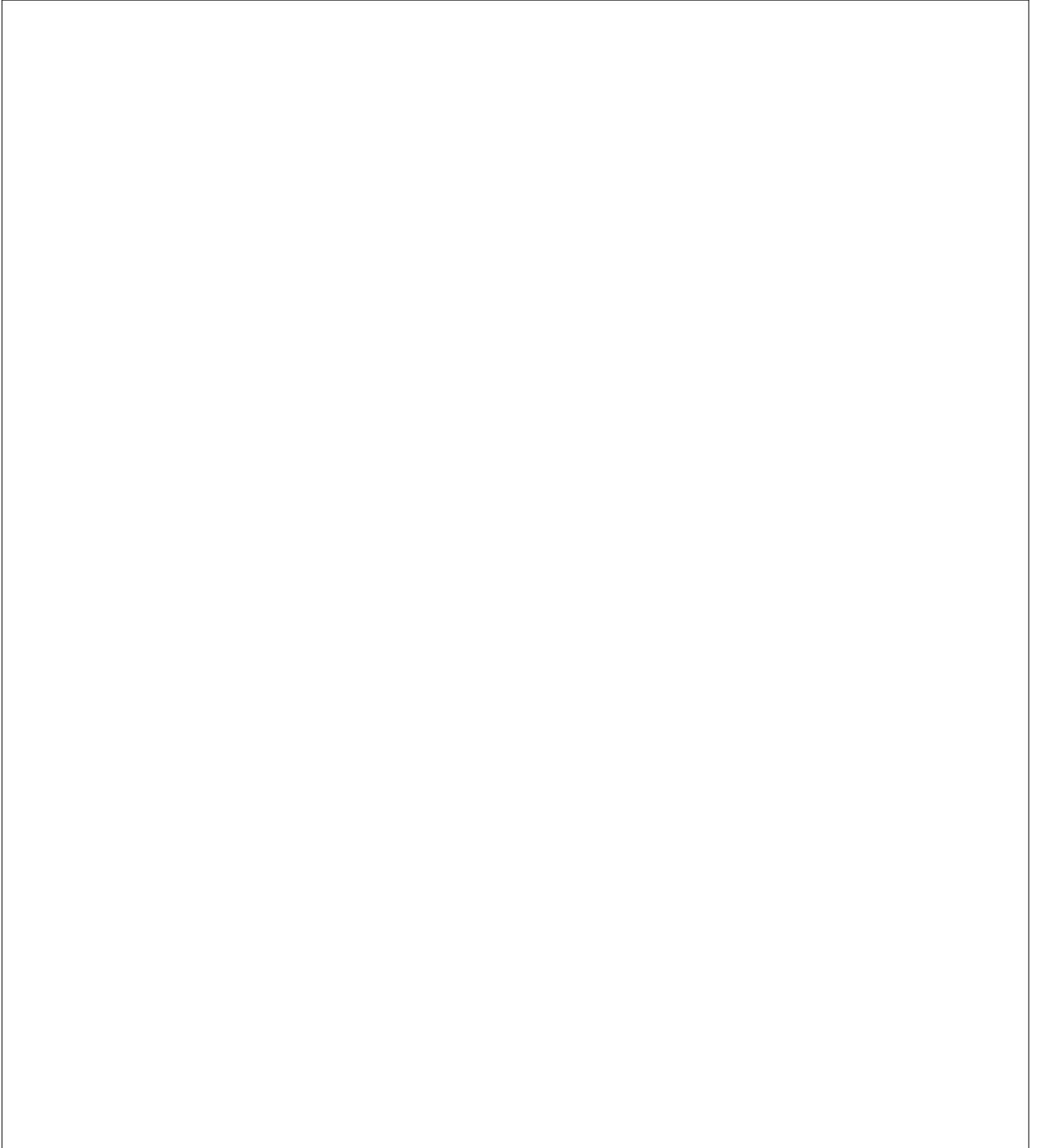
9. [Maximum mark: 6]

The first four terms of an arithmetic sequence are: $a + 1, a + b, 2b + 3$ and $4a + b$.

(a) Find a and b. [4]

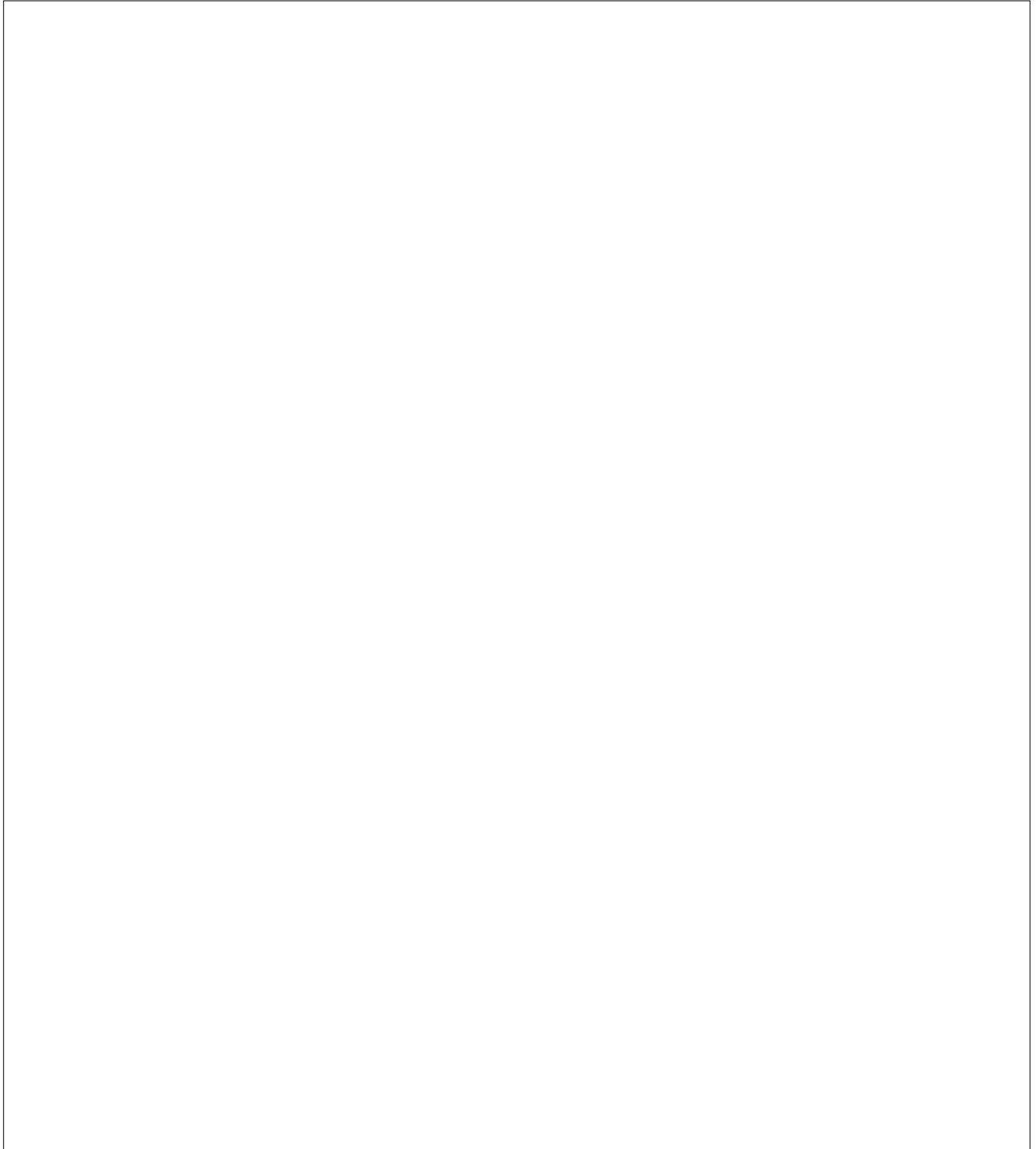(b) Find how many terms of this sequence are smaller than 100. [2]

10. [Maximum mark: 4]

The first term of an increasing arithmetic sequence is equal to 2. The first, second and fifth term of this sequence form (in the given order) a geometric sequence.

(a) Find the common difference of the arithmetic sequence. [3]

(b) State the common ratio of the geometric sequence. [1]

11. [Maximum mark: 6]

Consider a triangle ABC with $AB = 7$ cm, $AC = 5$ cm and $BC = 6$ cm.
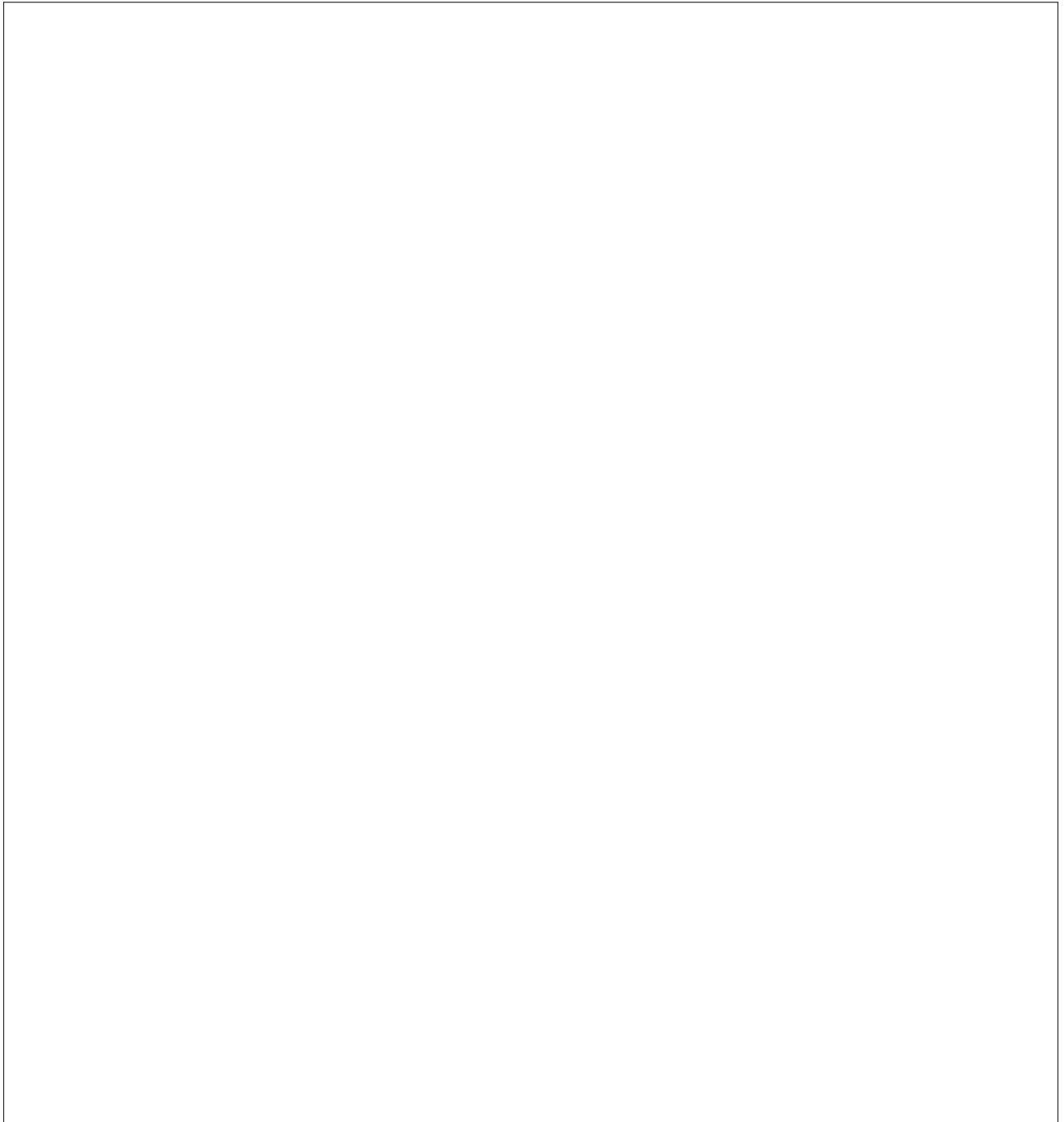
(a) Calculate $\cos \angle ACB$ [3]

Let D be the midpoint of BC.

(b) Find the length of AD. [3]

12. [Maximum mark: 6]

Consider the following data:

$$3, \ 1, \ 5, \ 5, \ 8, \ 4, \ 10, \ 8, \ a, \ b$$

Both the mean and the median of this data set are equal to 6 and $a < b$.
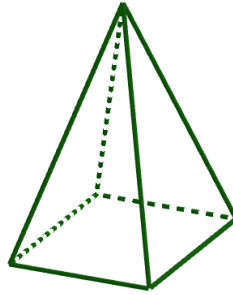
(a) Find a and b. [3]

The standard deviation is equal to 2.72 correct to 3 significant figures.

(b) Each element of the data set has been multiplied by 2 and then decreased by 3. Write down the mean and the standard deviation of the new set. [3]

13. [Maximum mark: 4]

A pyramid has a square base and a height of $4\ cm$.

Let θ be the angle between the edge of pyramid and its base. Find the length of the side of the base if $\sin\theta = \frac{2}{3}$.