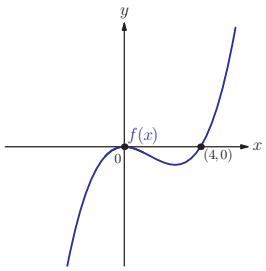
Mixed examination practice 6

Short questions

1. The graph of $y = f(x)$ is shown.



Sketch on separate diagrams the graphs of

(a) $y = 3f(x-2)$

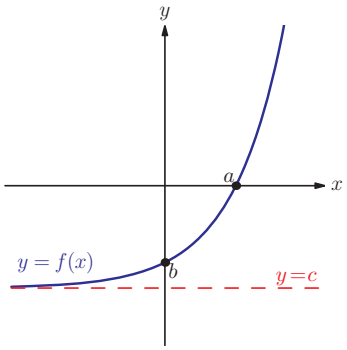
(b) $\dfrac{1}{f(x)}$

Indicate clearly the positions of any x-intercepts and asymptotes. *[6 marks]*

2. The graph of $y = x^3 - 1$ is transformed by applying a translation with vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ followed by a vertical stretch with scale factor 2. Find the equation of the resulting graph in the form $y = ax^3 + bx^2 + cx + d$. *[4 marks]*

3. Solve the inequality $|2x-1| < x$. *[6 marks]*

4. The diagram shows the graph of $y = f(x)$.



On separate diagrams sketch the following graphs, labelling appropriately.

(a) $y = |f(x)|$

(b) $y = f(|x|) - 1$ *[5 marks]*

5. (a) Sketch the graph of $y = -\dfrac{3}{x}$.

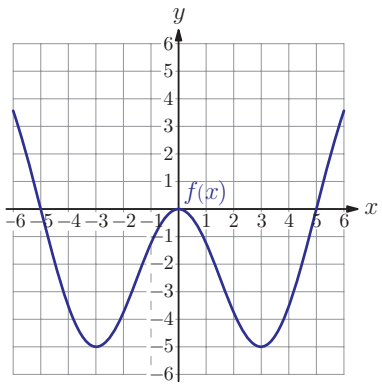
(b) Describe two transformations which transform the graph of $y = \dfrac{1}{x}$ to the graph of $y = -\dfrac{3}{x}$.

(c) Let $f(x) = -\dfrac{3}{x}, x \neq 0$. Write down an equation for $f^{-1}(x)$. *[4 marks]*

6. The graph of $y = f(x)$ is shown.

(a) On the same diagram sketch the graph of $y = \dfrac{1}{f(x)}$.

(b) State the coordinates of the maximum points. *[5 marks]*



7. Find two transformations whose composition transforms the graph of $y = (x-1)^2$ to the graph of $y = 3(x+2)^2$. *[4 marks]*

8. (a) Describe two transformations whose composition transforms the graph of $y = f(x)$ to the graph of $y = 3f\left(\dfrac{x}{2}\right)$.

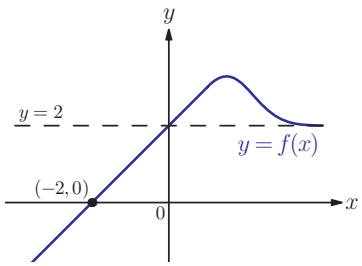
(b) Sketch the graph of $y = 3\ln\left(\dfrac{x}{2}\right)$.

(c) Sketch the graph of $y = 3\ln\left(\dfrac{x}{2}+1\right)$ marking clearly the positions of any asymptotes and x-intercepts. *[7 marks]*

9. The diagram shows a part of the graph of $y = f(x)$

On separate diagrams sketch the graphs of

(a) $y = \dfrac{1}{f(x)}$ (b) $y = xf(x)$ *[6 marks]*



10. For which values of the real number x is $|x+k| = |x| + k$, where k is a positive real number? *[4 marks]*

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Long questions

1 (a) Describe two transformations which transform the graph of $y = x^2$ to the graph of $y = 3x^2 - 12x + 12$.

(b) Describe two transformations which transform the graph of $y = x^2 + 6x - 1$ to the graph of $y = x^2$.

(c) Hence describe a sequence of transformations which transform the graph of $y = x^2 + 6x - 1$ to the graph of $y = 3x^2 - 12x + 12$.

(d) Sketch the graph of $y = \dfrac{1}{3x^2 - 12x + 12}$. *[12 marks]*

2. Given that $f(x) = \dfrac{3x - 5}{x - 2}$

(a) Write down the equation of the horizontal asymptote of the graph of $y = f(x)$.

(b) Find the value of constants p and q such that $f(x) = p + \dfrac{q}{x - 2}$.

(c) Hence describe a single transformation which transforms the graph of $y = \dfrac{1}{x}$ to the graph of $y = f(x)$.

(d) Find an expression for $f^{-1}(x)$ and state its domain.

(e) Describe the transformation which transforms the graph of $y = f(x)$ to the graph of $y = f^{-1}(x)$. *[11 marks]*

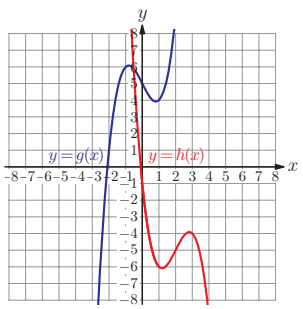
3. (a) Describe a transformation which transforms the graph of $y = f(x)$ to the graph of $y = f(x + 2)$.

(b) Sketch on the same diagram the graphs of

(i) $y = \ln(x + 2)$ (ii) $y = \dfrac{1}{\ln(x + 2)}$.

Mark clearly any asymptotes and x-intercepts on your sketches.

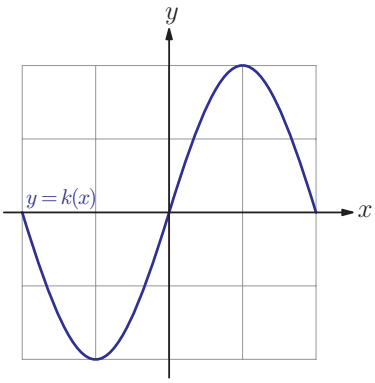
(c) The graph of the function $y = g(x)$ has been translated and then reflected in the x-axis to produce the graph of $y = h(x)$.



(i) State the translation vector.

(ii) If $g(x) = x^3 - 2x + 5$, find constants a, b, c and d such that $h(x) = ax^3 + bx^2 + cx + d$.

(d) The diagram below shows the graph of $y = k(x)$.



On the same diagram, sketch the graph of $y = (k(x))^2$. *[14 marks]*

4. $f(x) = x^2 - 7x + 10$ $\quad g(x) = x^2 - 7|x| + 10$

(a) Sketch the graph of $y = f(x)$.

(b) Show that $g(x) = f(|x|)$.

(c) Sketch the graph of $y = g(x)$.

(d) Solve the equation $g(x) = x^2$.

(e) Solve the equation $g(x) = -2$. *[12 marks]*

5. If $f(x) = 3x^2 + bx + 10$ and the graph $y = f(x)$ has a line of symmetry when $x = 3$

(a) find b.

(b) If $f(x) = f(d - x)$ for all x, find the value of d.

(c) $g(x) = f(x + p) + q$ and $g(x)$ is an even function which passes through the origin. Find p and q. *[14 marks]*

(d) Find the set values which satisfy $g(x) = g(|x|)$.