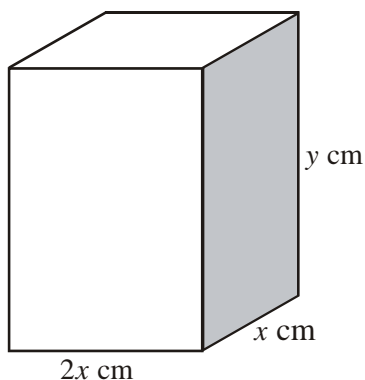


1. A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2 .



- (a) Show that $4x^2 + 6xy = 300$. (2)
- (b) Find an expression for y in terms of x . (2)
- (c) Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$. (2)
- (d) Find $\frac{dV}{dx}$. (2)
- (e) (i) Hence find the value of x and of y required to make the volume of the box a maximum.
- (ii) Calculate the maximum volume. (5)

(Total 13 marks)

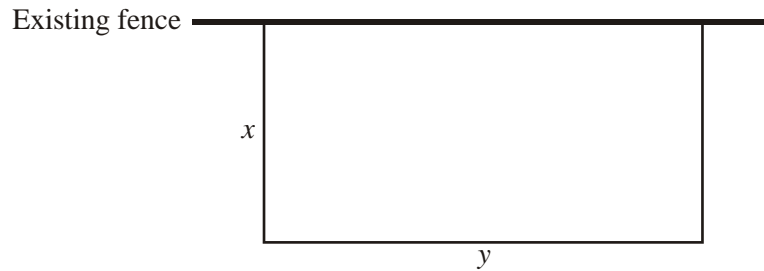
2. The cost per person, in euros, when x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x}$$

- (a) Find $C'(x)$. (3)
- (b) Show that the cost per person is a minimum when 10 people are invited to the party. (2)
- (c) Calculate the minimum cost per person. (2)

(Total 7 marks)

3. A farmer wishes to enclose a rectangular field using an existing fence for one of the four sides.



- (a) Write an expression in terms of x and y that shows the total length of the new fence. (1)

- (b) The farmer has enough materials for 2500 metres of new fence. Show that

$$y = 2500 - 2x \quad (1)$$

- (c) $A(x)$ represents the area of the field in terms of x .

- (i) Show that

$$A(x) = 2500x - 2x^2 \quad (2)$$

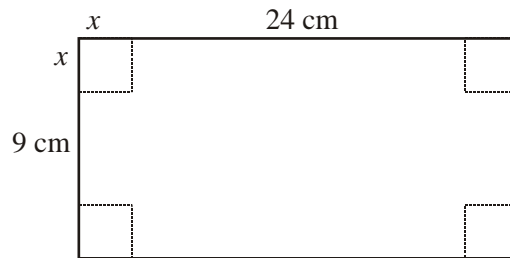
- (ii) Find $A'(x)$. (1)

- (iii) Hence or otherwise find the value of x that produces the maximum area of the field. (3)

- (iv) Find the maximum area of the field. (3)

(Total 11 marks)

4. A rectangular piece of card measures 24 cm by 9 cm. Equal squares of length x cm are cut from each corner of the card as shown in the diagram below. What is left is then folded to make an **open** box, of length l cm and width w cm.



- (a) Write expressions, in terms of x , for
- the length, l ;
 - the width, w .
- (2)
- (b) Show that the volume ($B \text{ m}^3$) of the box is given by $B = 4x^3 - 66x^2 + 216x$.
- (1)
- (c) Find $\frac{dB}{dx}$.
- (1)
- (d) (i) Find the value of x which gives the maximum volume of the box.
- (ii) Calculate the maximum volume of the box.

(4)
(Total 8 marks)

5. The cost of producing a mathematics textbook is \$15 (US dollars) and it is then sold for \$ x .

(a) Find an expression for the profit made on each book sold.

(1)

A total of $(100\,000 - 4000x)$ books is sold.

(b) Show that the profit made on all the books sold is

$$P = 160\,000x - 4000x^2 - 1500\,000.$$

(3)

(c) (i) Find $\frac{dP}{dx}$.

(2)

(ii) Hence calculate the value of x to make a maximum profit

(2)

(d) Calculate the number of books sold to make this maximum profit.

(2)

(Total 10 marks)

6. A closed box has a square base of side x and height h .

(a) Write down an expression for the volume, V , of the box. (1)

(b) Write down an expression for the total surface area, A , of the box. (1)

The volume of the box is 1000 cm^3

(c) Express h in terms of x . (2)

(d) Hence show that $A = 4000x^{-1} + 2x^2$. (2)

(e) Find $\frac{dA}{dx}$. (2)

(f) Calculate the value of x that gives a minimum surface area. (4)

(g) Find the surface area for this value of x . (3)

(Total 15 marks)

7. A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, h represents the height of the can in cm, and x represents the radius of the base of the can in cm.

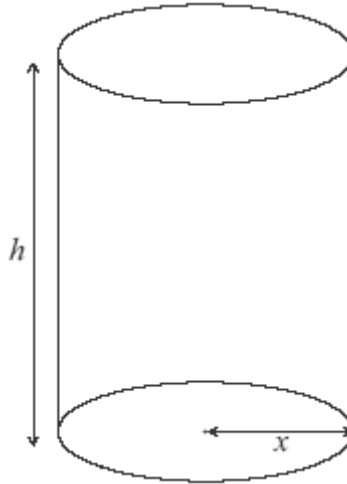


diagram not to scale

The volume of the dog food cans is 600 cm^3 .

- (a) Show that $h = \frac{600}{\pi x^2}$. (2)
- (b) (i) Find an expression for the curved surface area of the can, in terms of x .
Simplify your answer.
- (ii) Hence write down an expression for A , the total surface area of the can, in terms of x . (4)
- (c) Differentiate A in terms of x . (3)
- (d) Find the value of x that makes A a minimum. (3)
- (e) Calculate the minimum total surface area of the dog food can. (2)

(Total 14 marks)