6.

a **i** $u_5 = 217 \times (1.42)^5$
≈ 1252.86

The expected population size after 5 years is approximately 1250 birds.

ii $u_{10} = 217 \times (1.42)^{10}$
≈ 7233.41

The expected population size after 10 years is approximately 7230 birds.

b $217 \times (1.42)^n = 30\,000$

$\therefore\ (1.42)^n = \frac{30\,000}{217}$

$\therefore\ n \log 1.42 = \log\left(\frac{30\,000}{217}\right)$

$\therefore\ n = \frac{\log\left(\frac{30\,000}{217}\right)}{\log 1.42}$

$\therefore\ n \approx 14.1$

It will take approximately 14.1 years for the population to reach 30 000.

12 **a** The interest is calculated annually, so $n = 7$ time periods, and $i = 8.25\%$.

$u_7 = u_0 \times (1+i)^7$
$= 2000 \times (1.0825)^7$ $\{8.25\% = 0.0825\}$
≈ 3483.58

The total value of Kapil's investment on January 1st 2019 is 3484 rupees.

b There are $n = 7 \times 12 = 84$ time periods.

Each time period the investment increases by $i = \frac{8\%}{12} \approx 0.667\%$

$\therefore$ the value after 7 years is $u_{84} = u_0 \times (1+i)^{84}$
$\approx 2000 \times (1.006\,67)^{84}$ $\{0.667\% = 0.006\,67\}$
≈ 3494.84

The total value of Kapil's investment on January 1st 2019 for this account is 3495 rupees, which is 11 rupees more than the account in **a**.

$\therefore$ investing in the account paying 8% per annum interest compounded monthly is the better option.

19 **a** $5 \times 2^x = 160$

$\therefore\ 2^x = 32$

$\therefore\ 2^x = 2^5$

$\therefore\ x = 5$ {equating indices}

b $8^{2x-3} = 16^{2-x}$

$\therefore\ (2^3)^{2x-3} = (2^4)^{2-x}$

$\therefore\ 2^{6x-9} = 2^{8-4x}$

$\therefore\ 6x - 9 = 8 - 4x$ {equating indices}

$\therefore\ 10x = 17$

$\therefore\ x = \frac{17}{10}$

c $\left(\frac{1}{3}\right)^{2x-5} = 27$

$\therefore\ (3^{-1})^{2x-5} = 3^3$

$\therefore\ 3^{5-2x} = 3^3$

$\therefore\ 5 - 2x = 3$ {equating indices}

$\therefore\ -2x = -2$

$\therefore\ x = 1$

d $25^x + 2(5^x) = 35$

$\therefore\ (5^x)^2 + 2(5^x) - 35 = 0$

$\therefore\ (5^x - 5)(5^x + 7) = 0$

$\therefore\ 5^x = 5$ or $5^x = -7$

$\therefore\ 5^x = 5^1$ $\{5^x > 0$ for all $x\}$

$\therefore\ x = 1$ {equating indices}

20 **a** $\log_4 8$
$= \log_4(2 \times 4)$
$= \log_4(\sqrt{4} \times 4)$
$= \log_4(4^{\frac{1}{2}} \times 4^1)$
$= \log_4(4^{\frac{3}{2}})$
$= \frac{3}{2}$

b $\log_9(\frac{1}{27})$
$= \log_9\left(\frac{1}{9 \times 3}\right)$
$= \log_9\left(\frac{1}{9 \times \sqrt{9}}\right)$
$= \log_9\left(\frac{1}{9^{\frac{3}{2}}}\right)$
$= \log_9(9^{-\frac{3}{2}})$
$= -\frac{3}{2}$

c $\log_9\left(\frac{1}{3\sqrt{3}}\right)$
$= \log_9\left(\frac{1}{3^{\frac{3}{2}}}\right)$
$= \log_9\left(\frac{1}{(\sqrt{9})^{\frac{3}{2}}}\right)$
$= \log_9\left(\frac{1}{9^{\frac{3}{4}}}\right)$
$= \log_9(9^{-\frac{3}{4}})$
$= -\frac{3}{4}$

21 **a** $\log_3 x = 2$
$\therefore\ x = 3^2 = 9$

b $\log_x 27 = 3$
$\therefore\ 27 = x^3$
$\therefore\ x = 3$

c $\log_5(2x-1) = 1$
$\therefore\ 2x - 1 = 5^1$
$\therefore\ 2x - 1 = 5$
$\therefore\ 2x = 6$
$\therefore\ x = 3$

22 $\log_a(x+2) = \log_a x + 2$
$\therefore\ \log_a(x+2) - \log_a x = 2$
$\therefore\ \log_a\left(\frac{x+2}{x}\right) = 2$
$\therefore\ \frac{x+2}{x} = a^2$
$\therefore\ x + 2 = a^2 x$
$\therefore\ (1-a^2)x = -2$
$\therefore\ x = \frac{-2}{1-a^2} = \frac{2}{a^2-1} \quad \{a > 1\}$

23 **a** $\frac{1}{4}\ln 81 + \ln 12 - \ln 4$
$= \frac{1}{4}\ln(3^4) + \ln(3 \times 4) - \ln 4$
$= \frac{1}{4}(4\ln 3) + \ln 3 + \cancel{\ln 4} - \cancel{\ln 4}$
$= \ln 3 + \ln 3$
$= 2\ln 3$
$= \ln(3^2)$
$= \ln 9$

b $3\log_9 2 - \log_9 24$
$= 3\log_9 2 - \log_9(8 \times 3)$
$= 3\log_9 2 - \log_9(2^3 \times 9^{\frac{1}{2}})$
$= 3\log_9 2 - \left(\log_9(2^3) + \log_9(9^{\frac{1}{2}})\right)$
$= 3\log_9 2 - (3\log_9 2 + \frac{1}{2})$
$= -\frac{1}{2}$

c $5 + \log_2 3 - \frac{1}{2}\log_2 49 = 5 + \log_2 3 - \log_2(49^{\frac{1}{2}})$
$= \log_2(2^5) + \log_2 3 - \log_2 7$
$= \log_2(32 \times 3) - \log_2 7$
$= \log_2(\frac{96}{7})$

26 **a** $3\log_5 x = \log_5 24 + \log_5\left(\frac{1}{3}\right)$

$\therefore\ \log_5(x^3) = \log_5\left(\frac{24}{3}\right)$

$\therefore\ \log_5(x^3) = \log_5 8$

$\therefore\ x^3 = 8$

$\therefore\ x = 2$

b $\log_2 x = \log_2 12 - \log_2(7-x)$

$\therefore\ \log_2 x = \log_2\left(\frac{12}{7-x}\right)$

$\therefore\ x = \frac{12}{7-x}$

$\therefore\ x(7-x) = 12$

$\therefore\ 7x - x^2 = 12$

$\therefore\ x^2 - 7x + 12 = 0$

$\therefore\ (x-3)(x-4) = 0$

$\therefore\ x = 3$ or 4

c $\ln(x^2-3) - \ln(2x) = 0$

$\therefore\ \ln\left(\frac{x^2-3}{2x}\right) = 0$

$\therefore\ \frac{x^2-3}{2x} = e^0 = 1$

$\therefore\ x^2 - 3 = 2x$

$\therefore\ x^2 - 2x - 3 = 0$

$\therefore\ (x-3)(x+1) = 0$

$\therefore\ x = 3$ or -1

But $x = -1$ does not satisfy the original equation, as $\ln(-2)$ is undefined.

$\therefore$ the only solution is $x = 3$.

d $\log_3 x + \log_3(x-2) = 1$

$\therefore\ \log_3(x(x-2)) = 1$

$\therefore\ x(x-2) = 3$

$\therefore\ x^2 - 2x - 3 = 0$

$\therefore\ (x-3)(x+1) = 0$

$\therefore\ x = 3$ or -1

But $x = -1$ does not satisfy the original equation, as $\log_3(-1)$ is undefined.

$\therefore$ the only solution is $x = 3$.

30 **a** $9^x - 6(3^x) + 8 = 0$

$\therefore\ (3^x)^2 - 6(3^x) + 8 = 0$

$\therefore\ (3^x - 4)(3^x - 2) = 0$

$\therefore\ 3^x = 4$ or 2

$\therefore\ \log(3^x) = \log 4$ or $\log 2$

$\therefore\ x\log 3 = \log 4$ or $\log 2$

$\therefore\ x = \frac{\log 4}{\log 3}$ or $\frac{\log 2}{\log 3}$

b $25^x - 5^{x+1} + 6 = 0$

$\therefore\ (5^x)^2 - 5(5^x) + 6 = 0$

$\therefore\ (5^x - 2)(5^x - 3) = 0$

$\therefore\ 5^x = 2$ or 3

$\therefore\ \log(5^x) = \log 2$ or $\log 3$

$\therefore\ x\log 5 = \log 2$ or $\log 3$

$\therefore\ x = \frac{\log 2}{\log 5}$ or $\frac{\log 3}{\log 5}$

c $2 \times 3^{2x} + 3^{x+1} = 5$

$\therefore\ 2 \times (3^x)^2 + 3(3^x) - 5 = 0$

$\therefore\ 2 \times (3^x)^2 - 2(3^x) + 5(3^x) - 5 = 0$

$\therefore\ 2(3^x)(3^x - 1) + 5(3^x - 1) = 0$

$\therefore\ (3^x - 1)(2(3^x) + 5) = 0$

$\therefore\ 3^x = 1 \quad \{3^x > 0\}$

$\therefore\ \log(3^x) = \log 1$

$\therefore\ x\log 3 = 0$

$\therefore\ x = 0$

24 $x = \log_a 5$

a $\log_a(5a) = \log_a 5 + \log_a a$

$= x + 1$

b $\log_a\left(\frac{125}{a^2}\right)$

$= \log_a 125 - \log_a(a^2)$

$= \log_a(5^3) - 2$

$= 3\log_a 5 - 2$

$= 3x - 2$

51 $N = 120 \times (1.04)^t$

a When $t = 0$, $N = 120$

$\therefore$ there were 120 people who started the settlement.

b When $t = 4$,

$N = 120 \times (1.04)^4$

≈ 140

$\therefore$ there were about 140 people on the island after 4 years.

c When $N = 120 \times 2 = 240$,

$240 = 120 \times (1.04)^t$

$\therefore (1.04)^t = 2$

$\therefore t\ln(1.04) = \ln 2$

$\therefore t = \frac{\ln 2}{\ln(1.04)}$

≈ 17.7

$\therefore$ it will take about 17.7 years for the number of people to double.

52 $T(t) = A \times B^{-t} + 3$

a **i** The initial internal temperature of the refrigerator was 27°C.

So, $T(0) = 27$

$\therefore 27 = A + 3$

$\therefore A = 24$

ii After 3 hours, the internal temperature was 6°C.

So, $T(3) = 6$

$\therefore 6 = 24 \times B^{-3} + 3$ {using **i**}

$\therefore 24 \times B^{-3} = 3$

$\therefore B^{-3} = \frac{1}{8}$

$\therefore B^3 = 8$

$\therefore B = 2$

b $T(t) = 24 \times 2^{-t} + 3$ {using **a**}

$\therefore T(5) = 24 \times 2^{-5} + 3$

$= 3.75$

$\therefore$ the internal temperature is 3.75°C after 5 hours.

c As $t \to \infty$, $2^{-t} \to 0$

$\therefore T(t) \to 24 \times 0 + 3 = 3$

$\therefore$ the minimum temperature that the refrigerator could be expected to reach is 3°C.

54 $P = 1000 + ae^{kn}$

The initial population was 2000, so when $n = 0$, $P = 2000$

$\therefore 2000 = 1000 + ae^0$

$\therefore a = 1000$

$\therefore P = 1000 + 1000e^{kn}$

After 1 year, the population was 4000, so when $n = 1 \times 12 = 12$, $P = 4000$

$\therefore 4000 = 1000 + 1000e^{12k}$

$\therefore 1000e^{12k} = 3000$

$\therefore e^{12k} = 3$

$\therefore e^k = 3^{\frac{1}{12}}$

$\therefore P = 1000 + 1000 \times 3^{\frac{n}{12}}$

Now, when $P = 10\,000$, $10\,000 = 1000 + 1000 \times 3^{\frac{n}{12}}$

$\therefore 9000 = 1000 \times 3^{\frac{n}{12}}$

$\therefore 3^{\frac{n}{12}} = 9$

$\therefore 3^{\frac{n}{12}} = 3^2$

$\therefore \frac{n}{12} = 2$ {equating indices}

$\therefore n = 24$

$\therefore$ it will take 24 months, or 2 years, for the population to reach 10 000.