29 The polynomial $p(x) = (ax + b)^3$ leaves a remainder of -1 when divided by $x + 1$, and a remainder of 27 when divided by $x - 2$. Find the values of the real numbers a and b.

30 The quadratic polynomial $x^2 - 2x - 3$ is a factor of the quartic polynomial function $f(x) = 4x^4 - 6x^3 - 15x^2 - 8x - 3$. Find all of the zeros of the function f. Express the zeros exactly and completely simplified.

31 $x - 2$ and $x + 2$ are factors of $x^3 + ax^2 + bx + c$, and it leaves a remainder of 10 when divided by $x - 3$. Find the values of a, b and c.

32 Let $P(x) = x^3 + px^2 + qx + r$. Two of the zeros of $P(x) = 0$ are 3 and $1 + 4i$. Find the value of p, q and r.

33 When divided by $(x + 2)$ the expression $5x^3 - 3x^2 + ax + 7$ leaves a remainder of R. When the expression $4x^3 + ax^2 + 7x - 4$ is divided by $(x + 2)$ there is a remainder of $2R$. Find the value of the constant a.

34 The polynomial $x^3 + mx^2 + nx - 8$ is divisible by $(x + 1 + i)$. Find the value of m and n.

35 Given that the roots of the equation $x^3 - 9x^2 + bx - 216 = 0$ are consecutive terms in a geometric sequence, find the value of b and solve the equation.

39 One of the zeros of the equation $x^3 - 63x + 162 = 0$ is double another zero. Find all three zeros.

40 Find the three zeros of the equation $x^3 - 6x^2 - 24x + 64 = 0$ given that they are consecutive terms in a geometric sequence. [Hint: let the zeros be represented by $\frac{\alpha}{r}$, α, αr where r is the common ratio.]

41 Consider the equation $x^5 - 12x^4 + 62x^3 - 166x^2 + 229x - 130 = 0$. Given that two of the zeros of the equation are $x = 3 - 2i$ and $x = 2$, find the remaining three zeros.

42 Find the value of k such that the zeros of the equation $x^3 - 6x^2 + kx + 10 = 0$ are in arithmetic progression, that is, they can be represented by α, $\alpha + d$ and $\alpha + 2d$ for some constant d. [Hint: use the result from question 38.]

43 Find the value of k if the roots of the equation $x^3 + 3x^2 - 6x + k = 0$ are in geometric progression.

44 The roots of the equation $x^2 + x + 4 = 0$ are α and β.

a) Without solving the equation, find the value of the expression $\frac{1}{\alpha} + \frac{1}{\beta}$.

b) Find a quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

45 If α and β are roots of the quadratic equation $5x^2 - 3x - 1 = 0$, find a quadratic equation with integral coefficients which have the roots:

a) $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$

b) $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$