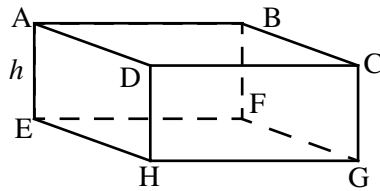


**EXERCISES 9.6**

1. A rectangular box is constructed as shown, with measurements  $HG = 10$  cm,  $\angle FHE = 30^\circ$ ,  $\angle CEG = 15^\circ$ . Find the height of the box.



2. From a point A due South of a vertical tower, the angle of elevation of the top of the tower is  $15^\circ$ , and from a point B due West of the tower it is  $32^\circ$ . If the distance from A to B is 50 metres, find the height of the tower.
3. From a point P, the angle of elevation of the top of a radio mast due North of P is  $17^\circ$ . From Q, due West of the radio mast, the angle of elevation is  $13^\circ$ . Given that P and Q are 130 m apart, show that  $h$ , the height of the mast, can be given by

$$h = \frac{130}{\sqrt{\tan^2 73 + \tan^2 77}}$$

and find  $h$  to the nearest m.

4. From a point due South of a radio tower, an observer measures the angle of elevation to the top of the tower to be  $41^\circ$ . A second observer is standing on a bearing of  $130^\circ$  from the base of the tower, and on a bearing of  $50^\circ$  from the first observer. If the height of the tower is 45m, find the distance between the two observers, and the angle of elevation of the top of the tower as measured by the second observer.
5. A small plane is flying due east at a constant altitude of 3 km and a constant speed of 120 km/h. It is approaching a small control tower that lies to the South of the plane's path. At time  $t_0$  the plane is on a bearing of  $300^\circ$  from the tower, and elevated at  $4.5^\circ$ . How long does it take for the plane to be due North of the tower, and what is its angle of elevation from the tower at this time?
6. A plane is flying at a constant altitude  $h$  with a constant speed of 250km/h. At 10:30 AM it passes directly over a town T heading towards a second town R. A fisherman located next to a river 50km due South of T observes the angle of elevation to the plane to be  $4.5^\circ$ . Town R lies on a bearing of  $300^\circ$  from where the fisherman is standing, and when the plane flies directly over R, the angle of depression to the fisherman is  $2.5^\circ$ . At what time does the plane pass directly over town R?
7. Frank and Stella are walking along a straight road heading North, when they spot the top of a tower in the direction  $N\theta^\circ E$ , behind low lying trees. The angle of elevation to the top of the tower is  $\alpha^\circ$ . After walking  $d$  m along the road they notice that the tower is now  $N\phi^\circ E$  of the road and that the angle of elevation of the top of the tower is now  $\beta^\circ$ . Let the height of the tower be  $h$  m.
- Find the distance of the tower from the
    - first sighting.
    - second sighting.
  - Find an expression of the height,  $h$  m, of the tower.
  - How much further must Stella and Frank walk before the tower is located in an easterly direction?

9	10.5	9.6	15.7	41°	37°	102°
10	21.7	36.0	36.2	35°	72°	73°
11	7.6	3.4	9.4	49°	20°	111°
12	7.2	15.2	14.3	28°	83°	69°
13	9.1	12.5	15.8	35°	52°	93°
14	14.9	11.2	16.2	63°	42°	75°
15	2.0	0.7	2.5	38°	13°	129°
16	7.6	3.7	9.0	56°	24°	100°
17	18.5	9.8	24.1	45°	22°	113°
18	20.7	16.3	13.6	87°	52°	41°
19	14.6	22.4	29.9	28°	46°	106°
20	7.0	6.6	9.9	45°	42°	93°
21	21.8	20.8	23.8	58°	54°	68°
22	1.1	1.7	1.3	41°	89°	50°
23	1.2	1.2	0.4	85°	76°	19°
24	23.7	27.2	29.7	49°	60°	71°
25	3.4	4.6	5.2	40°	60°	80°

### EXERCISE 9.5.5

1. (a) 10.14 km (b) 121°T 2. 7° 33' 3. 4.12 cm 4. 57.32 m 5. 315.5 m 6. (a) 124.3 km  
(b) W28° 47' S

### EXERCISE 9.5.6

1. 39.60m 52.84m 2. 30.2m 3. 54°, 42°, 84° 4. 37° 5. 028°T. 6. 108.1cm 7. (i) 135° (ii) 136cm  
8. 41°, 56°, 83° 9. (i) 158° left (ii) 43.22km 10. 264m 11. 53.33cm 12. 186m 13. 50.12cm  
14. 5.17cm 15. (a) 5950m (b) 13341m (c) 160° (d) 243° 17. (a) 20.70° (b) 2.578 m (c) 1.994 m<sup>3</sup>  
18. (a) 4243 m<sup>2</sup> (b) 86 m (c) 101 m

### EXERCISE 9.6

1. 5.36 cm 2. 12.3 m 3. 24 m 4. 40.3 m, 48.2° 5. 16.5 min, 8.9° 6. ~10:49 am

7. (a) i.  $\frac{d \sin \phi}{\sin(\phi - \theta)}$  ii.  $\frac{d \sin \theta}{\sin(\phi - \theta)}$  (b)  $\frac{d \sin \phi \tan \alpha}{\sin(\phi - \theta)}$  or  $\frac{d \sin \theta \tan \beta}{\sin(\phi - \theta)}$  (c)  $d \left( \frac{\sin \phi \cos \theta}{\sin(\phi - \theta)} - 1 \right)$

### EXERCISE 9.7

1. (i)  $\frac{169\pi}{150}$  cm<sup>2</sup>, 5.2 +  $\frac{13\pi}{15}$  cm (ii)  $\frac{529\pi}{32}$  cm<sup>2</sup>, 23 +  $\frac{23\pi}{8}$  cm (iii) 242 $\pi$ cm<sup>2</sup>, 88 + 11 $\pi$ cm

- (iv)  $\frac{1156\pi}{75}$  m<sup>2</sup>, 13.6 +  $\frac{68\pi}{15}$  m (v)  $\frac{96\pi}{625}$  cm<sup>2</sup>, 1.28 +  $\frac{12\pi}{25}$  cm (vi)  $\frac{361\pi}{15}$  cm<sup>2</sup>, 15.2 +  $\frac{19\pi}{3}$  cm

- (vii) 5248.8 $\pi$ m<sup>2</sup>, 648 + 32.4 $\pi$ cm (viii)  $\frac{12943\pi}{300}$  cm<sup>2</sup>, 17.2 +  $\frac{301\pi}{30}$  cm

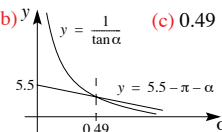
- (ix)  $\frac{1922\pi}{75}$  cm<sup>2</sup>, 12.4 +  $\frac{124\pi}{15}$  cm (x)  $\frac{15884\pi}{3}$  cm<sup>2</sup>, 152 +  $\frac{418\pi}{3}$  cm (xi) 12 $\pi$ cm<sup>2</sup>, 24 + 2 $\pi$ cm

- (xii)  $\frac{98\pi}{3}$  cm<sup>2</sup>, 28 +  $\frac{14\pi}{3}$  cm (xiii)  $\frac{196\pi}{75}$  cm<sup>2</sup>, 5.6 +  $\frac{28\pi}{15}$  cm (xiv)  $\frac{11532\pi}{25}$  cm<sup>2</sup>, 49.6 +  $\frac{186\pi}{5}$  cm

- (xv)  $\frac{3\pi}{50}$  cm<sup>2</sup>, 2.4 +  $\frac{\pi}{10}$  cm 2. 0.63°, 36° 3. 0.0942m<sup>3</sup> 4. 1.64° 5. 79cm 6. 5.25cm<sup>2</sup>

7. (a) 31.83m (b) 406.28m (c) 11° 8. 1.11° 9. 0.75° 10. (a) 1.85° (b) i. 37.09 cm ii. 88.57 cm  
(c) 370.92cm<sup>2</sup> 11. 26.57 cm<sup>2</sup> 12. 193.5 cm 13. (a) 105.22 cm (b) 118.83 cm 14. (a) 9 cm

- (b) 12 cm (c) 36°52' 15. (b)  $y = \frac{1}{\tan \alpha}$  (c) 0.49 16. 1439.16 cm<sup>2</sup>



### EXERCISE 10.1

1. (a) 120° (b) 108° (c) 216° (d) 50° 2. (a)  $\pi^c$  (b)  $\frac{3\pi^c}{2}$  (c)  $\frac{7\pi^c}{9}$  (d)  $\frac{16\pi^c}{9}$  3. (a)  $\frac{\sqrt{3}}{2}$  (b)  $-\frac{1}{2}$

- (c)  $-\sqrt{3}$  (d) -2 (e)  $-\frac{1}{2}$  (f)  $-\frac{\sqrt{3}}{2}$  (g)  $\frac{1}{\sqrt{3}}$  (h)  $\sqrt{3}$  (i)  $-\frac{1}{\sqrt{2}}$  (j)  $-\frac{1}{\sqrt{2}}$  (k) 1 (l)  $-\sqrt{2}$  (m)  $-\frac{1}{\sqrt{2}}$  (n)  $\frac{1}{\sqrt{2}}$

- (o) -1 (p)  $\sqrt{2}$  (q) 0 (r) 1 (s) 0 (t) undefined 4. (a) 0 (b) -1 (c) 0 (d) -1 (e)  $\frac{1}{\sqrt{2}}$  (f)  $-\frac{1}{\sqrt{2}}$  (g) -1

- (h)  $\sqrt{2}$  (i)  $-\frac{1}{2}$  (j)  $-\frac{\sqrt{3}}{2}$  (k)  $\frac{1}{\sqrt{3}}$  (l)  $\sqrt{3}$  (m)  $-\frac{\sqrt{3}}{2}$  (n)  $\frac{1}{2}$  (o)  $-\sqrt{3}$  (p) 2 (q)  $-\frac{1}{\sqrt{2}}$  (r)  $\frac{1}{\sqrt{2}}$  (s) -1

- (t)  $-\sqrt{2}$  5. (a)  $\frac{1}{2}$  (b)  $\frac{\sqrt{3}}{2}$  (c) 1 (d)  $\frac{1}{2}$  (e)  $-\frac{1}{\sqrt{3}}$  (f)  $-\frac{1}{2}$  (g)  $-\sqrt{2}$  (h)  $-\frac{2}{\sqrt{3}}$  6. (a)  $-\frac{1}{2}$  (b)  $-\frac{1}{\sqrt{2}}$

- (c)  $\sqrt{3}$  (d) -2 (e) 1 (f)  $\frac{1}{2}$  (g)  $-\frac{1}{\sqrt{3}}$  (h)  $-\frac{\sqrt{3}}{2}$  (i)  $-\frac{2}{\sqrt{3}}$  (j)  $\frac{1}{\sqrt{3}}$  (k)  $\frac{2}{\sqrt{3}}$  (l)  $-\frac{\sqrt{3}}{2}$  7. (a)  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

- (b)  $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  (c)  $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$  (d)  $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$  8. (a) 0 (b)  $\frac{\sqrt{3}}{2}$  (c)  $\frac{1}{\sqrt{3}}$  (d)  $\frac{1+\sqrt{3}}{2\sqrt{2}}$  10. (a)  $-\frac{2}{3}$

- (b)  $-\frac{2}{3}$  (c)  $-\frac{2}{3}$  11. (a)  $-\frac{2}{5}$  (b)  $\frac{5}{2}$  (c)  $\frac{2}{5}$  12. (a) k (b)  $-\frac{1}{k}$  (c) -k 13. (a)  $\frac{\sqrt{5}}{3}$  (b)  $\frac{3}{\sqrt{5}}$  (c)  $-\frac{\sqrt{5}}{3}$

14. (a)  $-\frac{3}{5}$  (b)  $\frac{3}{4}$  (c)  $\frac{4}{5}$  15. (a)  $\frac{4}{5}$  (b)  $\frac{3}{4}$  (c)  $-\frac{5}{3}$  16. (a) -k (b)  $-\sqrt{1-k^2}$  (c)  $-\frac{k}{\sqrt{1-k^2}}$

17. (a)  $-\sqrt{1-k^2}$  (b)  $\frac{k}{\sqrt{1-k^2}}$  (c)  $-\frac{1}{\sqrt{1-k^2}}$  18. (a)  $\sin \theta$  (b)  $\cot \theta$  (c) 1 (d) 1 (e)  $\cot \theta$  (f)  $\tan \theta$

19. (a)  $\frac{\pi}{3}, \frac{2\pi}{3}$  (b)  $\frac{\pi}{3}, \frac{5\pi}{3}$  (c)  $\frac{\pi}{3}, \frac{4\pi}{3}$  (d)  $\frac{5\pi}{6}, \frac{7\pi}{6}$  (e)  $\frac{5\pi}{6}, \frac{11\pi}{6}$  (f)  $\frac{7\pi}{6}, \frac{11\pi}{6}$

### EXERCISE 10.2.1

3. (a)  $x^2 + y^2 = k^2, -k \leq x \leq k$  (b)  $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, -b \leq x \leq b$  (c)  $(x-1)^2 + (2-y)^2 = 1, 0 \leq x \leq 2$

- (d)  $\frac{(1-x)^2}{b^2} + \frac{(y-2)^2}{a^2} = 1$  (e)  $5x^2 + 5y^2 + 6xy = 16$  4. (a) (i)  $-\frac{4}{5}$  (ii)  $-\frac{5}{3}$  (b) (i)  $\frac{4}{\sqrt{7}}$  (ii)  $-\frac{\sqrt{7}}{3}$

5. (a)  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$  (b)  $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$  (c) 0,  $\frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi$  (d)  $\frac{\pi}{2}, \frac{3\pi}{2}$  9. (a)  $\frac{2a}{a^2+1}$  (b)  $\frac{a^2-1}{a^2+1}$

10. (a) i. 1 ii. 1 (b) 1 11. (a)  $\frac{1-\sqrt{x^2-1}}{x}$  (b)  $\frac{1+\sqrt{x^2-1}}{x}$  (c)  $\frac{2}{x^2} - 1$  12. (a) i. 6 ii.  $\frac{5}{2}$  iii.  $\frac{9}{8}$

- (b) i. 5 ii. 1 iii. -2 13. (a)  $\pm 2$  (b)  $\frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}$  or  $\frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}$