Exponential equations

We will learn how to solve basic exponential equations.

We will deal with the equations of the form

$$a^{f(x)} = b^{g(x)}$$

where $a, b > 0$ and f, g are real-valued functions. In our example these will be polynomial functions and sometimes functions involving absolute value.

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$$3^{2x-1} = 243$$
$$3^{2x-1} = 3^5$$

Now we compare the exponents:

$$2x - 1 = 5$$
$$x = 3$$

Example 1

Solve

$$\left(\frac{1}{2}\right)^{x+1} = 4^{x+2}$$

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We write both sides as powers of 2:

$$\begin{aligned}\left(\frac{1}{2}\right)^{x+1} &= 4^{x+2}\\ 2^{-x-1} &= 2^{2x+4}\end{aligned}$$

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$$\left(\frac{1}{2}\right)^{x+1} = 4^{x+2}$$
$$2^{-x-1} = 2^{2x+4}$$

Compare exponents:

$$-x-1 = 2x+4$$
$$x = -\frac{5}{3}$$

Example 2

Solve:

$$\left(\frac{1}{9}\right)^{x-2} = (\sqrt{3})^{x+6}$$

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We write both sides as powers of 3:

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$$3^{-2x+4} = 3^{\frac{x}{2}+3}$$

Example 2

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We write both sides as powers of 3:

$$\left(\frac{1}{9}\right)^{x-2} = (\sqrt{3})^{x+6}$$
$$3^{-2x+4} = 3^{\frac{x}{2}+3}$$

Compare exponents:

$$-2x + 4 = \frac{x}{2} + 3$$
$$x = \frac{2}{5}$$

Example 3

Solve

$$4 \times 8^x = (2\sqrt{2})^{-x}$$

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Solution:

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$$4 \times 8^x = (2\sqrt{2})^{-x}$$

Solution:

$$\begin{aligned} 4 \times 8^x &= (2\sqrt{2})^{-x} \\ 2^2 \times 2^{3x} &= 2^{-\frac{3}{2}x} \\ 2^{3x+2} &= 2^{-\frac{3}{2}x} \\ 3x + 2 &= -\frac{3}{2}x \\ x &= -\frac{4}{9} \end{aligned}$$

Example 4

Solve

$$3 \times 81^{x-1} = (\sqrt[3]{3})^{-2x}$$

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Solution (try it on your own first):

$$\begin{aligned} 3 \times 81^{x-1} &= (\sqrt[3]{3})^{-2x} \\ 3 \times 3^{4x-4} &= 3^{-\frac{2x}{3}} \\ 3^{4x-3} &= 3^{-\frac{2x}{3}} \\ 4x - 3 &= -\frac{2x}{3} \\ x &= \frac{9}{14} \end{aligned}$$

Example 5

Solve:

$$4 \times \left(\frac{1}{\sqrt{2}}\right)^x = \frac{1}{2} \times 16^{x-1}$$

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$$4 \times \left(\frac{1}{\sqrt{2}}\right)^x = \frac{1}{2} \times 16^{x-1}$$

Solution:

$$\begin{aligned} 4 \times \left(\frac{1}{\sqrt{2}}\right)^x &= \frac{1}{2} \times 16^{x-1} \\ 2^2 \times 2^{-\frac{x}{2}} &= 2^{-1} \times 2^{4x-4} \\ 2^{2-\frac{x}{2}} &= 2^{4x-5} \\ 2 - \frac{x}{2} &= 4x - 5 \\ x &= \frac{14}{9} \end{aligned}$$

Example 6

Solve:

$$2^{|x+3|} = 1024$$

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Solution:

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$$2^{|x+3|} = 1024$$

Solution:

$$2^{|x+3|} = 1024$$
$$2^{|x+3|} = 2^{10}$$
$$|x+3| = 10$$
$$x+3=-10 \quad \vee \quad x+3=10$$
$$x=-13 \quad \vee \quad x=7$$

Example 7

Solve:

$$3^{|x-2|} = 9^x$$

Example 7

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Solution:

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$$3^{|x-2|} = 9^x$$

Solution:

$$3^{|x-2|} = 9^x$$
$$3^{|x-2|} = 3^{2x}$$
$$|x-2| = 2x$$

Now we need to solve $|x-2| = 2x$.

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$x < 2$	$x \geq 2$
$-(x-2) = 2x$	$x - 2 = 2x$
$x = \frac{2}{3}$	$x = -2$
$\frac{2}{3} < 2$	$-2 \ngeq 2$

Example 7

Solve:

$$3^{|x-2|} = 9^x$$

we need to solve $|x-2| = 2x$.

$x < 2$	$x \geq 2$
$-(x-2) = 2x$	$x - 2 = 2x$
$x = \frac{2}{3}$	$x = -2$
$\frac{2}{3} < 2$	$-2 \ngeq 2$

So the only solution is $x = \frac{2}{3}$.

Example 8

Solve:

$$(\sqrt[3]{2})^{3x^2-3} = 4^{x+1}$$

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Solution:

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$$(\sqrt[3]{2})^{3x^2-3} = 4^{x+1}$$

Solution:

$$\begin{aligned}(\sqrt[3]{2})^{3x^2-3} &= 4^{x+1}\\ 2^{x^2-1} &= 2^{2x+2}\\ x^2-1 &= 2x+2\\ x^2-2x-3 &= 0\\ (x-3)(x+1) &= 0\end{aligned}$$

Example 8

Solve:

$$(\sqrt[3]{2})^{3x^2-3} = 4^{x+1}$$

Solution:

$$\begin{aligned}(\sqrt[3]{2})^{3x^2-3} &= 4^{x+1} \\ 2^{x^2-1} &= 2^{2x+2} \\ x^2-1 &= 2x+2 \\ x^2-2x-3 &= 0 \\ (x-3)(x+1) &= 0\end{aligned}$$

We get $x = 3$ or $x = -1$.

In case of any question you can email me at T.J.Lechowski@gmail.com.