Exponential function

We will analyse functions $f(x) = a^x$, where $a \in \mathbb{R}^+$, i.e. a is a positive real number.

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x	-2	-1	0	1	2	3	4
f(x)	0.25	0.5	1	2	4	8	16
g(x)	0.(1)	0.(3)	1	3	9	27	81
h(x)	0.004	0.02	1	5	25	125	625

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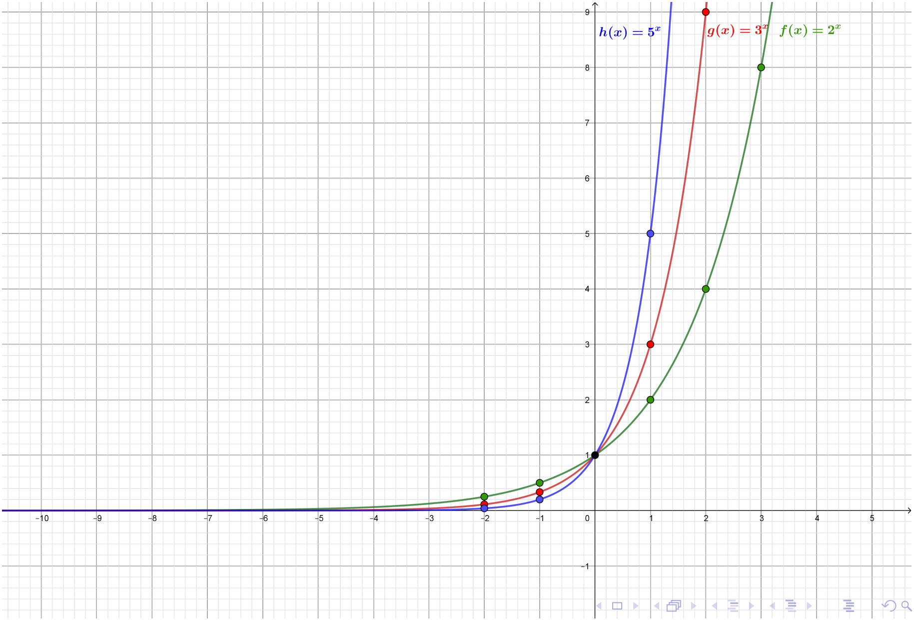

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So the range of values of the denominator is $]1, \infty[$. The denominator is then always positive, so the greater the denominator, the smaller the whole fraction and *vice versa*.

So the range of the function will be $]0, 2[$ (0 when the denominator approaches ∞, and 2 when the denominator approaches 1).

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We add 1 so in the end the range of the function is $]1, 4[$.

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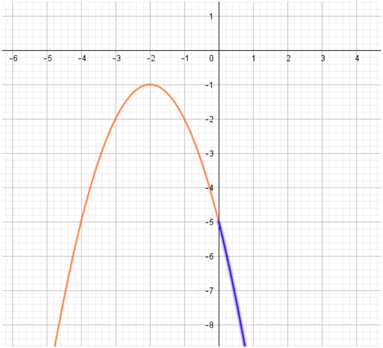

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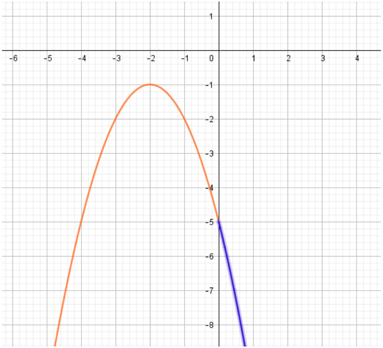

But we're interested in the blue part only (since $t \in]0, \infty[$), so in the end the range is $]-\infty, -5[$.

Short but important note must be made here. The blue part of the graph of the quadratic is **not** the graph of $f(x)$ (in particular the domain of $f(x)$ is all real numbers), but the ranges of these functions are the same.

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Since $x \in [-1, 1]$, then $t = -x^2 + 9 \in [8, 9]$ (this is a simple quadratic, if you struggle to understand, where these values came from, sketch the function with the domain $[-1, 1]$).

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We go back to $f(t) = 2^t$, the domain is $t \in [8, 9]$ and 2^t is an increasing function, so the range is $[2^8, 2^9]$, so $[256, 512]$.

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$$f_1(x) = \left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x} = f_2(-x)$$

What does this mean?

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What does this mean? It means that the graph of $f_1(x)$ is a reflection of the graph of $f_2(x)$ in the y-axis.

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So the graphs of $f(x) = (0.5)^x$, $g(x) = (\frac{1}{3})^x$, $h(x) = (0.2)^x$ look as follows (dotted lines represent graphs of 2^x, 3^x and 5^x):

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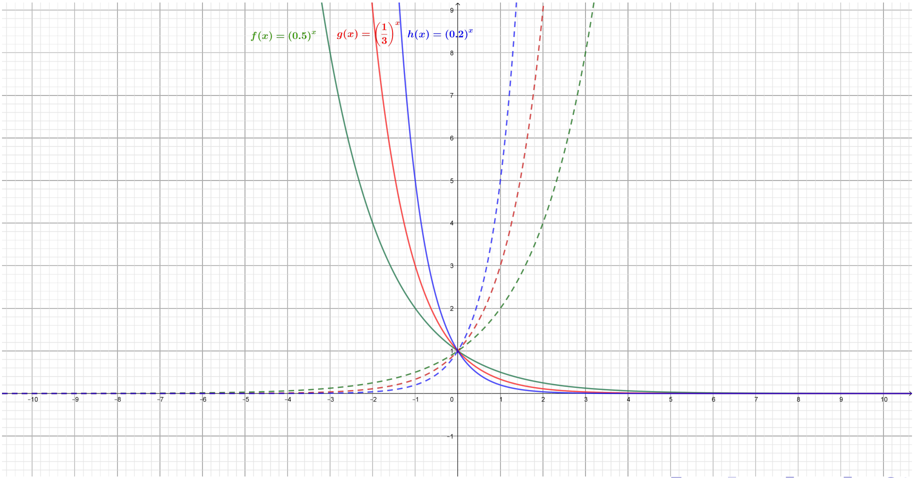

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The function is always positive The range of values is $]0, \infty[$.

Exercise 5

Arrange in ascending order:

$$\left(\frac{1}{4}\right)^{\sqrt{5}},\ \left(\frac{1}{4}\right)^{\sqrt{3}},\ \left(\frac{1}{4}\right)^{-1},\ \left(\frac{1}{4}\right)^{-\frac{1}{2}},\ \left(\frac{1}{4}\right)^{3},\ \left(\frac{1}{4}\right)^{2}$$

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Since the function is decreasing (the larger the argument, the smaller the value) we have:

$$\left(\frac{1}{4}\right)^{3} < \left(\frac{1}{4}\right)^{\sqrt{5}} < \left(\frac{1}{4}\right)^{2} < \left(\frac{1}{4}\right)^{\sqrt{3}} < \left(\frac{1}{4}\right)^{-\frac{1}{2}} < \left(\frac{1}{4}\right)^{-1}$$

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So in the end the range is $[\frac{1}{9}, 1]$.

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Finally the case $f(x) = a^x$, where $a = 1$. This is a trivial case $f(x) = a^x = 1^x = 1$. So we have a constant function, whose graph is a horizontal line $y = 1$. No more needs to be said about this case.

In case of any questions you can email me at t.j.lechowski@gmail.com.