Name:

Mathematics IB HL Test 5

December 20, 2021

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Calculators are **required** for this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [**72 marks**].
- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to **show all working**.
- Write your solutions in the space provided.

1. [Maximum mark: 5]

Given that $\log_2 3 = a$ and $\log_2 5 = b$ express the following in terms of a and b:

i) $\log_2 45$

ii) $\log_2 0.3$

iii) $\log_6 5$

2. [Maximum mark: 8]

The population of deer in the National Park can be modelled by the function $D(t) = 1500 \times e^{-0.09t}$, where t is the number of years since 2021.

a) State the population of dear in 2021. [1 *point*]

b) Calculate the estimated population of dear in 2031. Round your answer to the nearest integer. [2 *points*]

c) How long will it take for the population to decrease to 100 dear? [2 *points*]

The population of wolves in the same park can be modelled by $W(t) = 20 \times e^{0.08t}$

d) Calculate how long it will take for the population of wolves to reach one fifth the population of deer. [3 *points*]

3. [Maximum mark: 6]

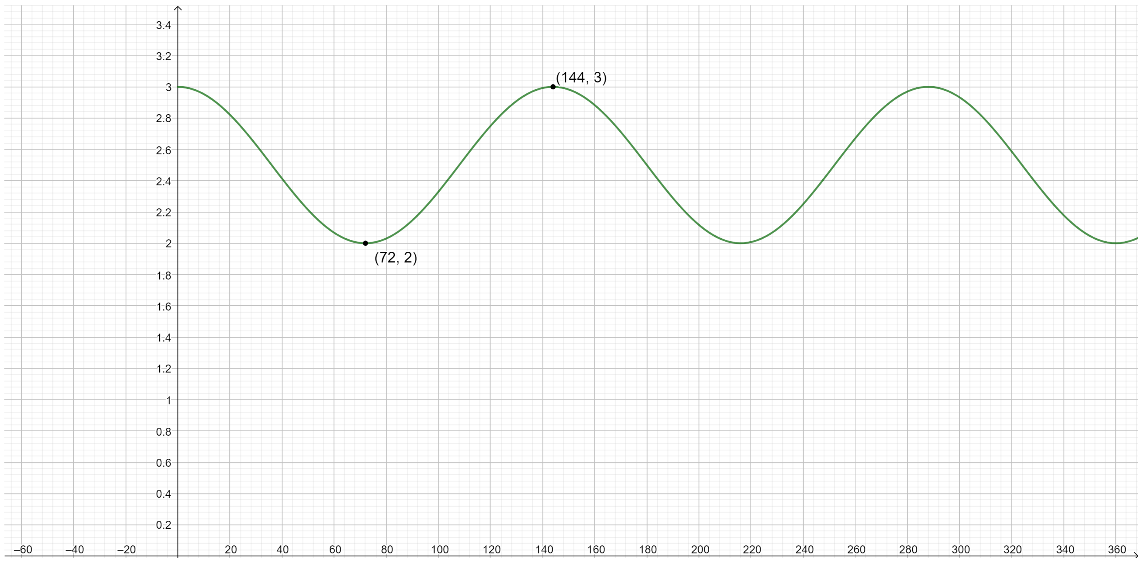
In a triangle ABC $AB = 11.2\ cm$ and $BC = 7.23\ cm$. Given that $\angle BAC = 27°$ find:

(i) the two possible values of $\angle ACB$ and hence possible lengths of AC, [4 *points*]

(ii) the difference between the areas of the two triangles satisfying the above conditions. [2 *points*]

4. [Maximum mark: 8]

The diagram below shows the graph of $T(t) = A\cos(Bt) + D$. The function has a maximum at $(144, 3)$ and a minimum at $(72, 2)$.



a) Find the values of A, B and D. [3 *points*]

T represents the temperature (in $°C$) in a fridge t minutes after the thermostat is switched on.

b) Find the temperature in the fridge after 2 hours. [1 *point*]

c) Find how long it takes for the thermostat to decrease the temperature to $2.1°C$. [1 *point*]

d) Find how long during the first 5 hours the temperature in the fridge is below $2.2°C$. [3 *points*]

5. [Maximum mark: 5]

Tomasz invests 10000 PLN into savings account that offers 2% annual interest rate compounded quarterly. Maria invests x PLN into a different savings account that pays an annual interest rate of 2.5% compounded annually. Calculate x if Tomasz and Maria has the same amount of money in their accounts after 10 years. Round your answer to 2 decimal places.

6. [Maximum mark: 4]

Let $f(x) = \frac{1}{abc}(x-a)^2(x-b)^2(x-c)$ with $a < 0 < b < c$.

a) Sketch the graph of $y = f(x)$. Clearly indicate the axes intercepts. [3 *points*]

b) Solve the inequality $f(x) > 0$ [1 *point*]

7. [Maximum mark: 6]

a) Prove that: [2 *points*]

$$\tan\theta + \cot\theta \equiv \frac{2}{\sin 2\theta}$$

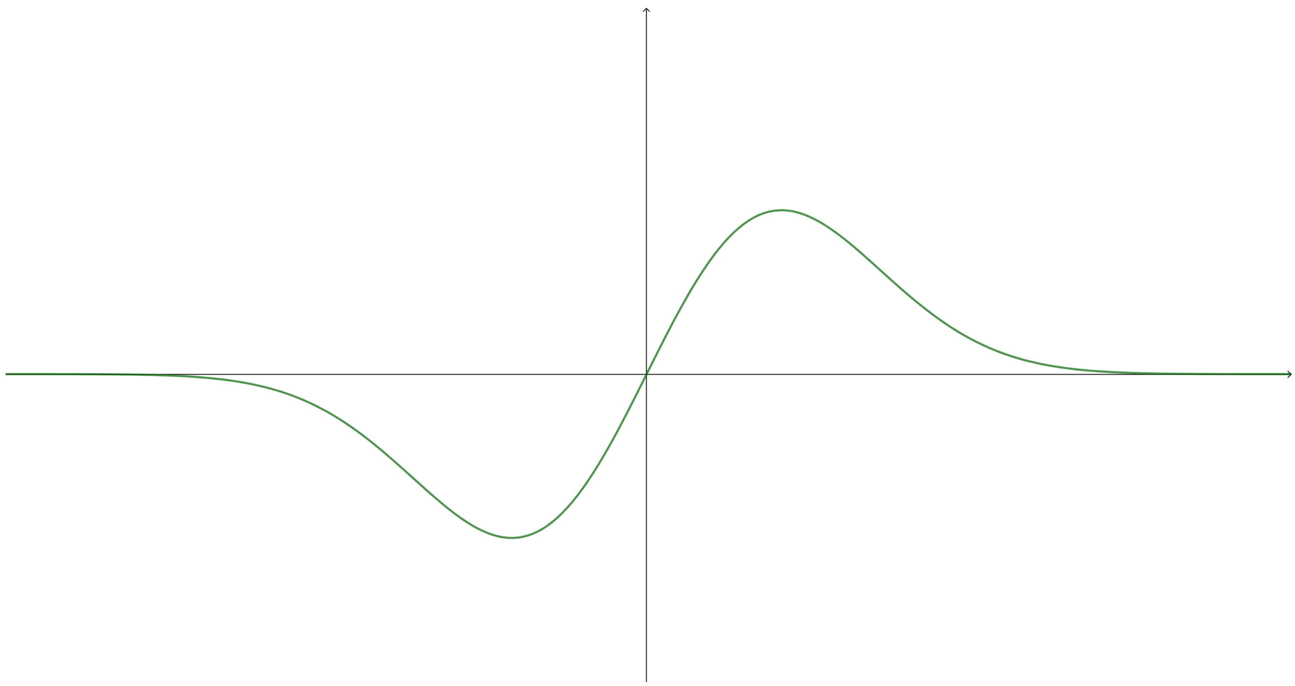
b) Hence, or otherwise, find the exact solutions to the equation: [4 *points*]

$$\tan 3\theta + \cot 3\theta = -4$$

for $0 < \theta < \frac{\pi}{2}$.

8. [Maximum mark: 5]

The graph of $y = f(x)$ is shown below. The graph has a minimum and maximum at $(-1, -1.5)$ and $(1, 1.5)$ respectively and a horizontal asymptote at $y = 0$.



On the same diagram sketch the graph of $y = [f(x)]^2 - 1$. Clearly show any asymptotes and coordinates of any local minima or maxima.

9. [Maximum mark: 10]

Let $f(x) = 3x^2 - x - 2$.

a) Write $f(x)$ in the factored form (as a product of two linear factors). [1 *point*]

b) Given that $(f(x))^7 = a_0 + a_1x + a_2x^2 + ... + a_{14}x^{14}$, find the value of a_2. [5 *points*]

c) By letting $x = 1$ show that $\sum_{k=0}^{14} a_k = 0$ [2 *points*]

d) Find the value of $\sum_{k=0}^{14} (-1)^k a_k$. [2 *points*]

10. [Maximum mark: 15]

There are 13 students (3 boys and 10 girls) in Maths HL class. 6 of these students are to be selected for Mathematics competition.

a) In how many ways can this be done if: [5 *points*]

i) there are no restrictions,

ii) at least one boy has to be selected,

iii) Tomasz or Maria has to be selected, but they cannot be both selected.

The six chosen students are to be seated in a room with nine 2-person desks. No two students can sit at the same desk.

b) In how many ways can the six students be seated? [3 *points*]

The students are asked to solve 3 Maths problems. Whoever solves a given problem first receives a point. All three problems were solved.

c) How many distributions of points are possible? [4 *points*]

The 7 students who were not selected for the competition were divided into 3 groups to work on revision questions. The groups were: two groups of 2 students and one group of 3 students.

d) Find the number of ways this can be done. [3 *points*]