1. Expand and simplify $\left(x^2 - \frac{2}{x}\right)^4$.

(Total 4 marks)

2. Given that tan $2\theta = \frac{3}{4}$, find the possible values of tan θ.

(Total 5 marks)

3. Given that 4 ln 2 – 3ln 4 = –ln k, find the value of k.

(Total 5 marks)

4. Given that $\frac{z}{z+2} = 2 - \mathrm{i}$, $z \in \mathbb{C}$, find z in the form $a + \mathrm{i}b$.

(Total 4 marks)

5. A geometric sequence $u_1, u_2, u_3, \ldots$ has $u_1 = 27$ and a sum to infinity of $\frac{81}{2}$.

(a) Find the common ratio of the geometric sequence.

(2)

An arithmetic sequence $v_1, v_2, v_3, \ldots$ is such that $v_2 = u_2$ and $v_4 = u_4$.

(b) Find the greatest value of N such that $\sum_{n=1}^{N} v_n > 0$.

(5)
(Total 7 marks)

6. The complex numbers $z_1 = 2 - 2\text{i}$ and $z_2 = 1 - \text{i}\sqrt{3}$ are represented by the points A and B respectively on an Argand diagram. Given that O is the origin,

(a) find AB, giving your answer in the form $a\sqrt{b-\sqrt{3}}$, where $a, b \in \mathbb{Z}^+$;

(3)

(b) calculate $\text{A}\hat{\text{O}}\text{B}$ in terms of π.

(3)
(Total 6 marks)

7. The common ratio of the terms in a geometric series is 2^x.

(a) State the set of values of x for which the sum to infinity of the series exists.

(2)

(b) If the first term of the series is 35, find the value of x for which the sum to infinity is 40.

(4)
(Total 6 marks)

8. Let $\sin x = s$.

(a) Show that the equation $4\cos 2x + 3\sin x \operatorname{cosec}^3 x + 6 = 0$ can be expressed as $8s^4 - 10s^2 + 3 = 0$.

(3)

(b) Hence solve the equation for x, in the interval $[0, \pi]$.

(6)
(Total 9 marks)

9. (a) Write down the quadratic expression $2x^2 + x - 3$ as the product of two linear factors.

(1)

(b) Hence, or otherwise, find the coefficient of x in the expansion of $(2x^2 + x - 3)^8$.

(4)
(Total 5 marks)

10. In the arithmetic series with n^{th} term u_n, it is given that $u_4 = 7$ and $u_9 = 22$.
Find the minimum value of n so that $u_1 + u_2 + u_3 + \ldots + u_n > 10\,000$.

(Total 5 marks)

11. Solve the equation $\log_3(x + 17) - 2 = \log_3 2x$.

(Total 5 marks)

12. Two players, A and B, alternately throw a fair six–sided dice, with A starting, until one of them obtains a six. Find the probability that A obtains the first six.

(Total 7 marks)

13. Consider the function f, where $f(x) = \arcsin(\ln x)$.

(a) Find the domain of f.

(3)

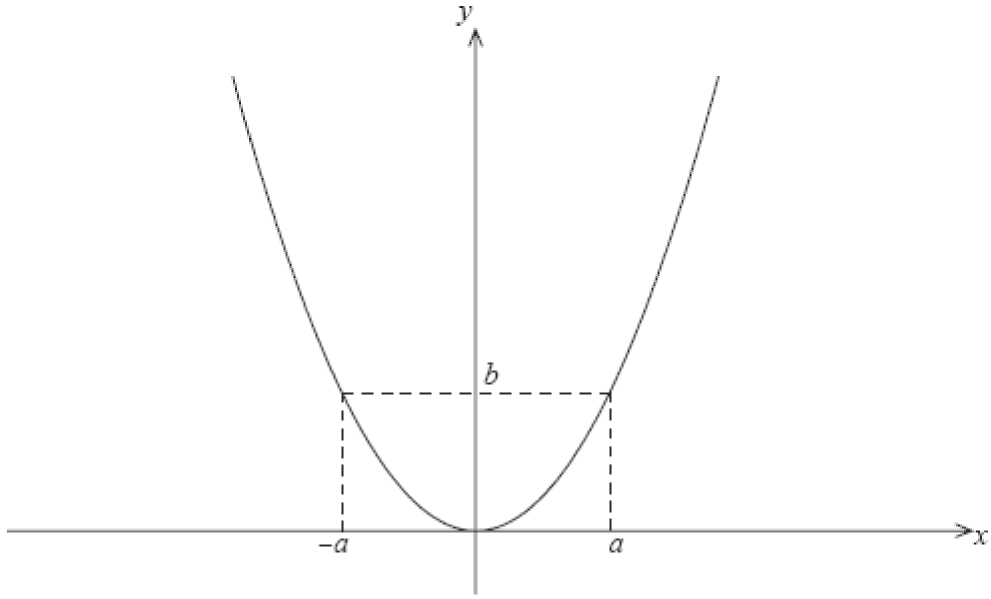
(b) Find $f^{-1}(x)$.

(3)
(Total 6 marks)

14. Solve the equation $2^{2x+2} - 10 \times 2^x + 4 = 0, x \in \mathbb{R}$.

(Total 6 marks)

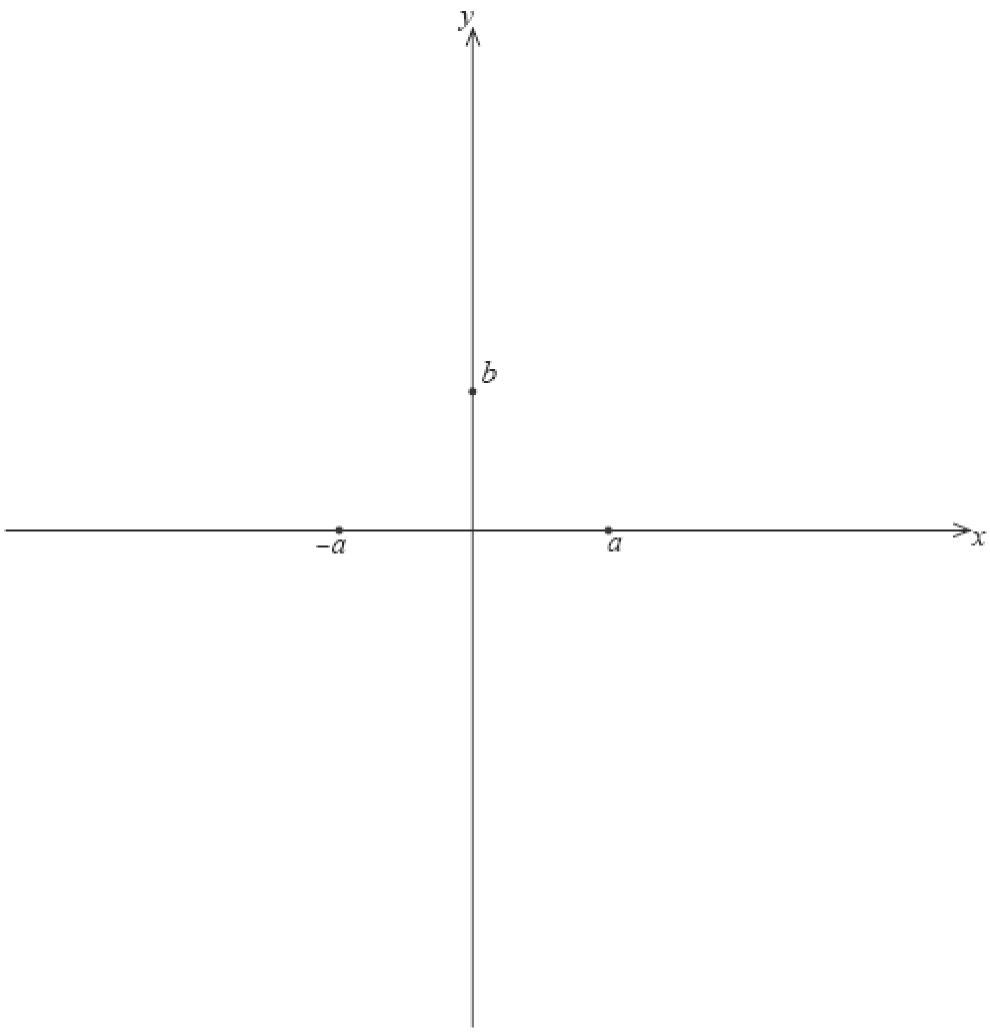
15. The diagram below shows the graph of the function $y = f(x)$, defined for all $x \in \mathbb{R}$, where $b > a > 0$.



Consider the function $g(x) = \dfrac{1}{f(x-a)-b}$.

(a) Find the largest possible domain of the function g. **(2)**

(b) On the axes below, sketch the graph of $y = g(x)$. On the graph, indicate any asymptotes and local maxima or minima, and write down their equations and coordinates.



(6)
(Total 8 marks)

16. Find the sum of all three-digit natural numbers that are not exactly divisible by 3.

(Total 5 marks)

17. When the function $q(x) = x^3 + kx^2 - 7x + 3$ is divided by $(x + 1)$ the remainder is seven times the remainder that is found when the function is divided by $(x + 2)$.

Find the value of k.

(Total 5 marks)

18. (a) If $\sin (x - \alpha) = k \sin (x + \alpha)$ express $\tan x$ in terms of k and α.

(3)

(b) Hence find the values of x between 0° and 360° when $k = \frac{1}{2}$ and $\alpha = 210°$.

(6)
(Total 9 marks)

19. Consider the arithmetic sequence 8, 26, 44, ...

(a) Find an expression for the n^{th} term.

(1)

(b) Write down the sum of the first n terms using sigma notation.

(1)

(c) Calculate the sum of the first 15 terms.

(2)
(Total 4 marks)

20. Solve the equation $4^{x-1} = 2^x + 8$.

(Total 5 marks)

21. (a) Simplify the difference of binomial coefficients

$$\binom{n}{3} - \binom{2n}{2}, \text{ where } n \geq 3.$$

(4)

(b) Hence, solve the inequality

$$\binom{n}{3} - \binom{2n}{2} > 32n, \text{ where } n \geq 3.$$

(2)
(Total 6 marks)

22. Determine the first three terms in the expansion of $(1 - 2x)^5 (1 + x)^7$ in ascending powers of x.
(Total 5 marks)

23. Solve $\sin 2x = \sqrt{2} \cos x$, $0 \leq x \leq \pi$.

(Total 6 marks)

24. (a) Show that the complex number i is a root of the equation

$$x^4 - 5x^3 + 7x^2 - 5x + 6 = 0.$$

(2)

(b) Find the other roots of this equation.

(4)
(Total 6 marks)

25. The functions f and g are defined as:

$$f(x) = e^{x^2}, \quad x \geq 0$$

$$g(x) = \frac{1}{x+3}, x \neq -3.$$

(a) Find $h(x)$ where $h(x) = g \circ f(x)$.

(2)

(b) State the domain of $h^{-1}(x)$.

(2)

(c) Find $h^{-1}(x)$.

(4)
(Total 8 marks)

26. When $\left(1 + \frac{x}{2}\right)^n$, $n \in \mathbb{N}$, is expanded in ascending powers of x, the coefficient of x^3 is 70.

(a) Find the value of n.

(5)

(b) Hence, find the coefficient of x^2.

(1)
(Total 6 marks)

27. The function $f(x) = 4x^3 + 2ax - 7a$, $a \in \mathbb{R}$ leaves a remainder of -10 when divided by $(x - a)$.

(a) Find the value of a.

(3)

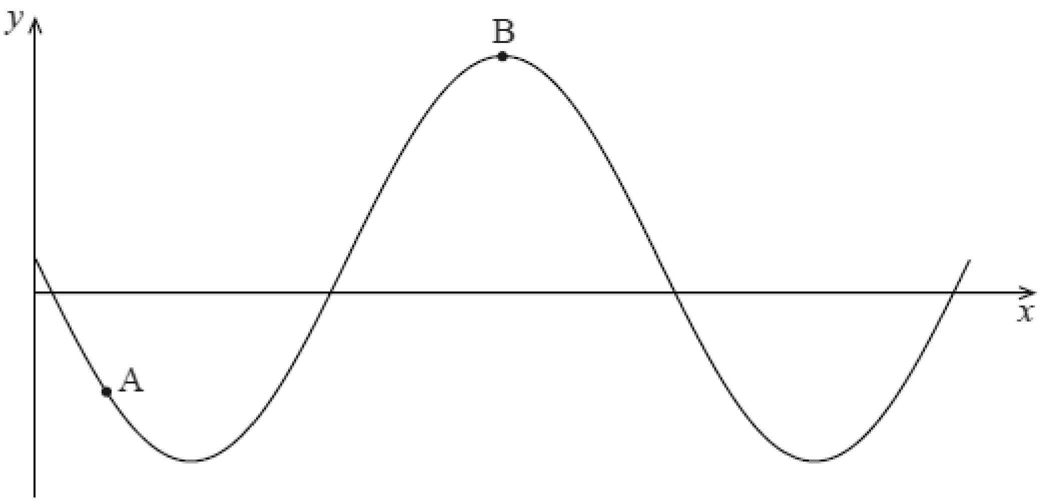
(b) Show that for this value of a there is a unique real solution to the equation $f(x) = 0$.

(2)
(Total 5 marks)

28. If x satisfies the equation $\sin\left(x + \frac{\pi}{3}\right) = 2\sin x \sin\left(\frac{\pi}{3}\right)$, show that $11 \tan x = a + b\sqrt{3}$, where $a, b \in \mathbb{Z}^+$.

(Total 6 marks)

29. The diagram below shows a curve with equation $y = 1 + k \sin x$, defined for $0 \le x \le 3\pi$.



The point $A\left(\frac{\pi}{6}, -2\right)$ lies on the curve and $B(a, b)$ is the maximum point.

(a) Show that $k = -6$.

(2)

(b) Hence, find the values of a and b.

(3)
(Total 5 marks)

30. Find all values of x that satisfy the inequality $\frac{2x}{|x-1|} < 1$.

(Total 5 marks)

31. Consider the equation $z^3 + az^2 + bz + c = 0$, where $a, b, c \in \mathbb{R}$. The points in the Argand diagram representing the three roots of the equation form the vertices of a triangle whose area is 9. Given that one root is $-1 + 3i$, find

(a) the other two roots;

(4)

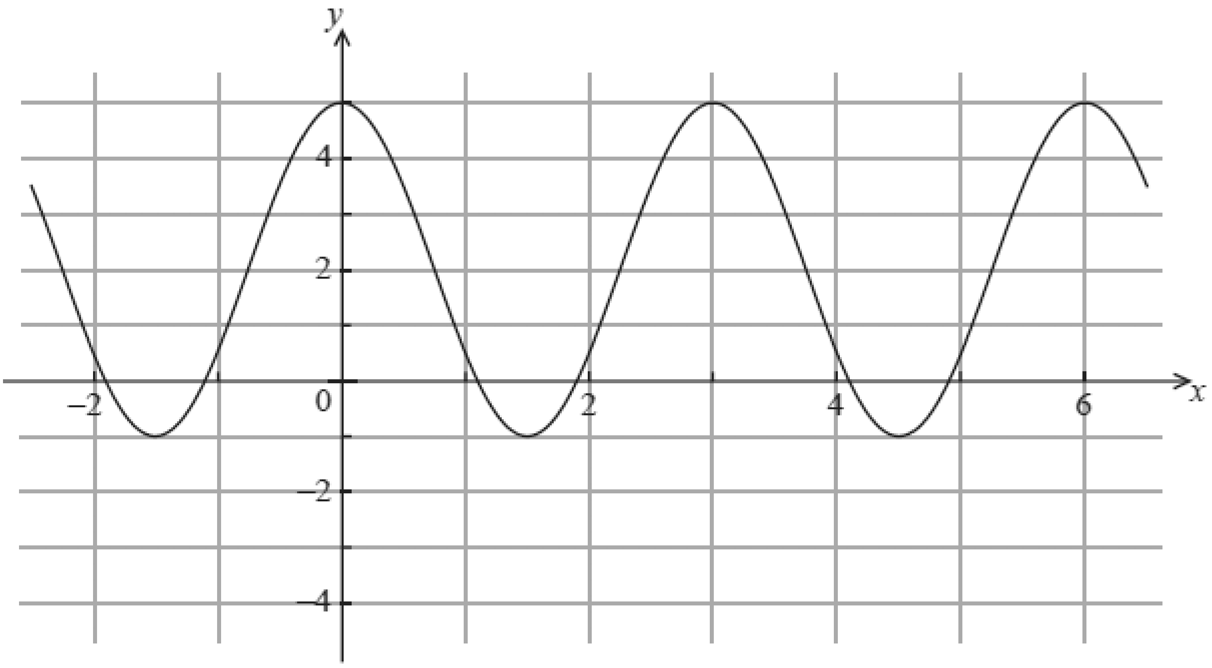
(b) a, b and c.

(3)
(Total 7 marks)

32. The mean of the first ten terms of an arithmetic sequence is 6. The mean of the first twenty terms of the arithmetic sequence is 16. Find the value of the 15^{th} term of the sequence.

(Total 6 marks)

33. The graph below shows $y = a \cos (bx) + c$.



Find the value of a, the value of b and the value of c.

(Total 4 marks)

34. Given that $Ax^3 + Bx^2 + x + 6$ is exactly divisible by $(x + 1)(x - 2)$, find the value of A and the value of B.

(Total 5 marks)

35. Let $f(x) = \frac{x+4}{x+1}$, $x \neq -1$ and $g(x) = \frac{x-2}{x-4}$, $x \neq 4$. Find the set of values of x such that $f(x) \leq g(x)$.

(Total 6 marks)

36. The sum, S_n, of the first n terms of a geometric sequence, whose n^{th} term is u_n, is given by

$$S_n = \frac{7^n - a^n}{7^n}, \text{ where } a > 0.$$

(a) Find an expression for u_n.

(2)

(b) Find the first term and common ratio of the sequence.

(4)

(c) Consider the sum to infinity of the sequence.

(i) Determine the values of a such that the sum to infinity exists.

(ii) Find the sum to infinity when it exists.

(2)
(Total 8 marks)

37. Consider the polynomial $p(x) = x^4 + ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbb{R}$.

Given that $1 + i$ and $1 - 2i$ are zeros of $p(x)$, find the values of a, b, c and d.

(Total 7 marks)

38. Consider the complex numbers $z = 1 + 2\text{i}$ and $w = 2 + a\text{i}$, where $a \in \mathbb{R}$.

Find a when

(a) $|w| = 2|z|$;

(3)

(b) $\text{Re}(zw) = 2\,\text{Im}(zw)$.

(3)
(Total 6 marks)

39. The depth, $h(t)$ metres, of water at the entrance to a harbour at t hours after midnight on a particular day is given by

$$h(t) = 8 + 4\sin\left(\frac{\pi t}{6}\right), 0 \leq t \leq 24.$$

(a) Find the maximum depth and the minimum depth of the water.

(3)

(b) Find the values of t for which $h(t) \geq 8$.

(3)
(Total 6 marks)

40. The obtuse angle B is such that $\tan B = -\dfrac{5}{12}$. Find the values of

(a) $\sin B$;

(1)

(b) $\cos B$;

(1)

(c) $\sin 2B$;

(2)

(d0 $\cos 2B$.

(2)
(Total 6 marks)

41. Given that $2 + \text{i}$ is a root of the equation $x^3 - 6x^2 + 13x - 10 = 0$ find the other two roots.

(Total 5 marks)

42. (a) Show that $p = 2$ is a solution to the equation $p^3 + p^2 - 5p - 2 = 0$.

(2)

(b) Find the values of a and b such that $p^3 + p^2 - 5p - 2 = (p - 2)(p^2 + ap + b)$.

(4)

(c) Hence find the other two roots to the equation $p^3 + p^2 - 5p - 2 = 0$.

(3)

(d) An arithmetic sequence has p as its common difference. Also, a geometric sequence has p as its common ratio. Both sequences have 1 as their first term.

(i) Write down, in terms of p, the first four terms of each sequence.

(ii) If the sum of the third and fourth terms of the arithmetic sequence is equal to the sum of the third and fourth terms of the geometric sequence, find the three possible values of p.

(iii) For which value of p found in (d)(ii) does the sum to infinity of the terms of the geometric sequence exist?

(iv) For the same value p, find the sum of the first 20 terms of the arithmetic sequence, writing your answer in the form $a + b\sqrt{c}$, where $a, b, c \in \mathbb{Z}$.

(13)
(Total 22 marks)

43. A sum of $ 5000 is invested at a compound interest rate of 6.3 % per annum.

(a) Write down an expression for the value of the investment after n full years.

(1)

(b) What will be the value of the investment at the end of five years?

(1)

(c) The value of the investment will exceed $10 000 after n full years.

(i) Write an inequality to represent this information.

(ii) Calculate the minimum value of n.

(4)
(Total 6 marks)

44. The angle θ satisfies the equation $2\tan^2\theta - 5\sec\theta - 10 = 0$, where θ is in the second quadrant. Find the value of $\sec\theta$.

(Total 6 marks)

45. Solve the equations

$$\ln\frac{x}{y} = 1$$
$$\ln x^3 + \ln y^2 = 5.$$

(Total 5 marks)