Name:

Mathematics IB HL Test 1

September 27, 2021

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Calculators are **not allowed** for this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [**72 marks**].
- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to **show all working**.
- Write your solutions in the space provided.

1. [Maximum mark: 4]

Solve the inequality

$$|x + 4| > |2x - 1|$$

2. [Maximum mark: 7]

a) Sketch the graph of the function

$$f(x) = \frac{x+2}{x^2 - 2x - 3}$$

State the equations of the asymptotes and the intersections with the axes.

b) Solve the inequality $f(x) > 0$.

3. [Maximum mark: 7]

Let $f(x) = \dfrac{2x-6}{x+1}$ with $x \neq -1$.

a) Sketch the graph of $y = f(x)$ clearly indicating all the asymptotes and axis intercepts.

b) State the range of values of f.

c) Find an expression for f^{-1}, the inverse of f.

d) State the range of values of f^{-1}.

4. [Maximum mark: 9]

Let $f(x) = \frac{1}{2}x^2 + x + 2$.

a) Show that the equation $f(x) = 0$ has no real solutions.

b) Write $f(x)$ in the form $f(x) = a(x + h)^2 + k$ and hence state the minimum value of $f(x)$.

c) Sketch the graph of $g(x) = \frac{1}{f(x+2)}$. Clearly indicate the asymptotes and any intercepts with the axes.

5. [Maximum mark: 8]

For each of the following functions decide if it is even, odd or neither. Justify your answers.

a) $f_1(x) = x^3 + 2x^2$

b) $f_2(x) = \frac{1}{x} - x^3$

c) $f_3(x) = \frac{2}{x^2 + 4}$

d) $f_4(x) = 5$

6. [Maximum mark: 8]

Numbers $a-b, a, 3b-4a$ and $6a$ are the first four terms of an arithmetic sequence.

a) Find a and b.

b) Find the sum of all terms of this sequence which are smaller than 50.

7. [Maximum mark: 7]

The grades of 16 students taking Maths HL test on transformations are given below. One of the grades is unknown and is denoted with x.

$$5,\ 2,\ 5,\ 5,\ 3,\ 7,\ 7,\ 3,\ 5,\ 6,\ 2,\ 5,\ 5,\ 6,\ 4,\ x$$

a) Given that the mean grade is 4.5 find x.

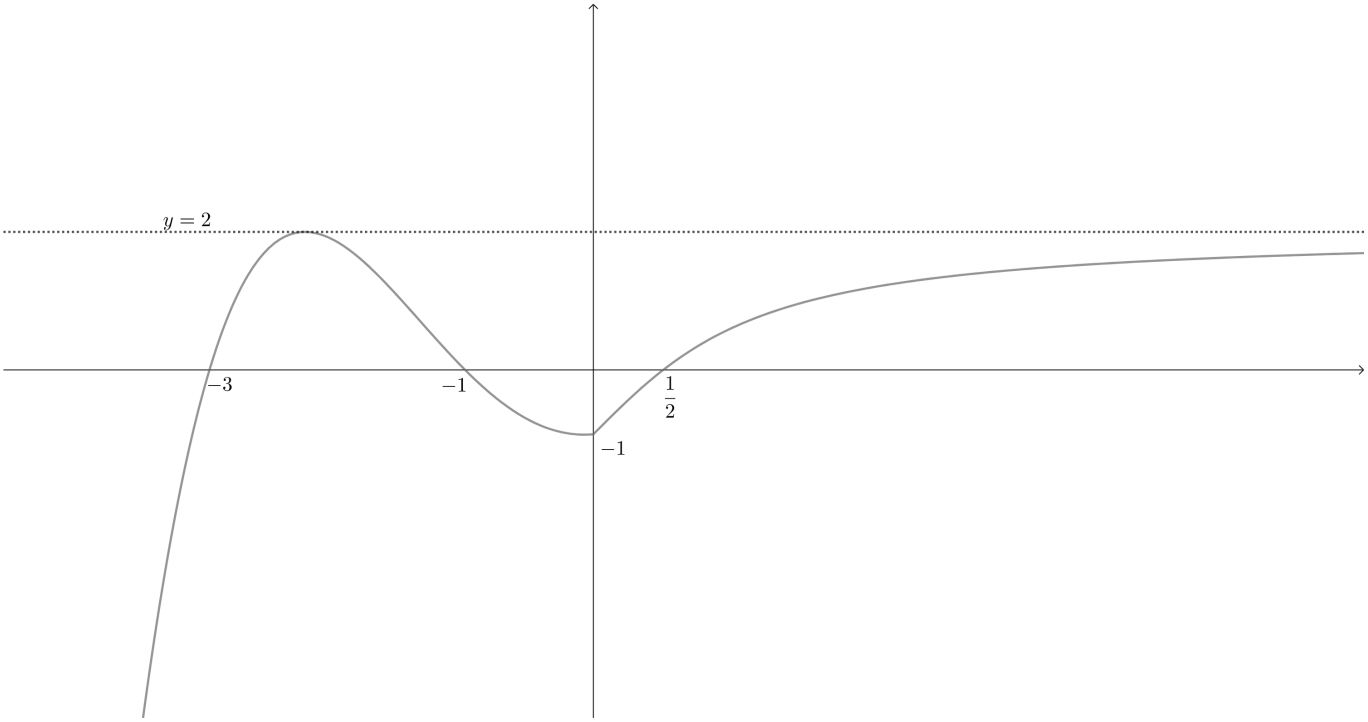
b) State the median grade and the modal grade.

c) 3 students are selected at random from this group. Find the probability that none of the selected students received a 7.

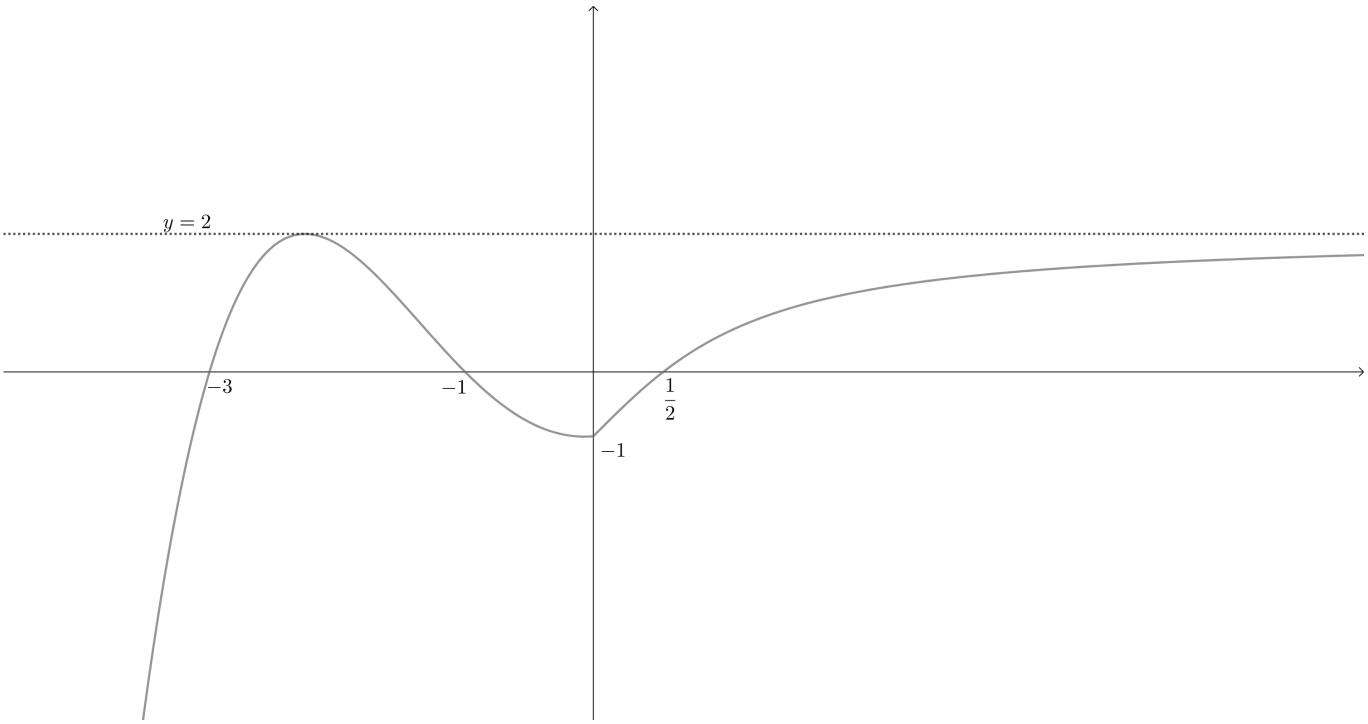
8. [Maximum mark: 12]

Four copies of the graph of $y = f(x)$ are shown below. Sketch (each on separate copy) the graphs of:

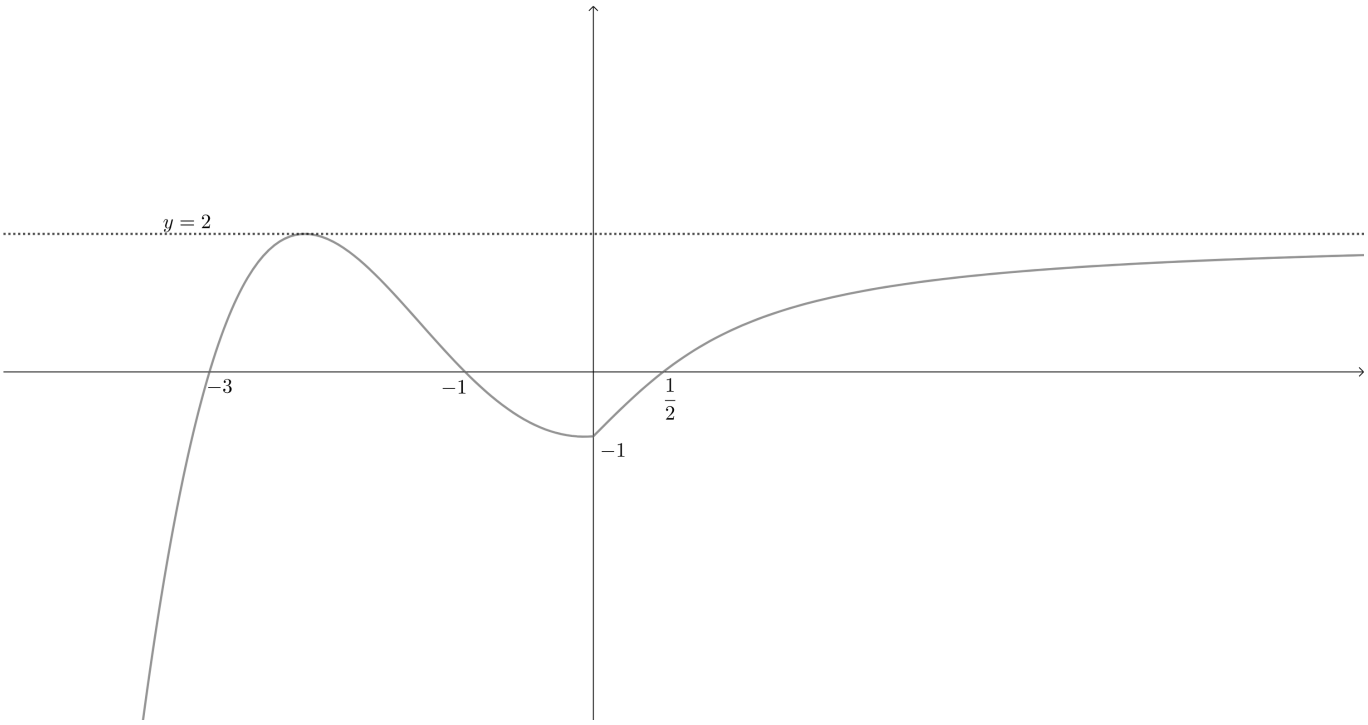
a) $y = -2f(x + 1)$



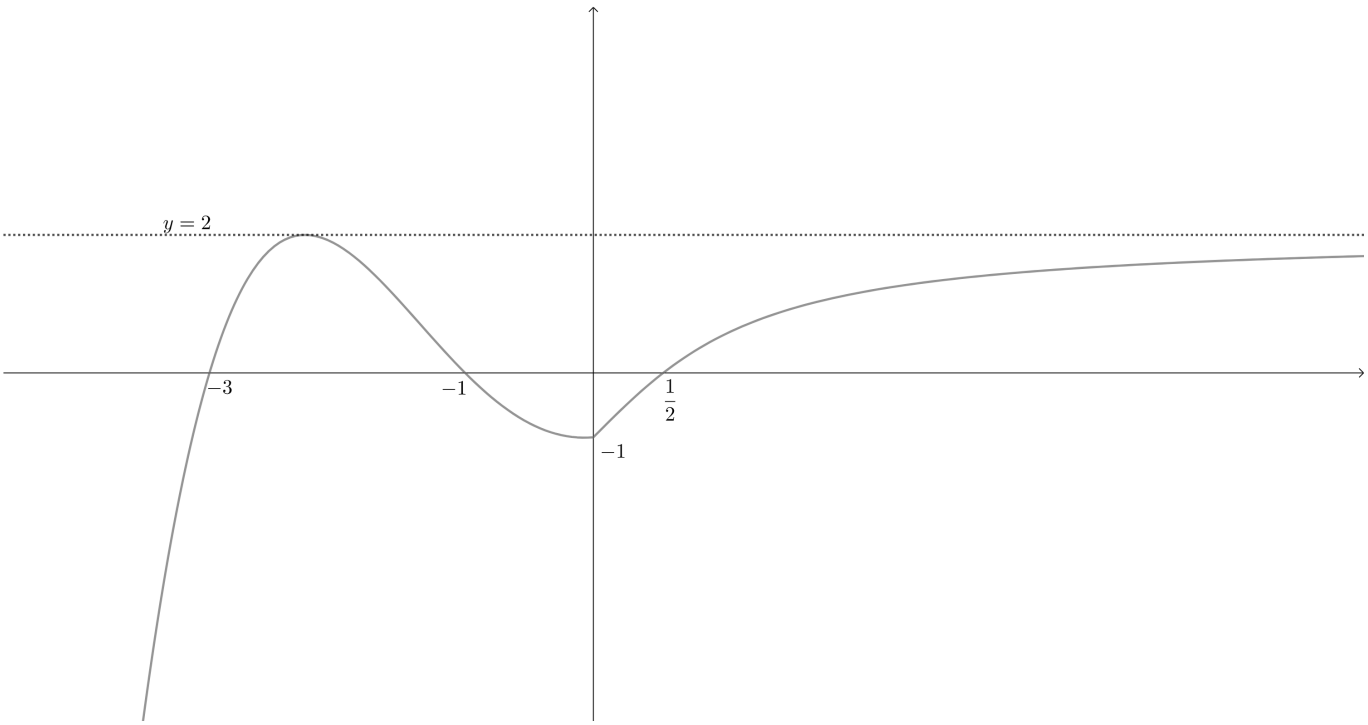
b) $y = |f(-x)|$



c) $y = \dfrac{1}{f(x)}$



d) $y = [f(x)]^2 - 1$



9. [Maximum mark: 10]

a) Sketch the graph of

$$f(x) = \frac{x^2 - x - 1}{x - 1}$$

b) Sketch the graph of

$$g(x) = f(|x|)$$

c) State the set of all possible values of k ($k \in \mathbb{R}$), for which the equation:

$$g(x) = k$$

has four solutions.