Name:

Mathematics IB HL Test 2 resit

November 4, 2021

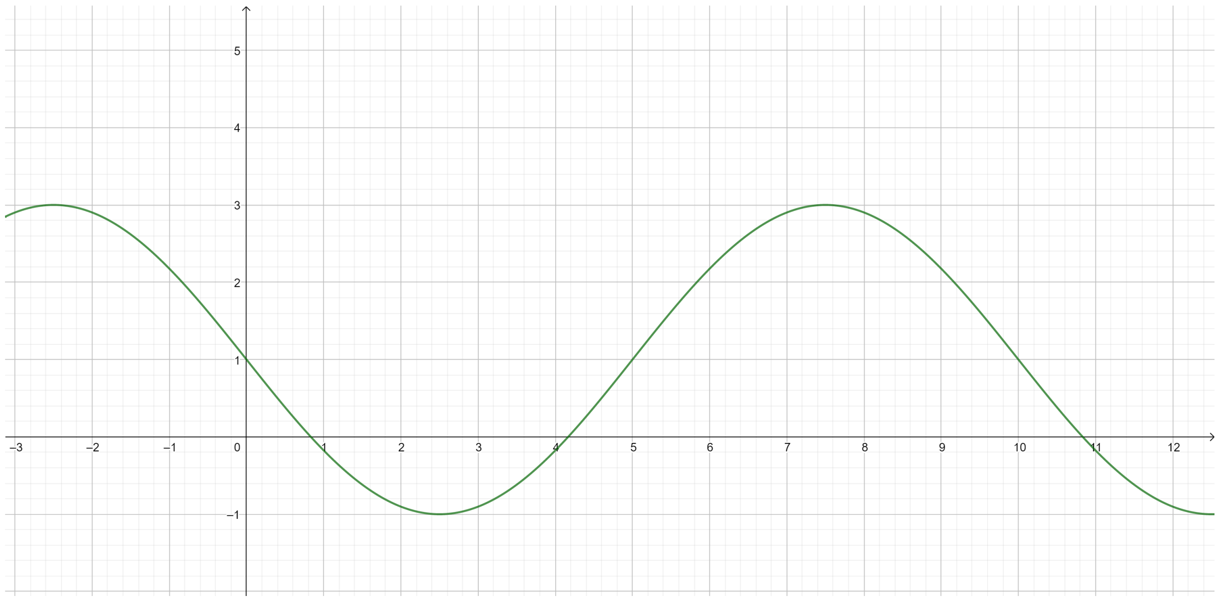
1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Calculators are **not allowed** for this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [**72 marks**].
- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to **show all working**.
- Write your solutions in the space provided.

1. [Maximum mark: 4]

Graph of $y = a\sin(bx) + d$ is shown below. The graph has a minimum at $(2.5, -1)$ and a maximum at $(7.5, 3)$.



Find the values of a, b and d.

2. [Maximum mark: 7]

Solve the equation

$$2\cos^2 3x + 5\sin 3x = 4$$

for $0 \leqslant x \leqslant \pi$.

3. [Maximum mark: 9]

A ball is attached to one end of an elastic string, with the other end held fixed above ground. When the ball is pulled down and released, it starts moving up and down, so that the height of the ball above the ground is given by the equation

$$h(t) = 120 - 10\cos(100\pi t)$$

where h is measured in cm and t is time in seconds.

(a) Find the least and greatest height of the ball above ground. [2]

(b) Find the time required to complete one full oscillation. [2]

(c) Find the first time after the ball is released when it reaches the greatest height. [1]

(d) Find for how long during the first oscillation will the ball be at least 125 cm above the ground. [4]

4. [Maximum mark: 4]

It is given that $\operatorname{cosec}\theta = \dfrac{3}{2}$ and $\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{2}$. Find the exact value of $\cot\theta$.

5. [Maximum mark: 5]

Let $f(x) = \sqrt{3}\sin x + \cos x$.

(a) Write f in the form $f(x) = A\sin(x + \alpha)$, where A and α are to be found and $A > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. [2]

(b) Solve the equation $f(x) = 1$ for $0 \leqslant x \leqslant 2\pi$. [3]

6. [Maximum mark: 8]

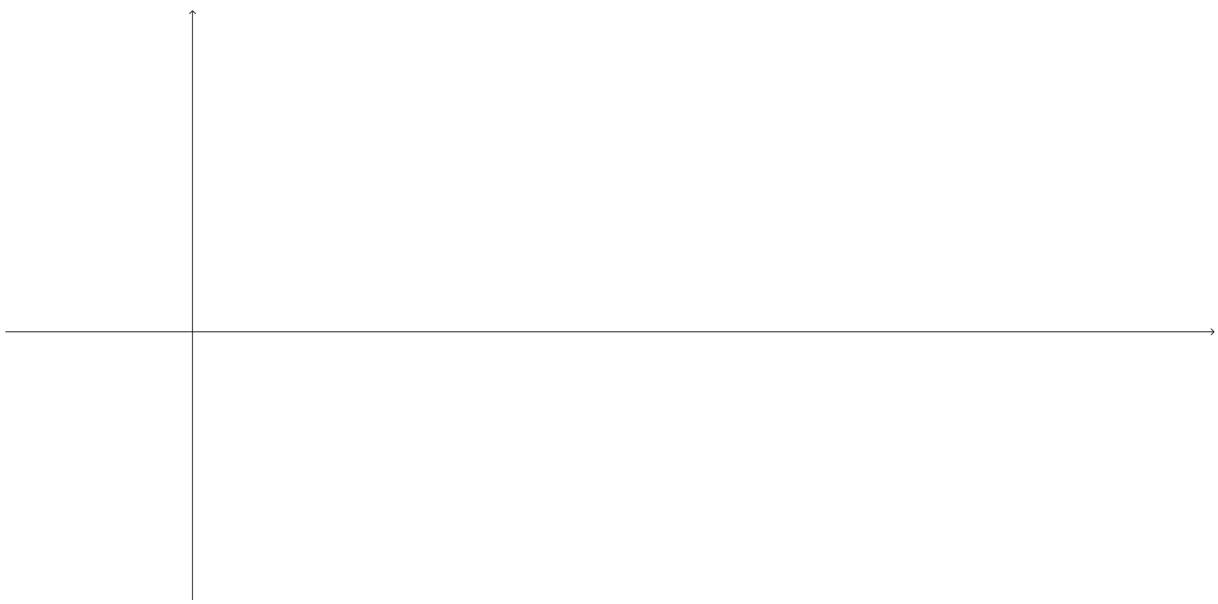
(a) Show that $\sin 2x + \cos 2x - 1 = 2\sin x(\cos x - \sin x)$. [2]

(b) Hence or otherwise, solve $\sin 2x + \cos 2x - 1 + \cos x - \sin x = 0$ for $0 < x < 2\pi$. [6]

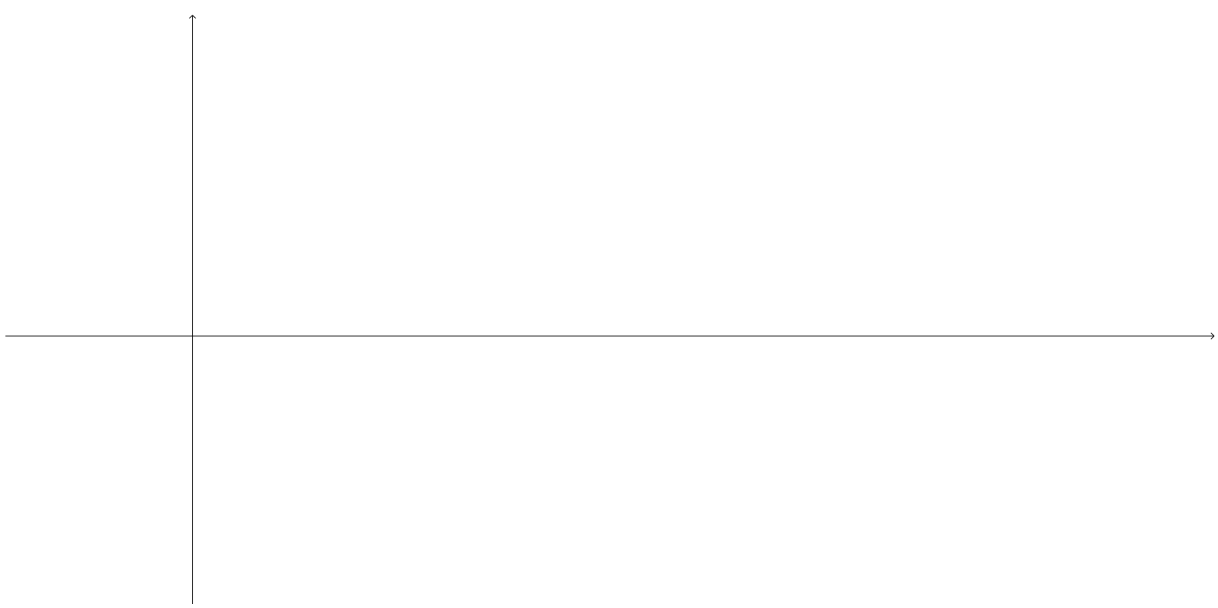
7. [Maximum mark: 8]

Let $f(x) = \dfrac{x-a}{(x-b)(x-c)}$, where a, b and c are positive constants. Sketch, on separate diagrams the graph of $y = f(x)$, when:

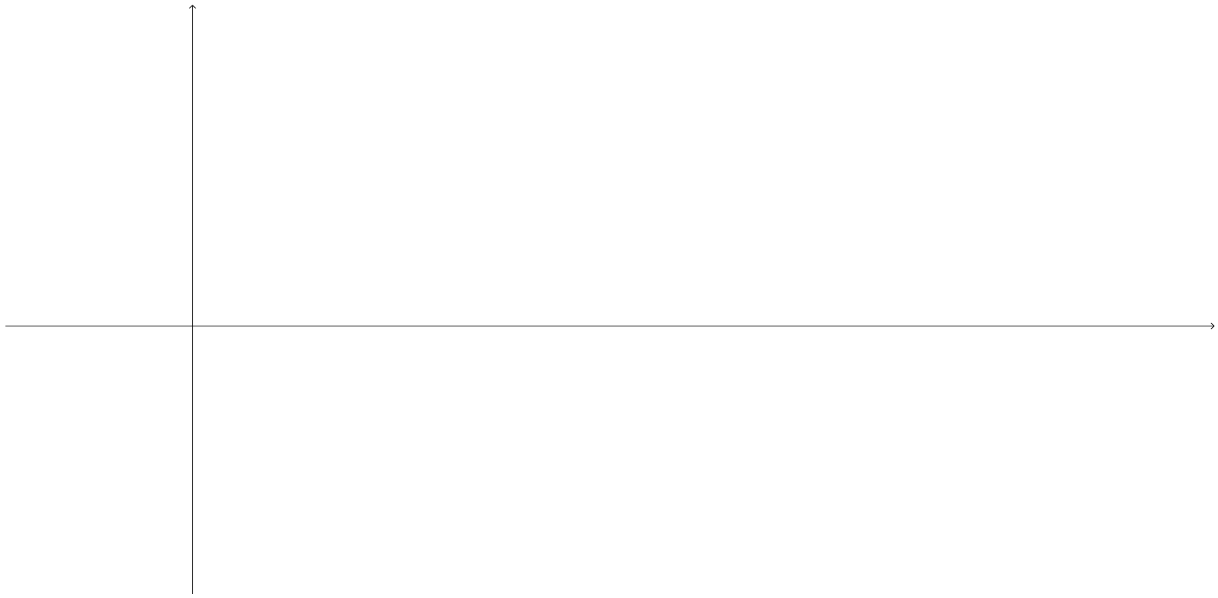
a) $a < b < c$,



b) $b < a < c$,



c) $b < c < a$.



8. [Maximum mark: 12]

Consider the function $f(x) = x^2 - 4x + 3$, $x \in \mathbb{R}$.

(a) Write $f(x)$ in the form $f(x) = (x - p)(x - q)$. [1]

(b) Write $f(x)$ in the form $f(x) = a(x - h)^2 + k$. [1]

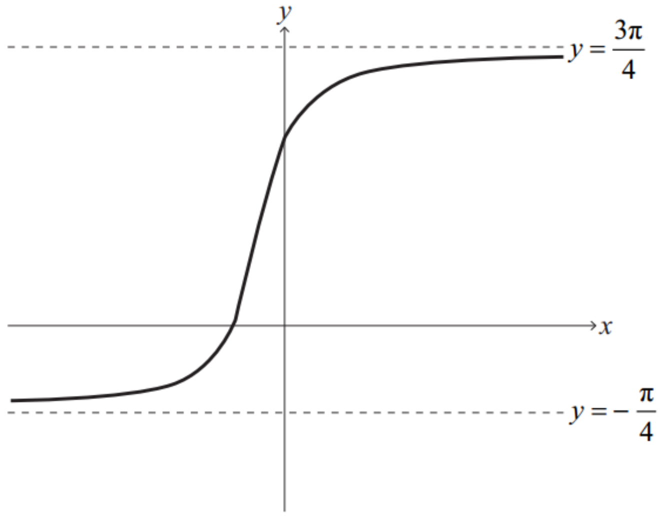
(c) Sketch the graph of $y = \dfrac{1}{f(x)}$. [3]

(d) Sketch the graph of $y = [f(x)]^2$. [3]

(e) The graph of $y = f(x)$ has been reflected in the line $x = 3$ and then in the line $y = 1$. The resulting graph is the graph of $y = g(x)$. Find $g(x)$ in the form $g(x) = ax^2 + bx + c$. [4]

9. [Maximum mark: 15]

The following diagram shows the graph of $y = \arctan(2x+1) + \frac{\pi}{4}$ for $x \in \mathbb{R}$ with asymptotes at $y = -\frac{\pi}{4}$ and $y = \frac{3\pi}{4}$.



(a) Describe a sequence of transformations that transforms the graph of $y = \arctan x$ to the graph of $y = \arctan(2x+1) + \frac{\pi}{4}$ for $x \in \mathbb{R}$. [3]

(b) Show that

$$\arctan p + \arctan q = \arctan\left(\frac{p+q}{1-pq}\right)$$

where $p, q > 0$ and $pq < 1$. [4]

(c) Verify that $\arctan(2x+1) = \arctan\left(\frac{x}{x+1}\right) + \frac{\pi}{4}$ for $x \in \mathbb{R}$, $x > 0$. [3]

(d) Hence, or otherwise, find the exact value of $\arctan\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$. [5]