Name:

Mathematics IB HL Test 2

October 21, 2021

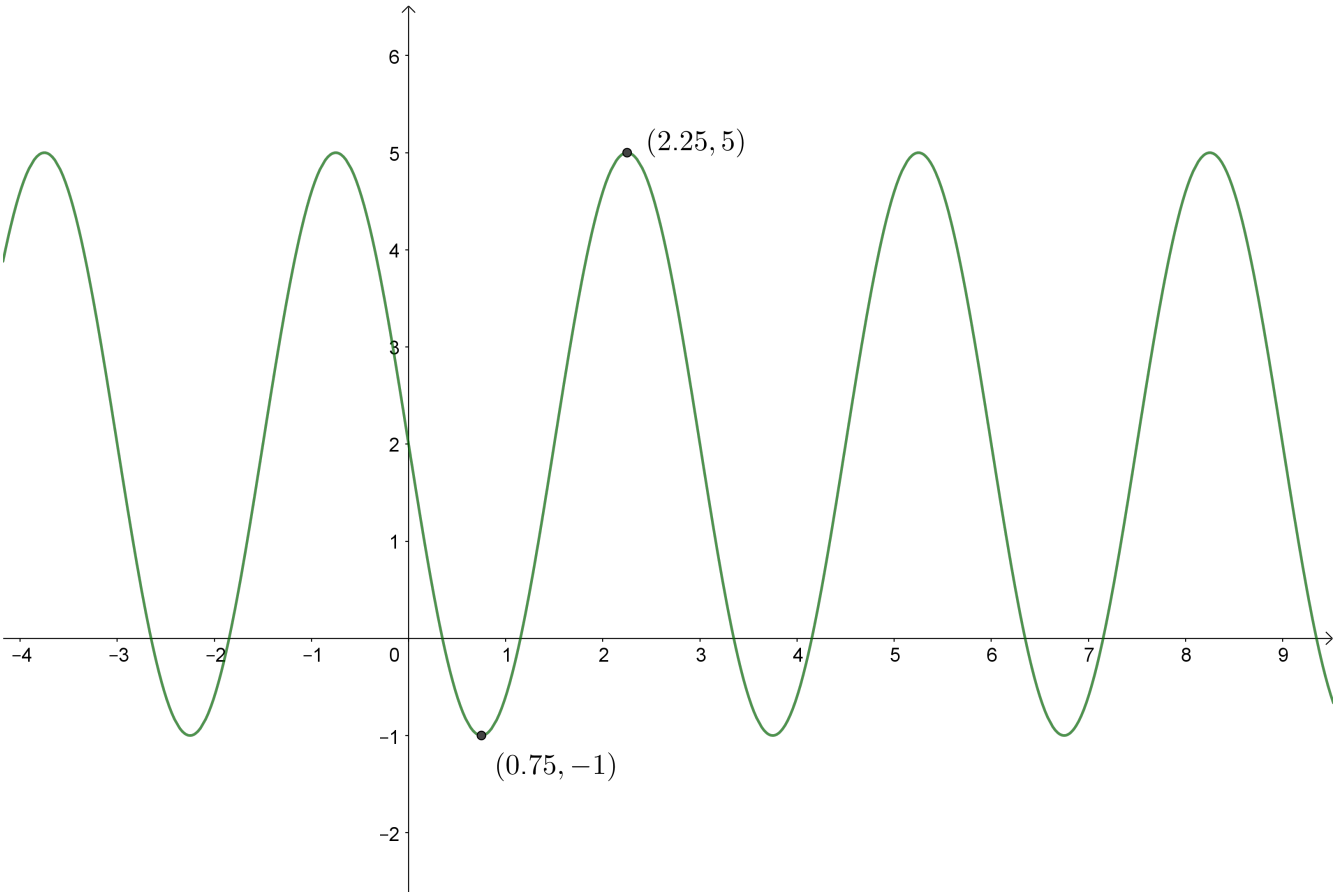
1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Calculators are **not allowed** for this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [**72 marks**].
- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to **show all working**.
- Write your solutions in the space provided.

1. [Maximum mark: 4]

The following diagram shows the graph of a function $f(x) = a\sin(bx) + c$, where $a, b, c, \in \mathbb{R}$.



Find the values of a, b and c.

2. [Maximum mark: 4]

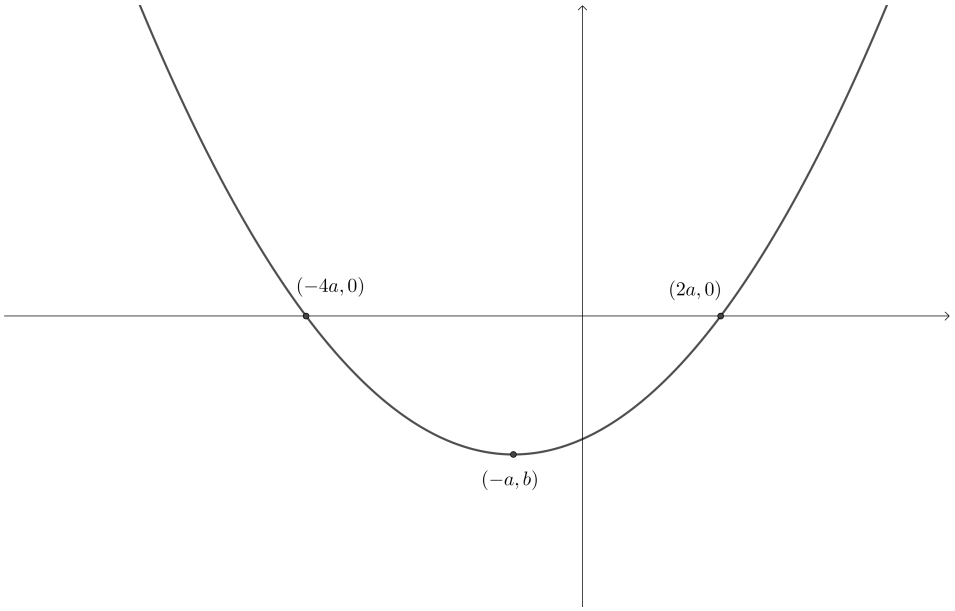
Solve:

$$3 + 3\cos x = 2\sin^2 x$$

for $0 \leqslant x \leqslant 4\pi$.

3. [Maximum mark: 5]

The diagram below shows the graph of a function $f(x)$.

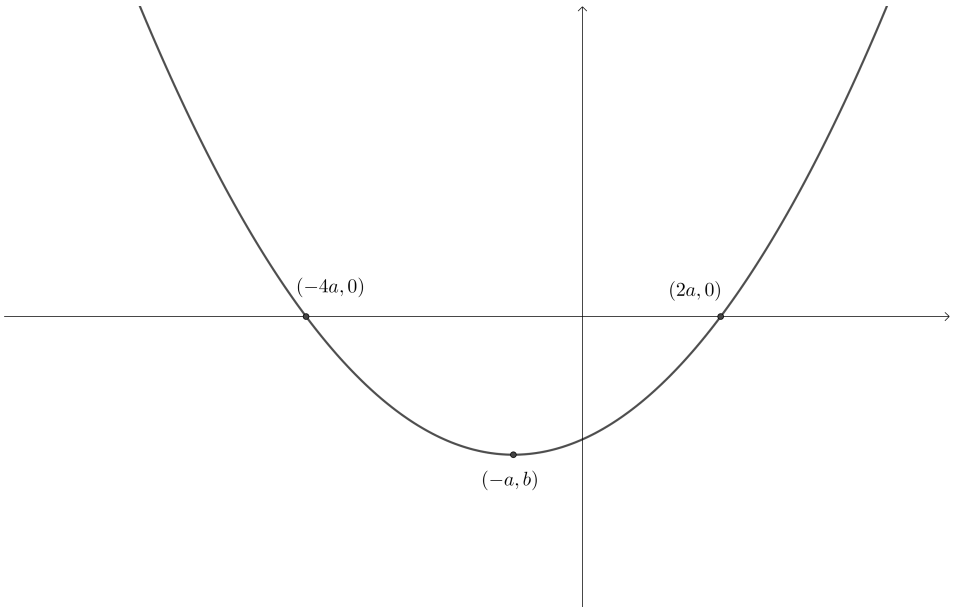


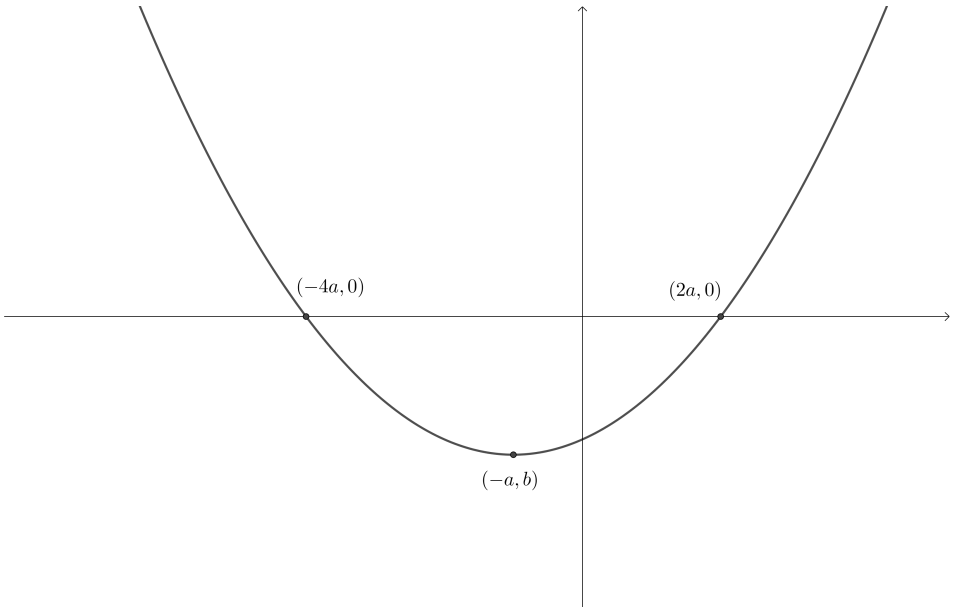
with $a > 0$ and $b < -1$.

On the following diagrams sketch the graphs of

(i) $g(x) = |f(-2x)|$ (ii) $h(x) = \dfrac{1}{f(x-2a)}$

Clearly indicate all the x-axis intercepts, maxima and minima and asymptotes.




$(-4a, 0)$
$(2a, 0)$
$(-a, b)$

4. [Maximum mark: 6]

Consider the function

$$f(x) = \sqrt{\arcsin x + \frac{\pi}{6}}$$

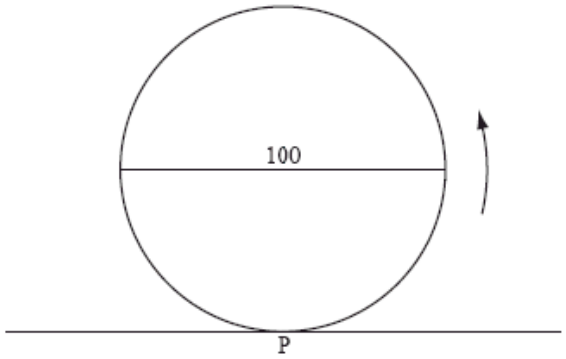
(a) Find the domain and range of $f(x)$.

(b) Find the $f^{-1}(x)$, the inverse of $f(x)$.

(c) Write down the domain and range of $f^{-1}(x)$.

5. [Maximum mark: 7]

The following diagram represents a large Ferris wheel, with a diameter of 100 metres.



Let P be a point on the wheel. The wheel starts with P at the lowest point, at ground level. The wheel rotates at a constant rate, in an counter-clockwise direction. One revolution takes 20 minutes.

(a) Write down the height of P above ground level after

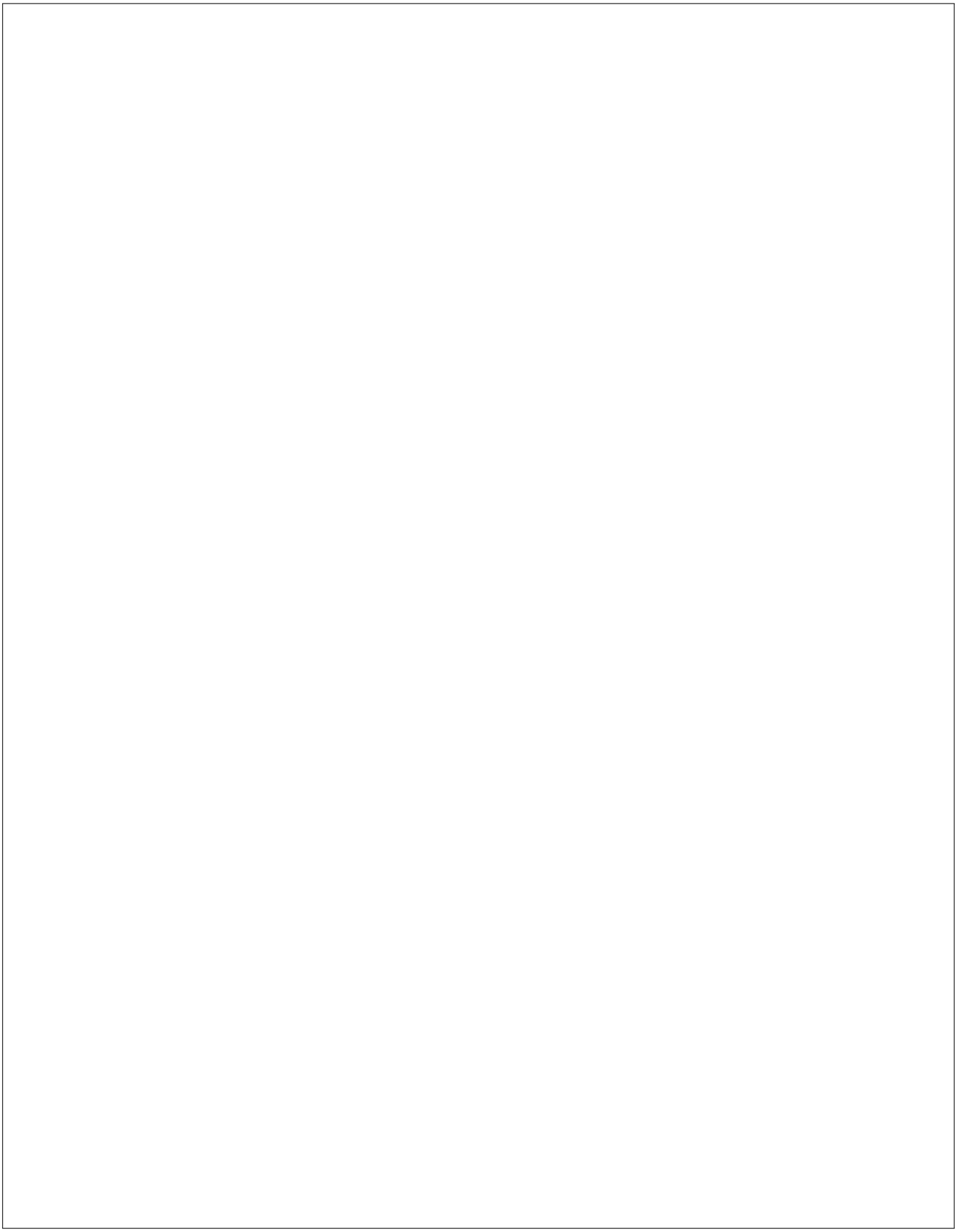
(i) 10 minutes;

(ii) 15 minutes;

Let $h(t)$ metres be the height of P above ground level after t minutes.

(b) Given that h can be expressed in the form $h(t) = a\cos bt + c$, find a, b and c.

(c) Sketch the graph of $h(t)$ for $0 \leqslant t \leqslant 40$.



6. [Maximum mark: 6]

(a) Solve:

$$\frac{x-3}{x+1} = -x$$

(b) Sketch the graphs of $y = \frac{x-3}{x+1}$ and $y = -x$ on the same set of axes. Indicate any asymptotes, axes intercepts and points of intersections of the graphs.

(c) State the solutions to:

$$\frac{x-3}{x+1} > -x$$

7. [Maximum mark: 6]

(a) Show that $\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \equiv \frac{4}{\sin^2 2x}$.

(b) Hence find the exact solutions to the equation

$$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{16}{3}$$

for $-\pi < x < \pi$.

8. [Maximum mark: 7]

Let $f(x) = \dfrac{x+1}{x^2+x-2}$.

(a) Sketch the graph $y = f(x)$. Clearly indicate all asymptotes and axes intercepts.

(b) Let $g(x) = f(|x|)$. State the domain and range of $g(x)$.

9. [Maximum mark: 7]

(a) Show that:

$$\cos\left(\arcsin x + \arcsin\left(\frac{x}{2}\right)\right) = \frac{\sqrt{1-x^2}\sqrt{4-x^2} - x^2}{2}$$

(b) **Hence** find the value of

$$\cos\left(\arcsin\left(\frac{3}{\sqrt{21}}\right) + \arcsin\left(\frac{3}{2\sqrt{21}}\right)\right)$$

(c) **Hence** write down the value of

$$\arcsin\left(\frac{3}{\sqrt{21}}\right) + \arcsin\left(\frac{3}{2\sqrt{21}}\right)$$

10. [Maximum mark: 6]

Let $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$ with $\tan\alpha = \frac{2}{3}$. Calculate:

(i) $\sin\alpha$

(ii) $\sin 2\alpha$

(iii) $\sin 3\alpha$

11. [Maximum mark: 14]

(a) By writing $3\theta = \theta + 2\theta$ show that:

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$$

(b) Hence solve the equation:

$$8\cos^3\theta - 6\cos\theta = 1$$

for $0 \leqslant \theta \leqslant 2\pi$.

(c) Show that:

$$\cos 5\theta \equiv 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

(d) Show that:

$$16\cos^5\theta \equiv \cos 5\theta + 5\cos 3\theta + 10\cos\theta$$

(e) Solve the equation:

$$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = -\frac{1}{2}$$

for $-2\pi \leqslant \theta \leqslant 2\pi$.