

11. [Maximum mark: 17]

(a) (i) Express  $x^2 + 3x + 2$  in the form  $(x + h)^2 + k$ .

(ii) Factorize  $x^2 + 3x + 2$ . [2]

Consider the function  $f(x) = \frac{1}{x^2 + 3x + 2}$ ,  $x \in \mathbb{R}$ ,  $x \neq -2$ ,  $x \neq -1$ .

(b) Sketch the graph of  $f(x)$ , indicating on it the equations of the asymptotes, the coordinates of the  $y$ -intercept and the local maximum. [5]

(e) Sketch the graph of  $y = f(|x|)$ . [2]

9. [Maximum mark: 14]

A quadratic function  $f$  can be written in the form  $f(x) = a(x - p)(x - 3)$ . The graph of  $f$  has axis of symmetry  $x = 2.5$  and  $y$ -intercept at  $(0, -6)$ .

(a) Find the value of  $p$ . [3]

(b) Find the value of  $a$ . [3]

(c) The line  $y = kx - 5$  is a tangent to the curve of  $f$ . Find the values of  $k$ . [8]

Hint: Tangent – there is exactly one point of intersection.

9. [Maximum mark: 17]

Consider the function  $f$  defined by  $f(x) = x^2 - a^2$ ,  $x \in \mathbb{R}$  where  $a$  is a positive constant.

(a) Showing any  $x$  and  $y$  intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes.

(i)  $y = f(x)$ ;

(ii)  $y = \frac{1}{f(x)}$ ;

(iii)  $y = \left| \frac{1}{f(x)} \right|$ . [8]

10. [Maximum mark: 15]

Let  $f(x) = \ln x$  and  $g(x) = 3 + \ln\left(\frac{x}{2}\right)$ , for  $x > 0$ .

The graph of  $g$  can be obtained from the graph of  $f$  by two transformations:

a horizontal stretch of scale factor  $q$  followed by  
a translation of  $\begin{pmatrix} h \\ k \end{pmatrix}$ .

(a) Write down the value of

(i)  $q$ ;

(ii)  $h$ ;

(iii)  $k$ .

[3]