

**INTERNATIONAL BACCALAUREATE****MATHEMATICS**

Higher Level

Tuesday 13 May 1997 (afternoon)

Paper 1

2 hours

This examination paper consists of 20 questions.

The maximum mark for each question is 4.

The maximum mark for this paper is 80.

This examination paper consists of 14 pages.

INSTRUCTIONS TO CANDIDATES

Write your candidate reference number in the box:

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DO NOT open this examination paper until instructed to do so.

Answer ALL questions in the spaces provided.

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

EXAMINATION MATERIALS

Required/Essential:

IB Statistical Tables
Electronic calculator
Ruler and compasses

Allowed/Optional:

A simple translating dictionary for candidates not working in their own language
Millimetre square graph paper

FORMULAE

Trigonometrical identities:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2}$$

$$\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta = \cos^2 \theta - \sin^2 \theta$$

$$\text{If } \tan \frac{\theta}{2} = t \text{ then } \sin \theta = \frac{2t}{1+t^2} \text{ and } \cos \theta = \frac{1-t^2}{1+t^2}$$

Integration by parts:

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Standard integrals:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c \quad (|x| < a)$$

Statistics:

If (x_1, x_2, \dots, x_n) occur with frequencies (f_1, f_2, \dots, f_n) then the mean m and standard deviation s are given by

$$m = \frac{\sum f_i x_i}{\sum f_i} \quad s = \sqrt{\frac{\sum f_i (x_i - m)^2}{\sum f_i}}, \quad i = 1, 2, \dots, n$$

Binomial distribution:

$$p_x = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

Maximum marks will be given for correct answers. Where an answer is wrong some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary, or on extra sheets of paper provided these are securely fastened to the cover sheet together with this examination paper.

1. Three 3×3 matrices A , B and C are given by

$$A = \begin{pmatrix} 1 & p & q \\ 1 & 0 & r \\ 0 & 0 & 2 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } C = \begin{pmatrix} -1 & -1 & 10 \\ 2 & 0 & -6 \\ 0 & 0 & 2 \end{pmatrix}.$$

Find p , q and r so that $AB = C$.

Working:

Answers:

2. The coordinates of points P and Q are $(-1, 5, 7)$ and $(1, 2, 3)$, respectively, and O is the origin.

(a) Find the vectors \vec{OP} and \vec{OQ} .

(b) Calculate $\vec{OP} \times \vec{OQ}$.

(c) Calculate the area of the parallelogram which has adjacent sides \vec{OP} and \vec{OQ} .

<i>Working:</i>	<i>Answers:</i>
	(a) _____
	(b) _____
	(c) _____

3. The complex numbers z and w are such that $z = -1 + 3i$ and $w = 3 + i$.

Express zw and $\frac{z}{w}$ in the form $a + ib$ where a and b are real numbers and $i = \sqrt{-1}$.

<i>Working:</i>	<i>Answers:</i>

4. A bag contains 5 white and 7 black balls. If two balls are drawn at random without replacement, what is the probability that one of them is black and the other is white.

Working:

Answer:

5. Find, in terms of k , the area bounded by the curve $y = \sqrt{x}$, the x -axis and the line $x = k$, where $k > 0$.

Working:

Answer:

6. Find $f'(x)$ where

$$f(x) = x \ln x + e^{\sin x} + \arctan x .$$

Working:

Answer:

7. Find the indefinite integral

$$\int (x^2 - 1)^3 x \, dx .$$

Working:

Answer:

8. If 30% of college students do not graduate, find the probability that out of 6 randomly selected college students exactly 4 of them will graduate.

Working:

Answer:

9. Find all values of θ , so that $3 \sin^2 \theta - 7 \sin \theta + 5 = 3 \cos^2 \theta$, $0^\circ \leq \theta \leq 90^\circ$.

Working:

Answers:

10. The complex number $z = 1 + 2i$ is a solution of the equation $z^3 - 3z^2 + 7z - 5 = 0$. Find the other two solutions.

Working:

Answers:

11. The coordinates of P and Q are $(3, -1)$ and $(\lambda, -4 - \lambda)$, respectively, where λ is a constant. If O is the origin, find all values of λ for which \vec{OP} is perpendicular to \vec{OQ} .

Working:

Answer:

12. The table below shows the number of components produced by machines *A* and *B*, and the probability of machines *A* and *B* producing faulty components.

Machine	Number of components produced	Probability of producing faulty components
<i>A</i>	2500	0.04
<i>B</i>	1500	0.05

- (a) If a component is chosen at random from the total number of components produced, what is the probability that it is faulty?
- (b) If a component is selected at random and it is found to be faulty, what is the probability that it is produced by machine *A*?

Working:

Answers:

- (a) _____
- (b) _____

13. Find the indefinite integral

$$\int x^2 e^{-2x} dx .$$

Working:

Answer:

14. A zoologist knows that the lengths of a certain type of tropical snake are normally distributed with mean length L metres and standard deviation 0.12 metres. If 20% of the snakes are longer than 0.70 metres, find the value of L .

Working:

Answer:

15. Find the term independent of x in the binomial expansion of

$$\left(\frac{9}{2}x^2 - \frac{1}{9x}\right)^9.$$

Working:

Answer:

16. A curve in the plane has the equation $xy^2 + x^2y = 2$.

- (a) Find the gradient (slope) of the curve at the point $(1, 1)$.
(b) Find the equation of the line which is perpendicular to the curve at the point $(1, 1)$.

Working:

Answers:

- (a) _____
(b) _____

17. Consider the differential equation

$$\cos^2 x \frac{dy}{dx} = 1 - y, \quad -\frac{\pi}{2} < x < \frac{\pi}{2},$$

with $y = 2$ when $x = 0$.

- (a) Find the integrating factor for the above differential equation.
- (b) Find the solution of the differential equation.

Working:

Answers:

(a) _____

(b) _____

18. An airplane flying horizontally at an altitude of 10 000 metres at a speed of 800 km per hour passes directly over a radar station.

Find the exact value of the rate at which the distance from the airplane to the radar station is increasing when it is 2 km away from the point vertically above the radar station.

Working:

Answer:

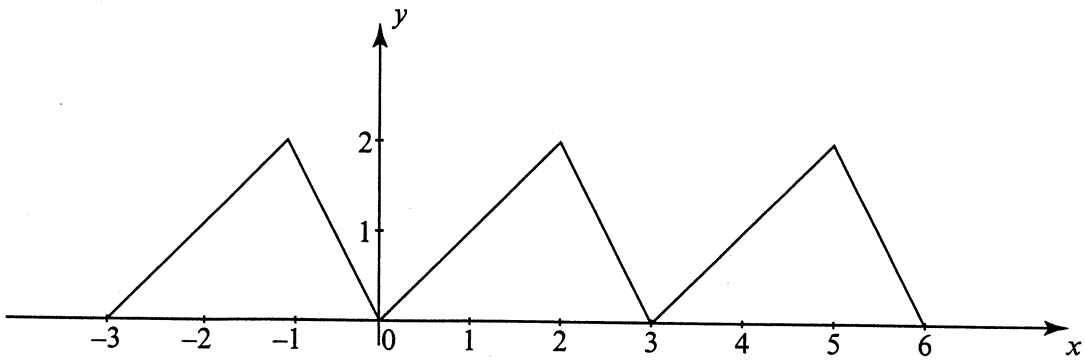
19. Find the exact value of

$$\sin \left(\arccos \frac{2}{3} + \arcsin \frac{1}{4} \right).$$

Working:

Answer:

20.



The diagram above shows the graph of a periodic function over the interval $-3 \leq x \leq 6$.

- (a) What is its period?
- (b) Assuming that the graph of the function continues to have the same form over the interval $-30 \leq x \leq 60$, find $f(52)$.
- (c) Find $f'(26\frac{1}{2})$.

Working:

Answers:

- (a) _____
- (b) _____
- (c) _____