

SL 5.5 Integration

SL 5.8 Area under curve

Introduction

The purpose of this presentation is to review the chapters on integration.

The presentation is divided into three parts:

- Approximation of the area under the curve using trapezoidal rule.
- Calculating the anti-derivatives (indefinite integrals) of a given function.
- Using the definite integrals to calculate the exact area under the curve.

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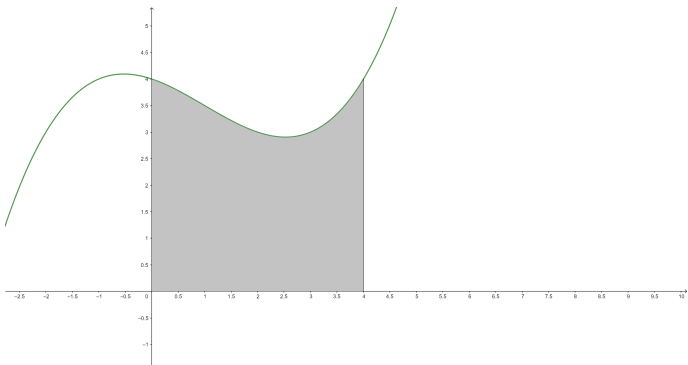
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TRAPEZOIDAL RULE

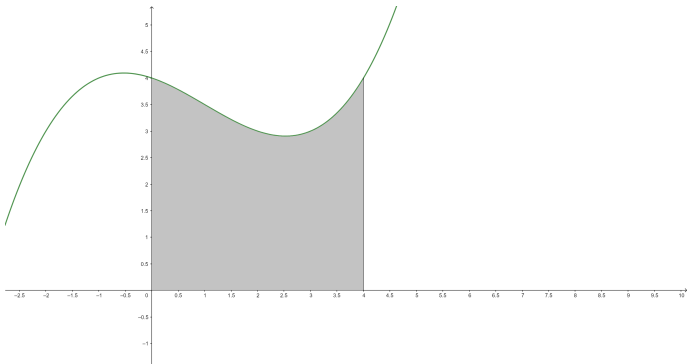
Suppose we want to calculate the shaded area:



Which is the area under the graph of $y = \frac{1}{12}x^3 - \frac{1}{4}x^2 - \frac{1}{3}x + 4$ between $x = 0$ and $x = 4$.

The trapezoidal rule is a rule for approximating this area using (yes, you guessed it) trapeziums.

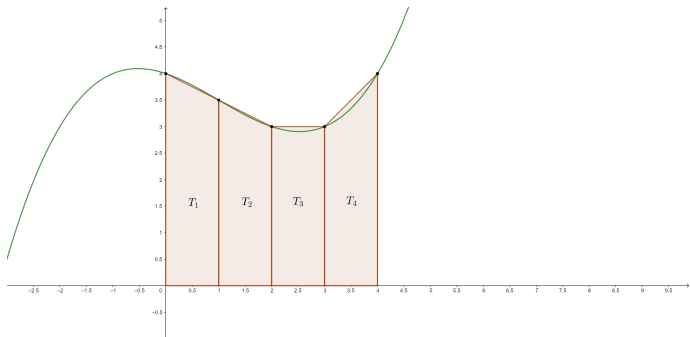
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We will start by dividing the area into 4 trapeziums as follows

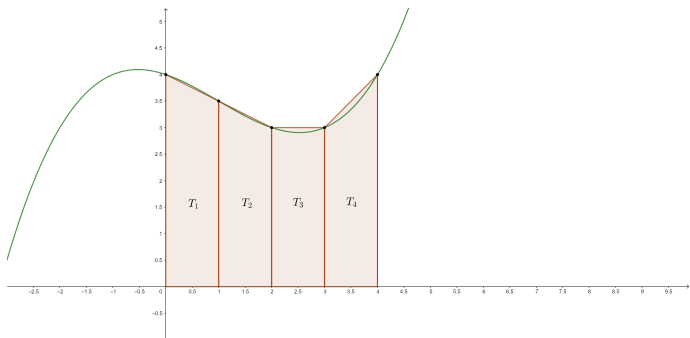


We can then say that:

$$\text{Area} \approx T_1 + T_2 + T_3 + T_4$$

Note that I've used \approx because the area is not exactly equal to the sum of the areas of the trapeziums. You can see that on the diagram.

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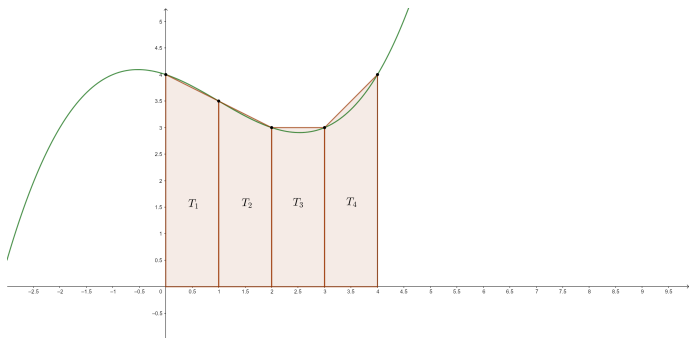


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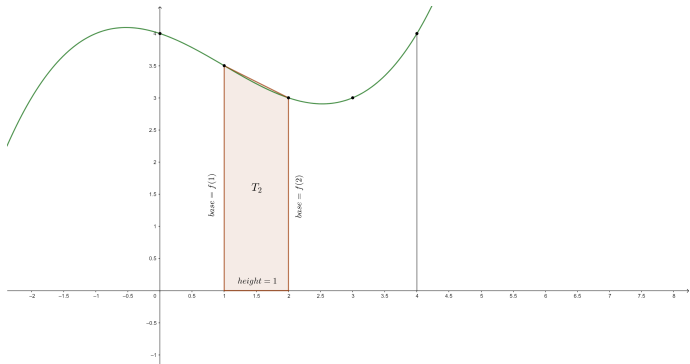


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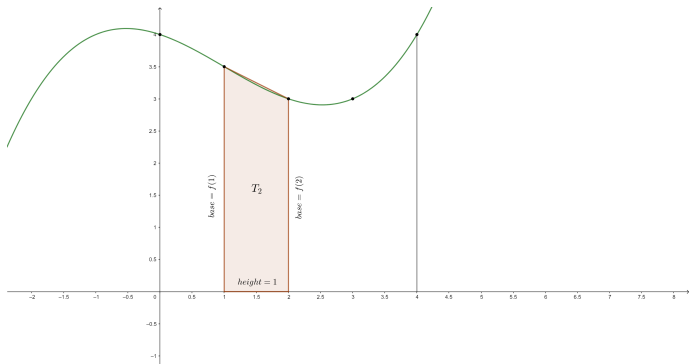
The area of each trapezium is easy to calculate. Let's take a look at T_2 :



We should think of it as a trapezium rotated by 90° .

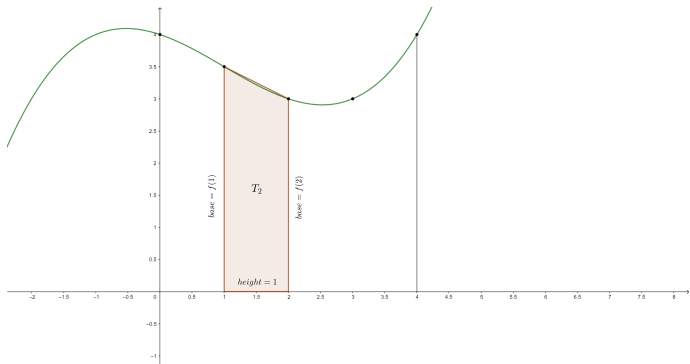
The formula for the area of the trapezium is:

$$A = \frac{(a + b) \cdot h}{2}$$



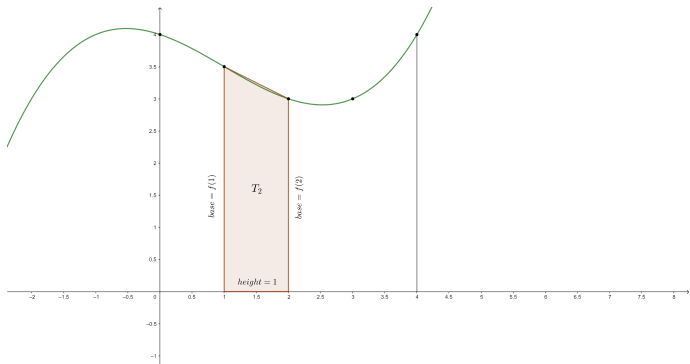
Note that in our case of T_2 we have $a = f(1)$, $b = f(2)$ and $h = 1$. We can calculate a and b by substituting 1 and 2 into the formula for our function. We have $a = 3.5$ and $b = 3$.

$$T_2 = \frac{(3.5 + 3) \cdot 1}{2} = 3.25$$



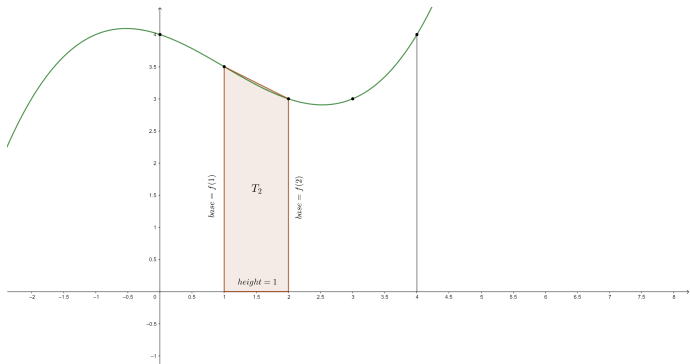
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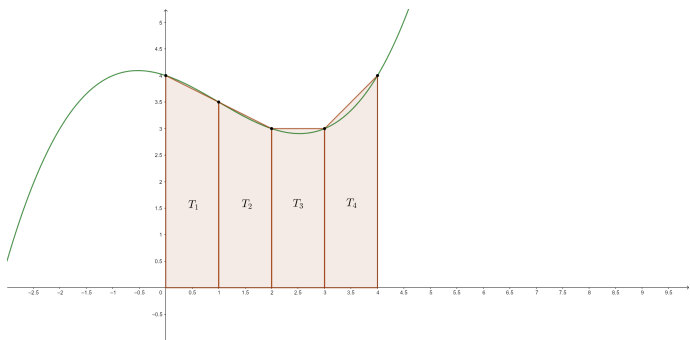
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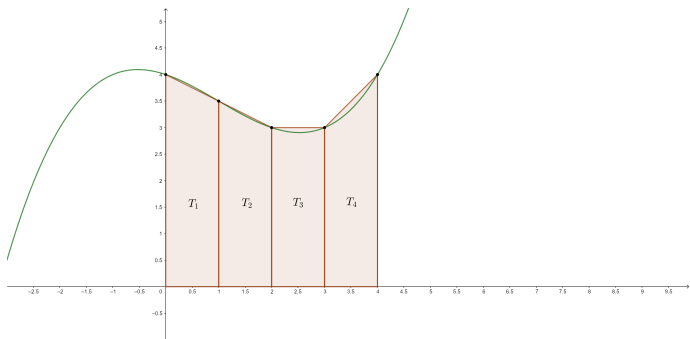
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We can calculate the remaining trapeziums in an analogous way and we get $T_1 = 3.75$, $T_3 = 3$ and $T_4 = 3.5$.

So the area under the curve is approximately equal to:

$$\text{Area} \approx 3.75 + 3.25 + 3 + 3.5 = 13.5$$



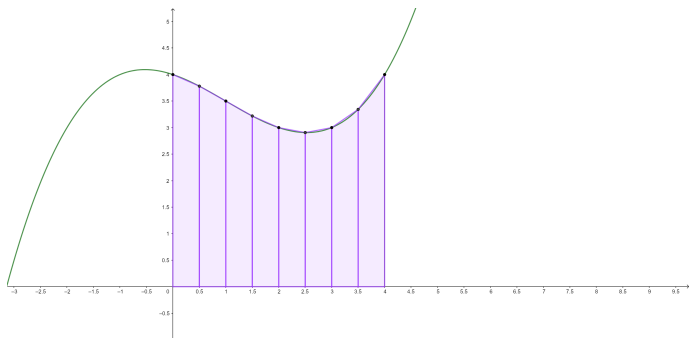
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So the area under the curve is approximately equal to:

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This is an approximation of the area and this approximation can be improved by considering more trapeziums. We chose 4, but we could have chosen 8 or any other number.

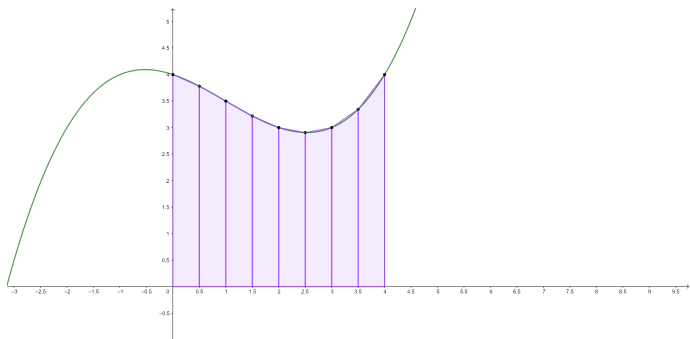
Note that with 8 trapeziums the area fits much better:



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Formula

Instead of calculating the area of each trapezium separately we can apply the formula:

$$\text{Area} \approx \frac{h}{2} \left(f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n) \right)$$

Here h is the height of each trapezium and x_i s are the x -coordinates of the vertices of the trapezium on the horizontal axis.

Note that this formula is what we would get if we added the formulas for the areas of each trapezium, so there is no need to calculate the areas separately.

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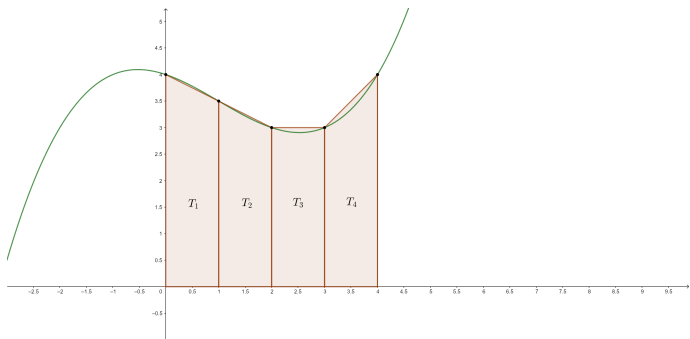
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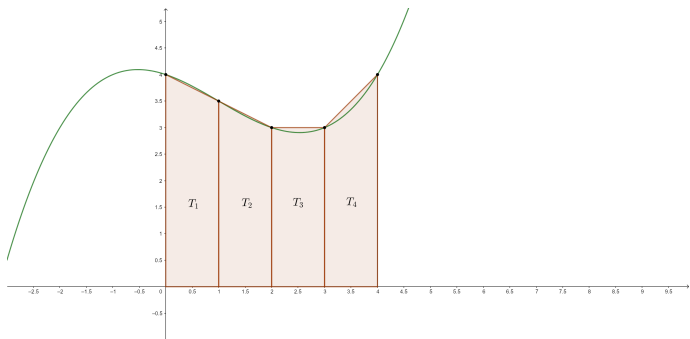
In our example:



We have $h = 1$ and the x_i s are 0, 1, 2, 3 and 4, so the area is:

$$\begin{aligned} \text{Area} &\approx \frac{1}{2} \left(f(0) + 2f(1) + 2f(2) + 2f(3) + f(4) \right) = \\ &= \frac{1}{2} \left(4 + 7 + 6 + 6 + 4 \right) = 13.5 \end{aligned}$$

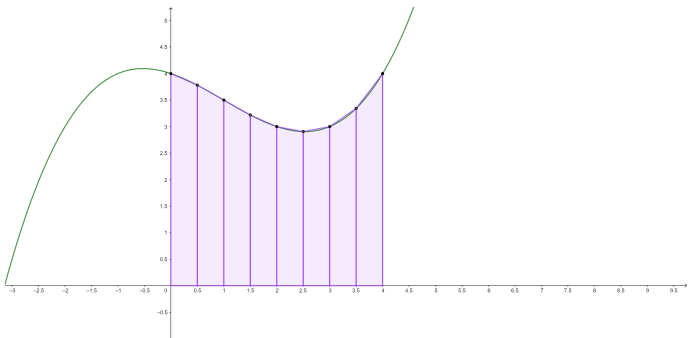
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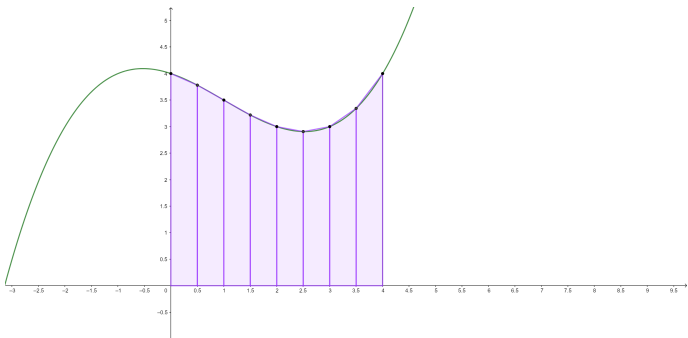
Had we used 8 trapezium:



We would have $h = 0.5$ and the x 's are $0, 0.5, 1, 1.5, \dots, 4$ and the area would be:

$$\begin{aligned} \text{Area} &\approx \frac{0.5}{2} \left(f(0) + 2f(0.5) + 2f(1) + 2f(1.5) + \dots + 2f(3.5) + f(4) \right) = \\ &= \frac{1}{4} \left(4 + 7.5625 + 7 + 6.4375 + \dots + 6.6875 + 4 \right) = 13.375 \end{aligned}$$

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GDC

Quick tip on the use of GDC.

If you need to evaluate function several times (i.e. calculate $f(1)$, then $f(2)$, then $f(3)$ etc.), it makes sense to store the function under $Y1$ (in the GRAPH option) and then use $\text{VARS} \rightarrow \text{GRPH} \rightarrow Y1$. Then you can do $Y1(1)$ to evaluate the function at $x = 1$, $Y1(2)$ to evaluate at $x = 2$ etc.

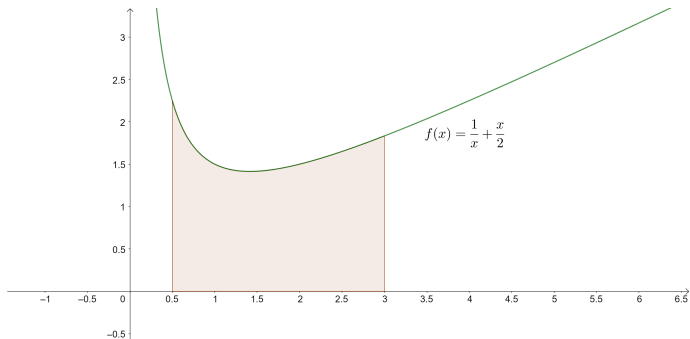
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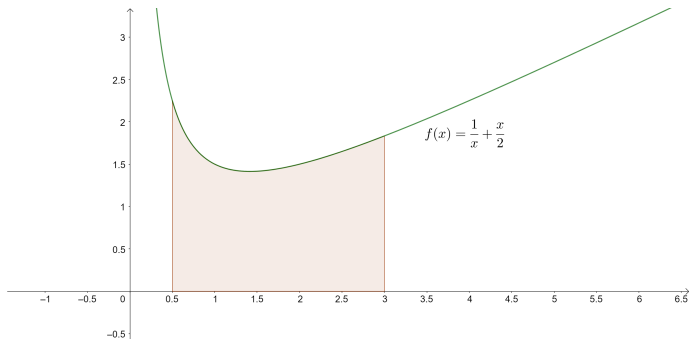
Approximate the given area using 5 trapeziums.



The x is between 0.5 and 3 and we want 5 trapeziums so each trapezium will have height of 0.5 and the x s that we will use will be 0.5, 1, 1.5, 2, 2.5 and 3.

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We now use the formula:

$$\begin{aligned} \text{Area} &\approx \frac{h}{2} \left(f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n) \right) = \\ &= \frac{0.5}{2} \left(f(0.5) + 2f(1) + 2f(1.5) + 2f(2) + 2f(2.5) + f(3) \right) = \\ &= 4.0541(6) \end{aligned}$$

and this is our approximation.

Now we want to review how to calculate the exact area.

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INDEFINITE INTEGRALS

Anti-differentiation

Given a function $f(x)$ its anti-derivative is a function $F(x)$ such that $F'(x) = f(x)$.

Note that the anti-derivative is not unique.

For example:

$$(2x^3)' = 6x^2 \text{ but also } (2x^3 + 17)' = 6x^2 \text{ and } (2x^3 - 100)' = 6x^2.$$

So the anti-derivative of $f(x) = 6x^2$ can be written as $F(x) = 2x^3 + c$, where c is any constant.

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Terminology

Another name for anti-derivative is **integral** and the process of finding the anti-derivative is called anti-differentiation or more commonly **integration**. We use \int to denote the process of integration.

Important formula

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c$$

Examples:

- $\int x^2 dx = \frac{1}{3}x^3 + c,$
- $\int x^5 dx = \frac{1}{6}x^6 + c,$
- $\int x dx = \int x^1 dx = \frac{1}{2}x^2 + c,$
- $\int 1 dx = \int x^0 dx = x + c,$
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Integration rules

Recall that we have the following rules for differentiation:

$$(cf(x))' = cf'(x) \quad \text{and} \quad (f(x) \pm g(x))' = f'(x) \pm g'(x)$$

So for example: $(7x^3)' = 21x^2$ and $(5x^2 - x^4)' = 10x - 4x^3$.

It follows from these two that we also must have:

$$\int cf(x) dx = c \int f(x) dx$$

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$$\begin{aligned}\int x^3 + 4x - 5x^2 \, dx &= \int x^3 \, dx + 4 \int x \, dx - 5 \int x^2 \, dx = \\ &= \frac{1}{4}x^4 + 4 \cdot \frac{1}{2}x^2 - 5 \cdot \frac{1}{3}x^3 + c = \\ &= \frac{1}{4}x^4 + 2x^2 - \frac{5}{3}x^3 + c.\end{aligned}$$

Of course when you calculate the integral you do not need to write down all the intermediate step, you can go directly to the last line.

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Of course when you calculate the integral you do not need to write down all the intermediate step, you can go directly to the last line.

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Calculate the following integrals:

$$\int 3x^2 - 2x \, dx = x^3 - x^2 + c$$

$$\int 2x^4 + x^{-2} \, dx = \frac{2}{5}x^5 - x^{-1} + c$$

$$\int x - 2 + x^3 \, dx = \frac{1}{2}x^2 - 2x + \frac{1}{4}x^4 + c$$

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Note that in all our examples we had to include c .

This is because the derivative of a constant is 0, so the derivative of $x^2 + 7$ is the same as that of $x^2 - 5$ or simply x^2 . In all these cases the derivative is $2x$, so the anti-derivative (integral) of $2x$ can be any of these.

That is why we write $\int 2x \, dx = x^2 + c$, where c can be 7 or -5 or any other constant.

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So our function is $f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + c$, but we also know that $f(2) = 5$. So we substitute this into our formula and get:

$$5 = \frac{1}{4} \cdot 2^4 - \frac{1}{2} \cdot 2^2 + c$$

This allows us to calculate that $c = 3$.

Integrals with initial conditions

Consider the following example.

Find $f(x)$ if $f'(x) = x^3 - x$ and $f(2) = 5$.

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Integrals with initial conditions

So the answer to the problem:

Find $f(x)$ if $f'(x) = x^3 - x$ and $f(2) = 5$.

is the function $f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + 3$.

Integrals with initial conditions - practice

Find $f(x)$ if $f'(x) = 4x^3 - 5$ and $f(1) = 2$.

$$f(x) = \int 4x^3 - 5 \, dx = x^4 - 5x + c$$

Using $f(1) = 2$:

$$2 = 1^4 - 5 \cdot 1 + c$$

So $c = 6$, which means that $f(x) = x^4 - 5x + 6$.

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Integrals with initial conditions - practice

Find y as a function of x given that $\frac{dy}{dx} = x^2 - 4$ and $y = 2$, when $x = 3$.

Note that this is exactly the same problem, only different notation is used.

$$y = \int x^2 - 4 \, dx = \frac{1}{3}x^3 - 4x + c$$

Using $y = 2$ when $x = 3$:

$$2 = \frac{1}{3} \cdot 3^3 - 4 \cdot 3 + c$$

So $c = 5$, which means that $y = \frac{1}{3}x^3 - 4x + 5$.

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Integrals with initial conditions - practice

Find y as a function of x given that $\frac{dy}{dx} = \frac{2}{x^3} + x$ and the point $(1, 5)$ lies on the graph of this function.

$$y = \int \frac{2}{x^3} + x \, dx = \int 2x^{-3} + x \, dx = -x^{-2} + \frac{1}{2}x^2 + c = -\frac{1}{x^2} + \frac{1}{2}x^2 + c$$

Using the point $(1, 5)$, so $y = 5$ when $x = 1$:

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DEFINITE INTEGRALS

Definite integrals

A definite integral is an integral over a specific interval. It is denoted by including the boundaries of the interval above and below the integral sign. The definite integral of x^2 between 1 and 3 (so over the interval $[1, 3]$) is denoted:

$$\int_1^3 x^2 dx$$

A definite integral is calculated as follows:

$$\int_1^3 x^2 dx = \left[\frac{1}{3} x^3 \right]_1^3 = \left(\frac{1}{3} \cdot 3^3 \right) - \left(\frac{1}{3} \cdot 1^3 \right) = 26 \frac{2}{3}$$

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Definite integrals

Let's go through this step by step:

$$\int_1^3 x^2 dx = \left[\frac{1}{3} x^3 \right]_1^3$$

First we calculate the indefinite integral. Note that c is omitted, we will come back to this later.

$$\left(\frac{1}{3} \cdot 3^3 \right)$$

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Then we substitute the boundary values into the indefinite integrals. And finally we subtract these values:

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Definite integrals - GDC

The definite integrals can be calculated on the GDC:

CASIO

MENU → RUN-MAT → MATH (F4) → F6 → \int (F1)

Ti-84

MATH → 9:fnInt()

Then you need to enter the function you want to integrate and the boundaries. Check our answer to $\int_1^3 x^2 dx$.

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On the next slide we will practice definite integrals.

In each case you should try doing the calculations by hand and then check the result on GDC.

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Calculate the following:

$$\int_0^2 x^3 dx = \left[\frac{1}{4}x^4 \right]_0^2 = \left(\frac{1}{4} \cdot 2^4 \right) - \left(\frac{1}{4} \cdot 0^4 \right) = 4$$

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Fundamental Theorem of Calculus

Now we come to the part that will link all these pieces together.

Recall that what we want to be able to do is to calculate the area under the graph of some function $f(x)$ (here we will assume that the graph of the function lies above the x -axis).

The Fundamental Theorem of Calculus states that the area under the graph of $f(x)$ between $x = a$ and $x = b$ is equal to the definite integral

$$\int_a^b f(x) dx$$

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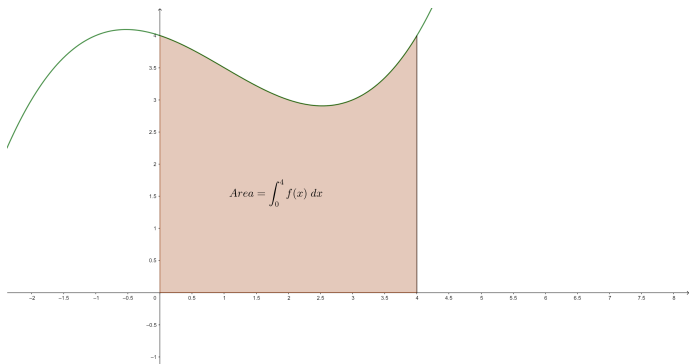
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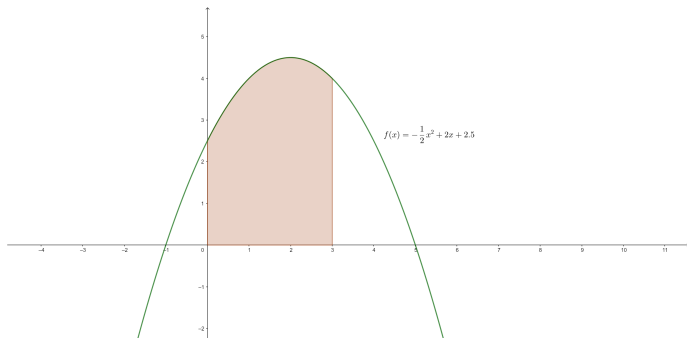
Coming back to our initial example:



The area can be calculated **exactly** using integral. We can do this by hand (or better using GDC) and we get that the area is equal to $13\frac{1}{3}$ units².

Practice problems

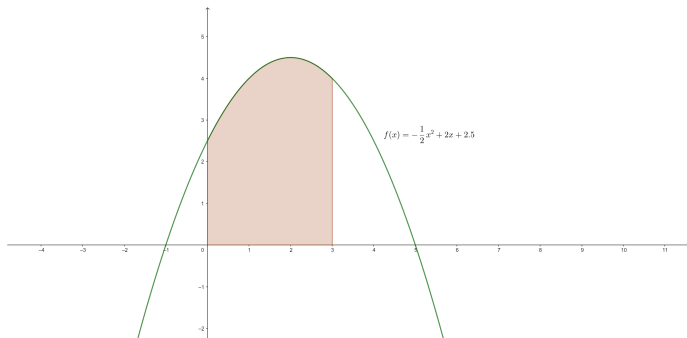
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The area is given by $Area = \int_0^3 -\frac{1}{2}x^2 + 2x + 2.5 \, dx = 12$.

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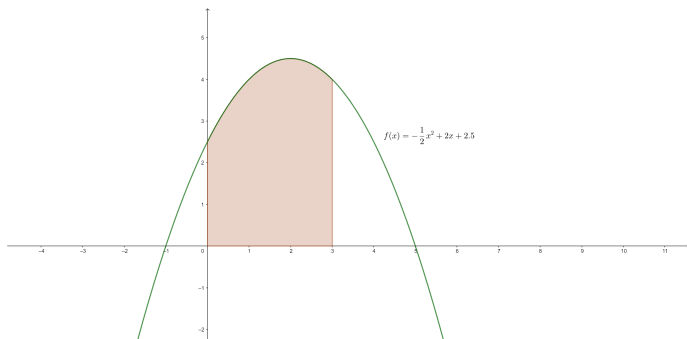
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Practice problems

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GDC

We can calculate the areas on the GDC directly, but we can also use graphs to do this.

CASIO MENU \rightarrow GRAPH \rightarrow $y1 = \dots$ \rightarrow DRAW (F6) \rightarrow G-SOLVE (F5)
 \rightarrow F6 \rightarrow \int (F3)

Then you need to input the boundaries for example: 0 EXE, 3 EXE.

TI - 84

$Y=$ \rightarrow $y1 = \dots$ \rightarrow GRAPH \rightarrow CALC \rightarrow 7: $\int f(x) dx$

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→ F6 → \int (F3)

Then you need to input the boundaries for example: 0 EXE, 3 EXE.

T1 - 84

$Y=$ → $y1=\dots$ → GRAPH → CALC → 7: $\int f(x)dx$

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GDC

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Ti - 84

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On the next slides you will be asked to calculate the shaded areas.

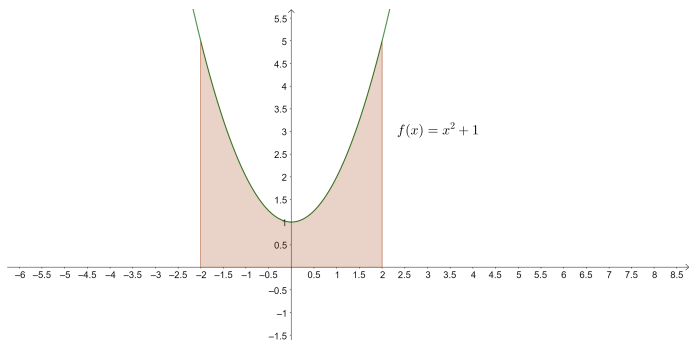
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Area under graph - practice

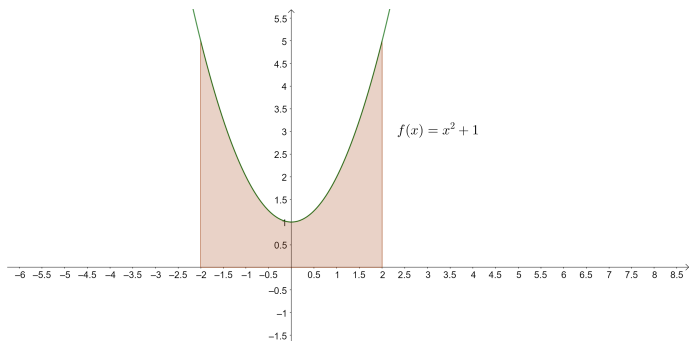
Calculate shaded area:



$$\text{Area} = \int_{-2}^2 x^2 + 1 \, dx = \frac{28}{3}$$

Area under graph - practice

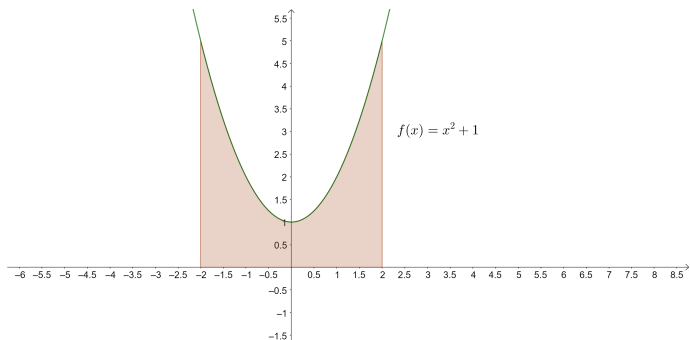
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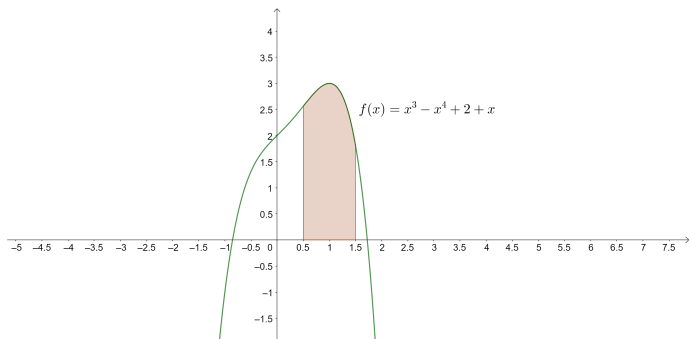
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Area under graph - practice

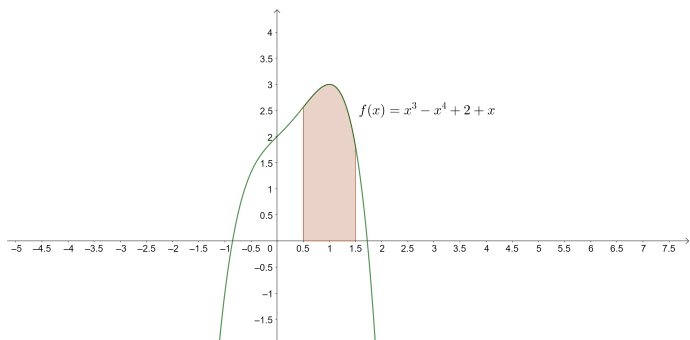
Calculate shaded area:



$$\text{Area} = \int_{0.5}^{1.5} x^3 - x^4 + 2 + x \, dx = 2.7375$$

Area under graph - practice

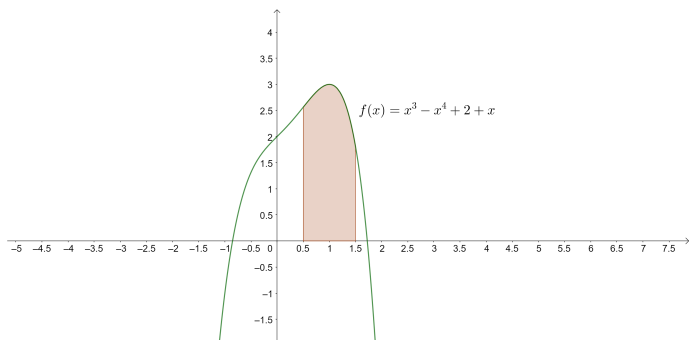
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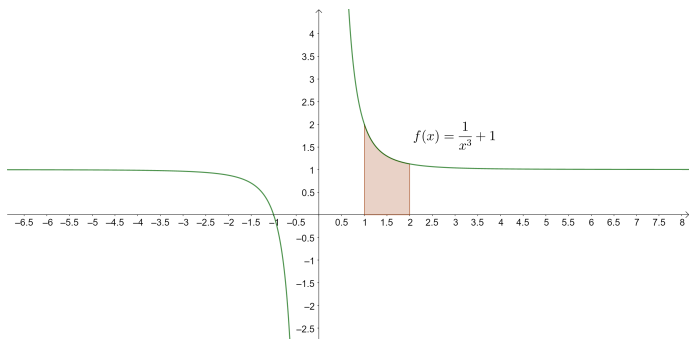
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Area under graph - practice

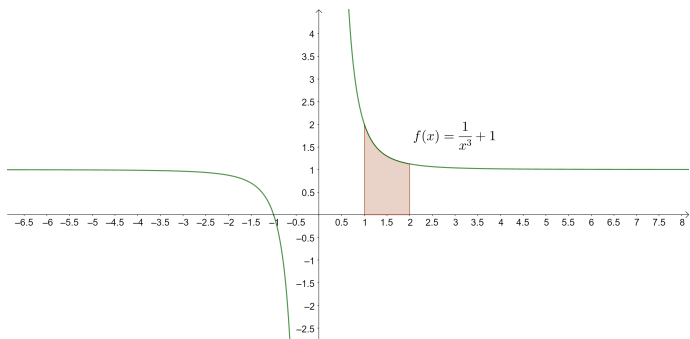
Calculate shaded area:



$$\text{Area} = \int_1^2 \left(\frac{1}{x^3} + 1 \right) dx = \frac{11}{8}$$

Area under graph - practice

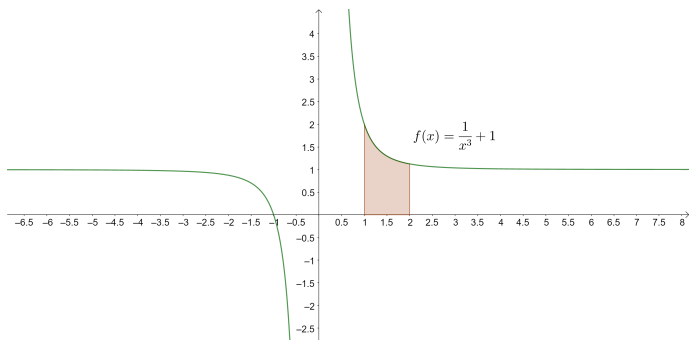
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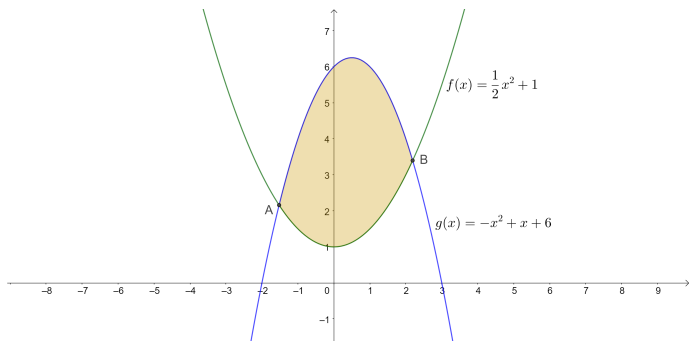
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Area between graphs

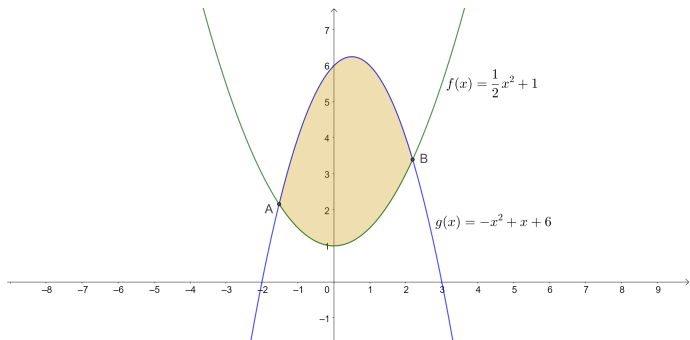
You may also be asked to calculate the area between graphs:



Here we need to do 4 steps.

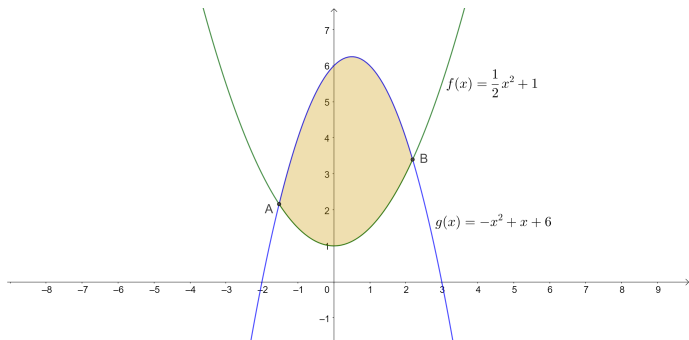
Area between graphs

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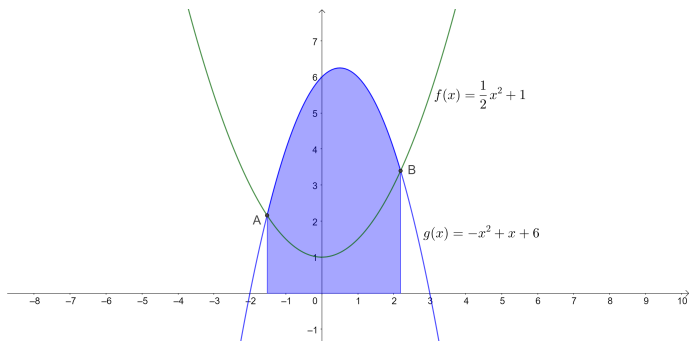
Here we need to do 4 steps.

Area between graphs - step 1



Step 1: Calculate where the graphs intersect. This can be done using graphs on GDC. In this example the graphs intersect at $x \approx -1.5226$ and at $x \approx 2.1893$.

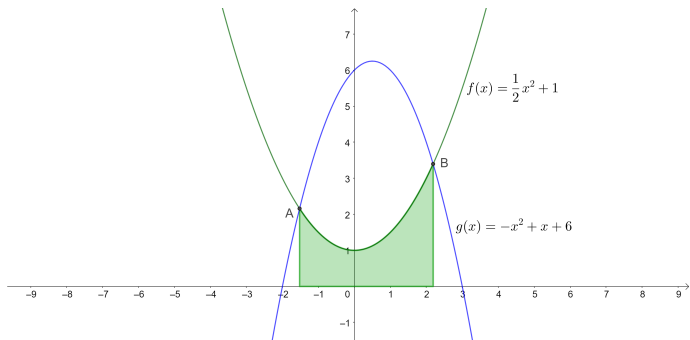
Area between graphs - step 2



Step 2: Calculate the area under the top function between the values you found in step 1.:

$$\text{Area}_g = \int_{-1.5226}^{2.1893} -x^2 + x + 6 \, dx \approx 18.83435$$

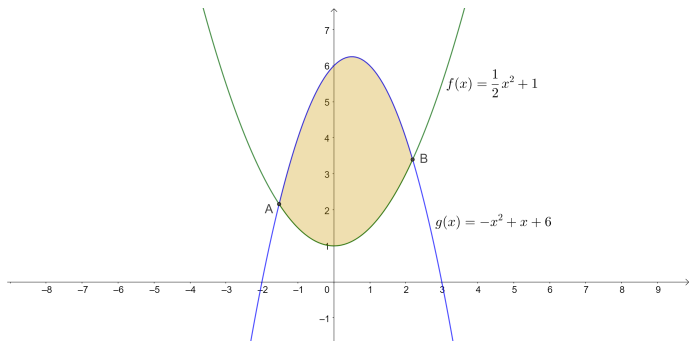
Area between graphs - step 3



Step 3: Calculate the area under the bottom function between the values you found in step 1.:

$$Area_f = \int_{-1.5226}^{2.1893} \frac{1}{2}x^2 + 1 \, dx \approx 6.049108$$

Area between graphs - step 4

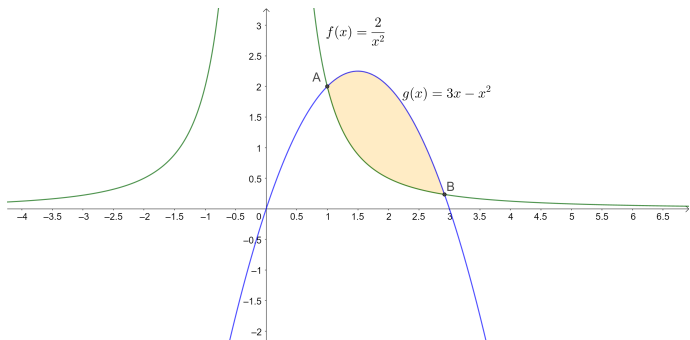


Step 4: The required area is the difference between the areas found in steps 2 and 3:

$$\text{Area} \approx 18.83435 - 6.049108 \approx 12.8$$

Area between graphs - practice

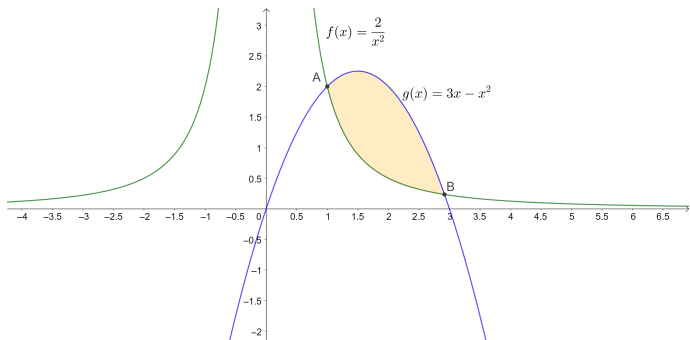
Find the shaded area:



$$\text{Area} = \int_1^{2.9196} (3x - x^2) dx - \int_1^{2.9196} \frac{2}{x^2} dx \approx 2.01$$

Area between graphs - practice

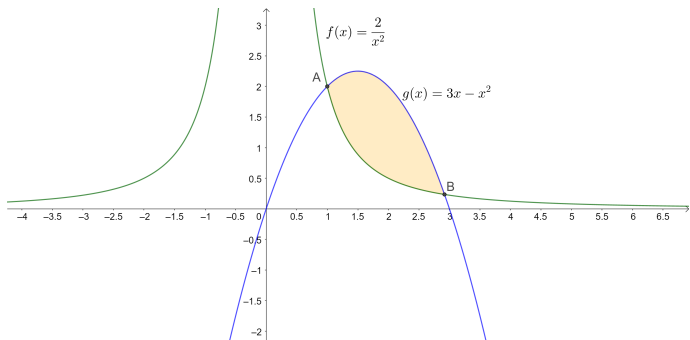
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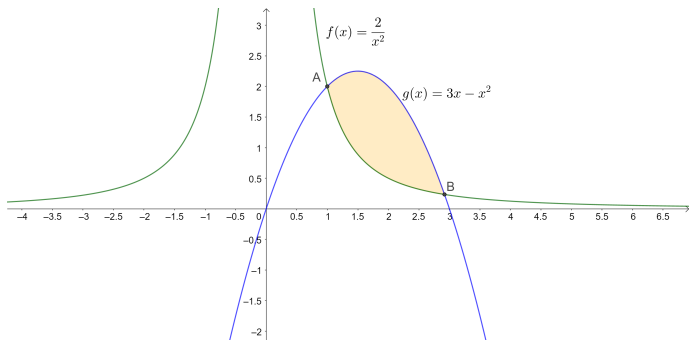
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Area between graphs - practice

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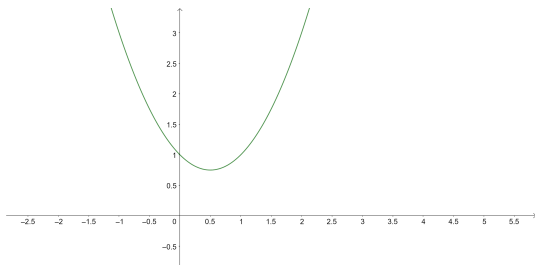


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EXAM-STYLE QUESTION

Exam-style question

Let $f(x) = x^2 - x + 1$. The graph of $y = f(x)$ is shown below.



Let A be the area under the graph between $x = 0$ and $x = 2$.

- Approximate A using trapezoidal rule using 4 trapeziums.
- Write down the integral that represents the exact value of A and calculate it.
- Calculate the percentage error of your approximation in part (a).

Exam-style question (a)

Because we have to use 4 trapeziums and the area is between $x = 0$ and $x = 2$, this means that the height of each trapezium is 0.5.

We can use the formula:

$$\begin{aligned} A &\approx \frac{h}{2}(f(0) + 2f(0.5) + 2f(1) + 2f(1.5) + f(2)) \\ &= \frac{1}{4}(1 + 1.5 + 2 + 3.5 + 3) = 2.75 \end{aligned}$$

Exam-style question (b)

We want the definite integral of $f(x)$ between $x = 0$ and $x = 2$:

$$A = \int_0^2 x^2 - x + 1 \, dx = \frac{8}{3}$$

Exam-style question (b)

We want the definite integral of $f(x)$ between $x = 0$ and $x = 2$:

$$A = \int_0^2 x^2 - x + 1 \, dx = \frac{8}{3}$$

Exam-style question (c)

The exact area was calculated in part (b), the approximated area (using trapezoidal rule) was calculated in part (a).

The percentage error of the approximation is equal to:

$$\epsilon_{\%} = \left| \frac{2.75 - \frac{8}{3}}{\frac{8}{3}} \right| \cdot 100\% = 3.125\%$$

Make sure you go through all the examples in this presentation.

If there are any questions, ask me in person or on Teams chat.

We will do some more exam-style questions in class.