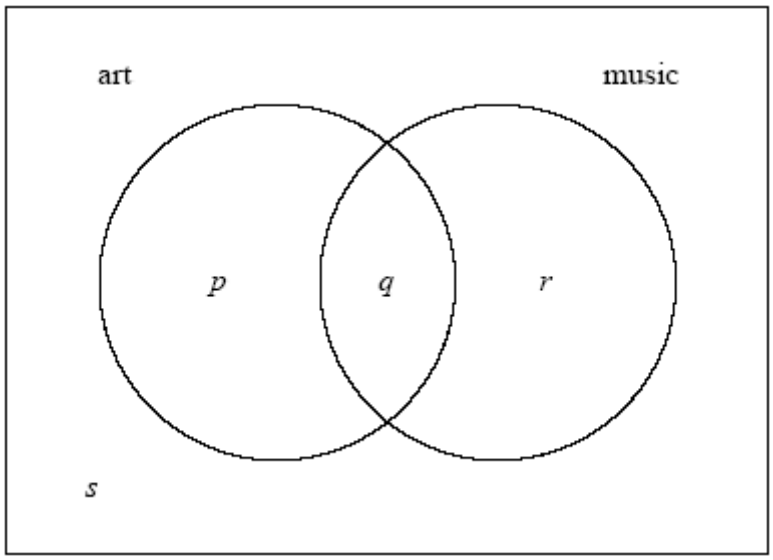
1. In a group of 16 students, 12 take art and 8 take music. One student takes neither art nor music. The Venn diagram below shows the events art and music. The values p, q, r and s represent numbers of students.



(a) (i) Write down the value of s.

(ii) Find the value of q.

(iii) Write down the value of p and of r.

(5)

(b) (i) A student is selected at random. Given that the student takes music, write down the probability the student takes art.

(ii) **Hence**, show that taking music and taking art are **not** independent events.

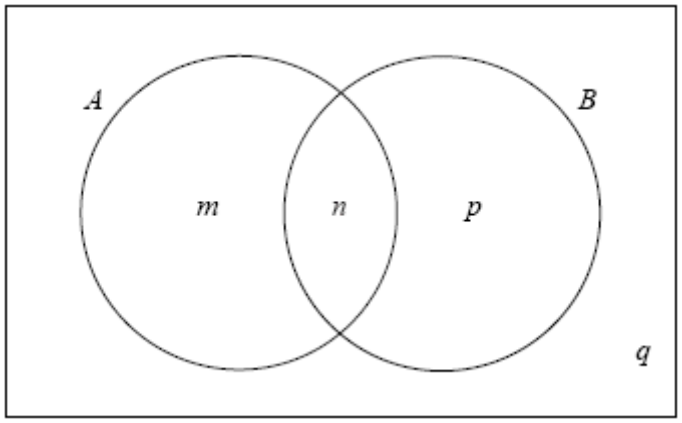
(4)

(c) Two students are selected at random, one after the other. Find the probability that the first student takes **only** music and the second student takes **only** art.

(4)

(Total 13 marks)

2. The Venn diagram below shows events A and B where $P(A) = 0.3$, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.1$. The values m, n, p and q are probabilities.



(a) (i) Write down the value of n.

(ii) Find the value of m, of p, and of q.

(4)

(b) Find $P(B')$.

(2)
(Total 6 marks)

3. A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.
90 % of the boxes made by machine B pass inspection.

A box is selected at random.

(a) Find the probability that it passes inspection.

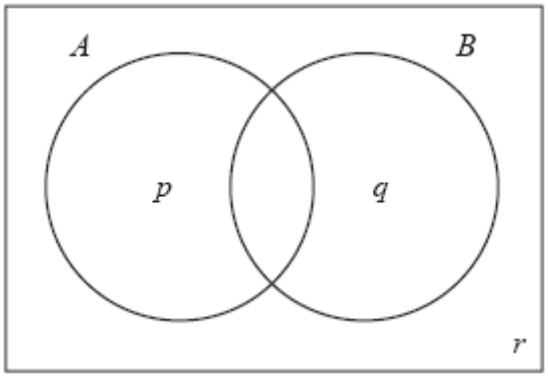
(3)

(b) The company would like the probability that a box passes inspection to be 0.87.
Find the percentage of boxes that should be made by machine B to achieve this.

(4)
(Total 7 marks)

4. Consider the events A and B, where $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$.

The Venn diagram below shows the events A and B, and the probabilities p, q and r.



(a) Write down the value of

(i) p;

(ii) q;

(iii) r.

(3)

(b) Find the value of $P(A \mid B')$.

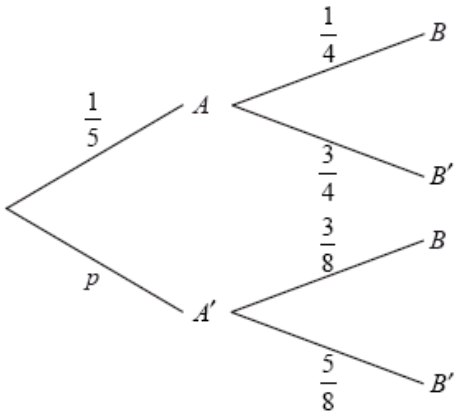
(2)

(c) Hence, or otherwise, show that the events A and B are **not** independent.

(1)

(Total 6 marks)

5. The diagram below shows the probabilities for events A and B, with $P(A') = p$.



(a) Write down the value of p.

(1)

(b) Find $P(B)$.

(3)

(c) Find $P(A' \mid B)$.

(3)

(Total 7 marks)

6. The letters of the word PROBABILITY are written on 11 cards as shown below.

Two cards are drawn at random without replacement.
Let A be the event the first card drawn is the letter A.
Let B be the event the second card drawn is the letter B.

(a) Find P(A).

(1)

(b) Find P($B \mid A$).

(2)

(c) Find P($A \cap B$).

(3)

(Total 6 marks)

7. In any given season, a soccer team plays 65 % of their games at home.
When the team plays at home, they win 83 % of their games.
When they play away from home, they win 26 % of their games.

The team plays one game.

(a) Find the probability that the team wins the game.

(4)

(b) If the team does not win the game, find the probability that the game was played at home.

(4)

(Total 8 marks)

8. In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.

(a) (i) Find the number of boys who play both sports.

(ii) Write down the number of boys who play only rugby.

(3)

(b) One boy is selected at random.

(i) Find the probability that he plays only one sport.

(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby.

(4)

Let A be the event that a boy plays football and B be the event that a boy plays rugby.

(c) Explain why A and B are **not** mutually exclusive.

(2)

(d) Show that A and B are **not** independent.

(3)

(Total 12 marks)

9. Consider the independent events A and B. Given that $P(B) = 2P(A)$, and $P(A \cup B) = 0.52$, find $P(B)$.

(Total 7 marks)

10. There are 20 students in a classroom. Each student plays only one sport. The table below gives their sport and gender.

	Football	**Tennis**	**Hockey**
Female	5	3	3
Male	4	2	3

(a) One student is selected at random.

(i) Calculate the probability that the student is a male or is a tennis player.

(ii) Given that the student selected is female, calculate the probability that the student does not play football.

(4)

(b) Two students are selected at random. Calculate the probability that neither student plays football.

(3)

(Total 7 marks)

11. Consider the events A and B, where $P(A) = \frac{2}{5}$, $P(B') = \frac{1}{4}$ and $P(A \cup B) = \frac{7}{8}$.

(a) Write down $P(B)$.

(b) Find $P(A \cap B)$.

(c) Find $P(A \mid B)$.

(Total 6 marks)

12. Let A and B be independent events, where $P(A) = 0.6$ and $P(B) = x$.

(a) Write down an expression for $P(A \cap B)$.

(1)

(b) Given that $P(A \cup B) = 0.8$,

(i) find x;

(ii) find $P(A \cap B)$.

(4)

(c) **Hence**, explain why A and B are **not** mutually exclusive.

(1)

(Total 6 marks)

13. The eye colour of 97 students is recorded in the chart below.

	Brown	Blue	Green
Male	21	16	9
Female	19	19	13

One student is selected at random.

(a) Write down the probability that the student is a male.

(b) Write down the probability that the student has green eyes, given that the student is a female.

(c) Find the probability that the student has green eyes or is male.

(Total 6 marks)

14. Two restaurants, *Center* and *New*, sell fish rolls and salads.

Let F be the event a customer chooses a fish roll.
Let S be the event a customer chooses a salad.
Let N be the event a customer chooses neither a fish roll nor a salad.

In the *Center* restaurant $P(F) = 0.31$, $P(S) = 0.62$, $P(N) = 0.14$.

(a) Show that $P(F \cap S) = 0.07$.

(3)

(b) Given that a customer chooses a salad, find the probability the customer also chooses a fish roll.

(3)

(c) Are F and S independent events? Justify your answer.

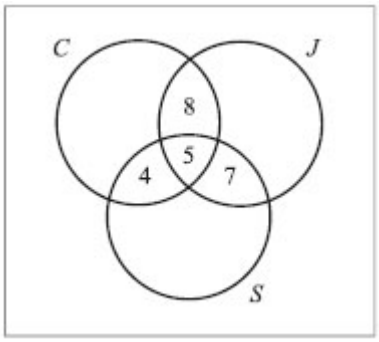
(3)

At *New* restaurant, $P(N) = 0.14$. Twice as many customers choose a salad as choose a fish roll. Choosing a fish roll is **independent** of choosing a salad.

(d) Find the probability that a fish roll is chosen.

(7)
(Total 16 marks)

15. The Venn diagram below shows information about 120 students in a school. Of these, 40 study Chinese (C), 35 study Japanese (J), and 30 study Spanish (S).



A student is chosen at random from the group. Find the probability that the student

(a) studies exactly two of these languages;

(1)

(b) studies only Japanese;

(2)

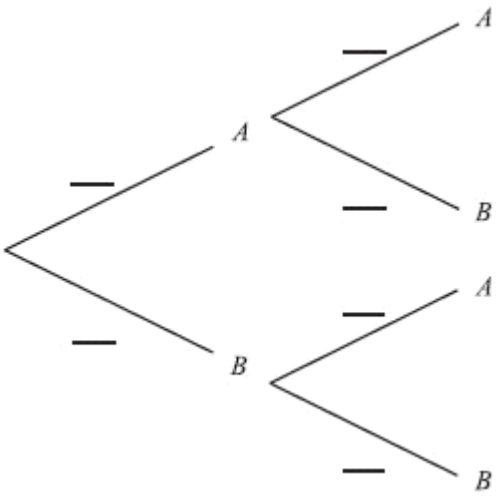
(c) does not study any of these languages.

(3)

(Total 6 marks)

16. A bag contains four apples (A) and six bananas (B). A fruit is taken from the bag and eaten. Then a second fruit is taken and eaten.

(a) Complete the tree diagram below by writing probabilities in the spaces provided.



(3)

(b) Find the probability that one of each type of fruit was eaten.

(3)

(Total 6 marks)

17. Let A and B be independent events such that $P(A) = 0.3$ and $P(B) = 0.8$.

(a) Find $P(A \cap B)$.

(b) Find $P(A \cup B)$.

(c) Are A and B mutually exclusive? Justify your answer.

(Total 6 marks)

18. In a class, 40 students take chemistry only, 30 take physics only, 20 take both chemistry and physics, and 60 take neither.

(a) Find the probability that a student takes physics given that the student takes chemistry.

(b) Find the probability that a student takes physics given that the student does **not** take chemistry.

(c) State whether the events "taking chemistry" and "taking physics" are mutually exclusive, independent, or neither. Justify your answer.

(Total 6 marks)

19. Events E and F are independent, with $P(E) = \frac{2}{3}$ and $P(E \cap F) = \frac{1}{3}$. Calculate

(a) $P(F)$;

(b) $P(E \cup F)$.

(Total 6 marks)

20. Two unbiased 6-sided dice are rolled, a red one and a black one. Let E and F be the events

E : the same number appears on both dice;

F : the sum of the numbers is 10.

Find

(a) $P(E)$;

(b) $P(F)$;

(c) $P(E \cup F)$.

(Total 6 marks)

21. The table below shows the subjects studied by 210 students at a college.

	Year 1	Year 2	Totals
History	50	35	85
Science	15	30	45
Art	45	35	80
Totals	110	100	210

(a) A student from the college is selected at random.
Let A be the event the student studies Art.
Let B be the event the student is in Year 2.

(i) Find $P(A)$.

(ii) Find the probability that the student is a Year 2 Art student.

(iii) Are the events A and B independent? Justify your answer.

(6)

(b) Given that a History student is selected at random, calculate the probability that the student is in Year 1.

(2)

(c) Two students are selected at random from the college. Calculate the probability that one student is in Year 1, and the other in Year 2.

(4)
(Total 12 marks)

22. A class contains 13 girls and 11 boys. The teacher randomly selects four students. Determine the probability that all four students selected are girls.

(Total 6 marks)

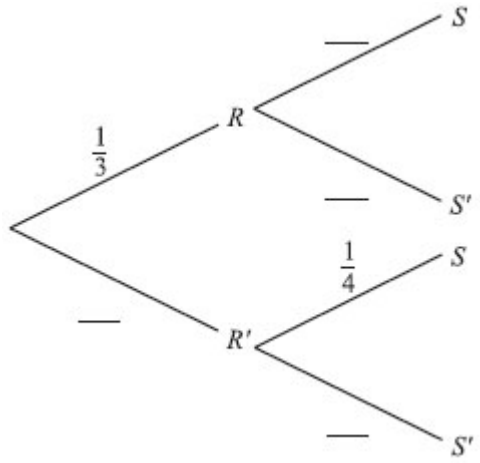
23. The events A and B are independent such that $P(B) = 3P(A)$ and $P(A \cup B) = 0.68$. Find $P(B)$

(Total 6 marks)

24. The following probabilities were found for two events R and S.

$$P(R) = \frac{1}{3},\ P(S \mid R) = \frac{4}{5},\ P(S \mid R') = \frac{1}{4}.$$

(a) **Copy** and **complete** the tree diagram.



(3)

(b) Find the following probabilities.

(i) $P(R \cap S)$.

(ii) $P(S)$.

(iii) $P(R \mid S)$.

(7)
(Total 10 marks)

25. Let A and B be events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{7}{8}$.

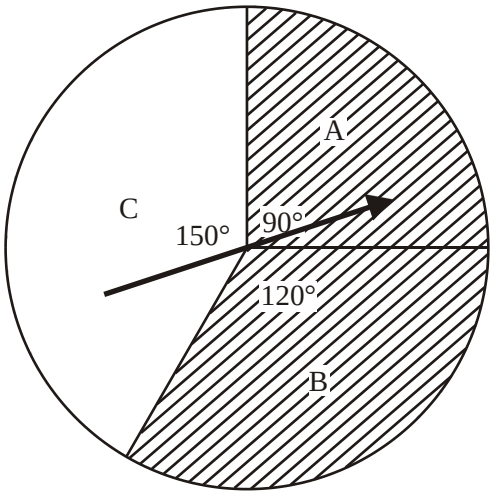
(a) Calculate $P(A \cap B)$.

(b) Calculate $P(A \mid B)$.

(c) Are the events A and B independent? Give a reason for your answer.

(Total 6 marks)

26. The following diagram shows a circle divided into three sectors A, B and C. The angles at the centre of the circle are 90°, 120° and 150°. Sectors A and B are shaded as shown.



The arrow is spun. It cannot land on the lines between the sectors. Let *A*, *B*, *C* and *S* be the events defined by

A: Arrow lands in sector A
B: Arrow lands in sector B
C: Arrow lands in sector C
S: Arrow lands in a shaded region.

Find

(a) P(*B*);

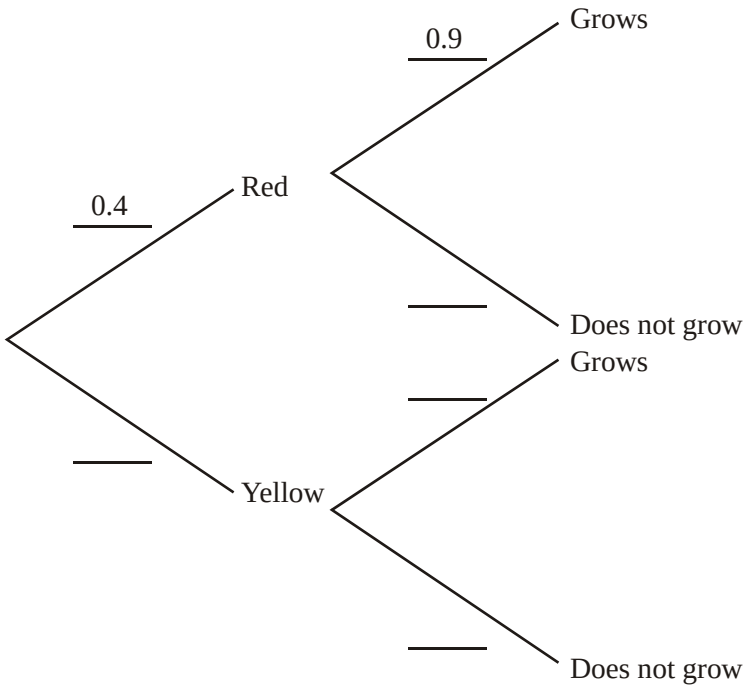
(b) P(*S*);

(c) P($A \mid S$).

(Total 6 marks)

27. A packet of seeds contains 40% red seeds and 60% yellow seeds. The probability that a red seed grows is 0.9, and that a yellow seed grows is 0.8. A seed is chosen at random from the packet.

(a) Complete the probability tree diagram below.



(3)

(b) (i) Calculate the probability that the chosen seed is red and grows.

(ii) Calculate the probability that the chosen seed grows.

(iii) Given that the seed grows, calculate the probability that it is red.

(7)
(Total 10 marks)

28. Consider events *A*, *B* such that P (*A*) ≠ 0, P (*A*) ≠ 1, P (B) ≠ 0, and P (B) ≠ 1.

In each of the situations (a), (b), (c) below state whether *A* and *B* are

mutually exclusive (M);
independent (I);
neither (N).

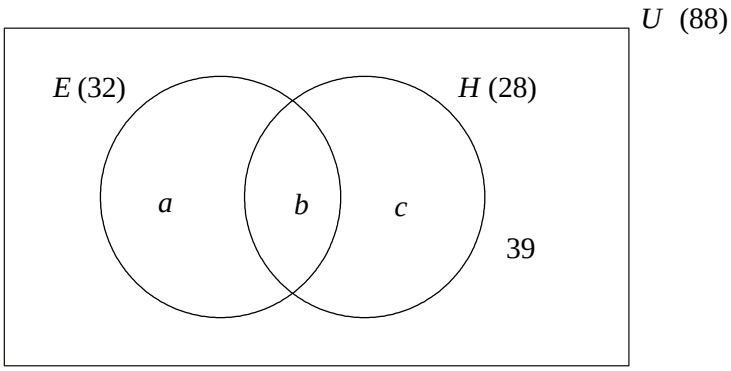
(a) $P(A|B) = P(A)$

(b) $P(A \cap B) = 0$

(c) $P(A \cap B) = P(A)$

(Total 6 marks)

29. In a school of 88 boys, 32 study economics (E), 28 study history (H) and 39 do not study either subject. This information is represented in the following Venn diagram.



(a) Calculate the values a, b, c.

(4)

(b) A student is selected at random.

(i) Calculate the probability that he studies **both** economics and history.

(ii) Given that he studies economics, calculate the probability that he does **not** study history.

(3)

(c) A group of three students is selected at random from the school.

(i) Calculate the probability that none of these students studies economics.

(ii) Calculate the probability that at least one of these students studies economics.

(5)

(Total 12 marks)

30. A painter has 12 tins of paint. Seven tins are red and five tins are yellow. Two tins are chosen at random. Calculate the probability that both tins are the same colour.

(Total 6 marks)

31. A box contains 22 red apples and 3 green apples. Three apples are selected at random, one after the other, without replacement.

(a) The first two apples are green. What is the probability that the third apple is red?

(b) What is the probability that exactly two of the three apples are red?

(Total 6 marks)

32. Two fair dice are thrown and the number showing on each is noted. The sum of these two numbers is *S*. Find the probability that

(a) *S* is less than 8;

(2)

(b) at least one die shows a 3;

(2)

(c) at least one die shows a 3, given that *S* is less than 8.

(3)

(Total 7 marks)

33. For events *A* and *B*, the probabilities are P (*A*) = $\frac{3}{11}$, P (*B*) = $\frac{4}{11}$.

Calculate the value of P ($A \cap B$) if

(a) P ($A \cup B$) = $\frac{6}{11}$;

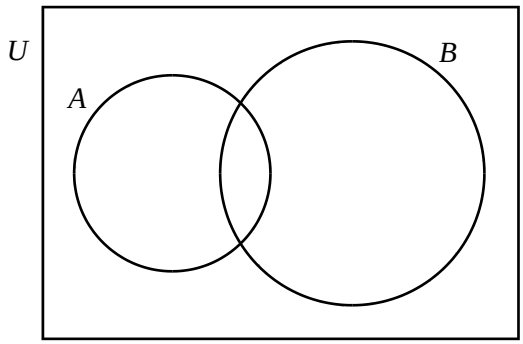
(b) events *A* and *B* are independent.

(Total 6 marks)

34. A bag contains 10 red balls, 10 green balls and 6 white balls. Two balls are drawn at random from the bag without replacement. What is the probability that they are of different colours?

(Total 4 marks)

35. The following Venn diagram shows the universal set U and the sets A and B.



(a) Shade the area in the diagram which represents the set $B \cap A'$.

$n(U) = 100$, $n(A) = 30$, $n(B) = 50$, $n(A \cup B) = 65$.

(b) Find $n(B \cap A')$.

(c) An element is selected at random from U. What is the probability that this element is in $B \cap A'$?

(Total 4 marks)

36. In a survey of 200 people, 90 of whom were female, it was found that 60 people were unemployed, including 20 males.

(a) Using this information, complete the table below.

	Males	Females	Totals
Unemployed			
Employed			
Totals			200

(b) If a person is selected at random from this group of 200, find the probability that this person is

(i) an unemployed female;

(ii) a male, given that the person is employed.

(Total 4 marks)