

Name:

## Mathematics IB HL Test 4

January 20, 2022

1 hour 30 minutes

### Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Calculators are **not allowed** for this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [**72 marks**].
- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to **show all working**.
- Write your solutions in the space provided.

1. [Maximum mark: 5]

Complex number  $z$  satisfies the equation:

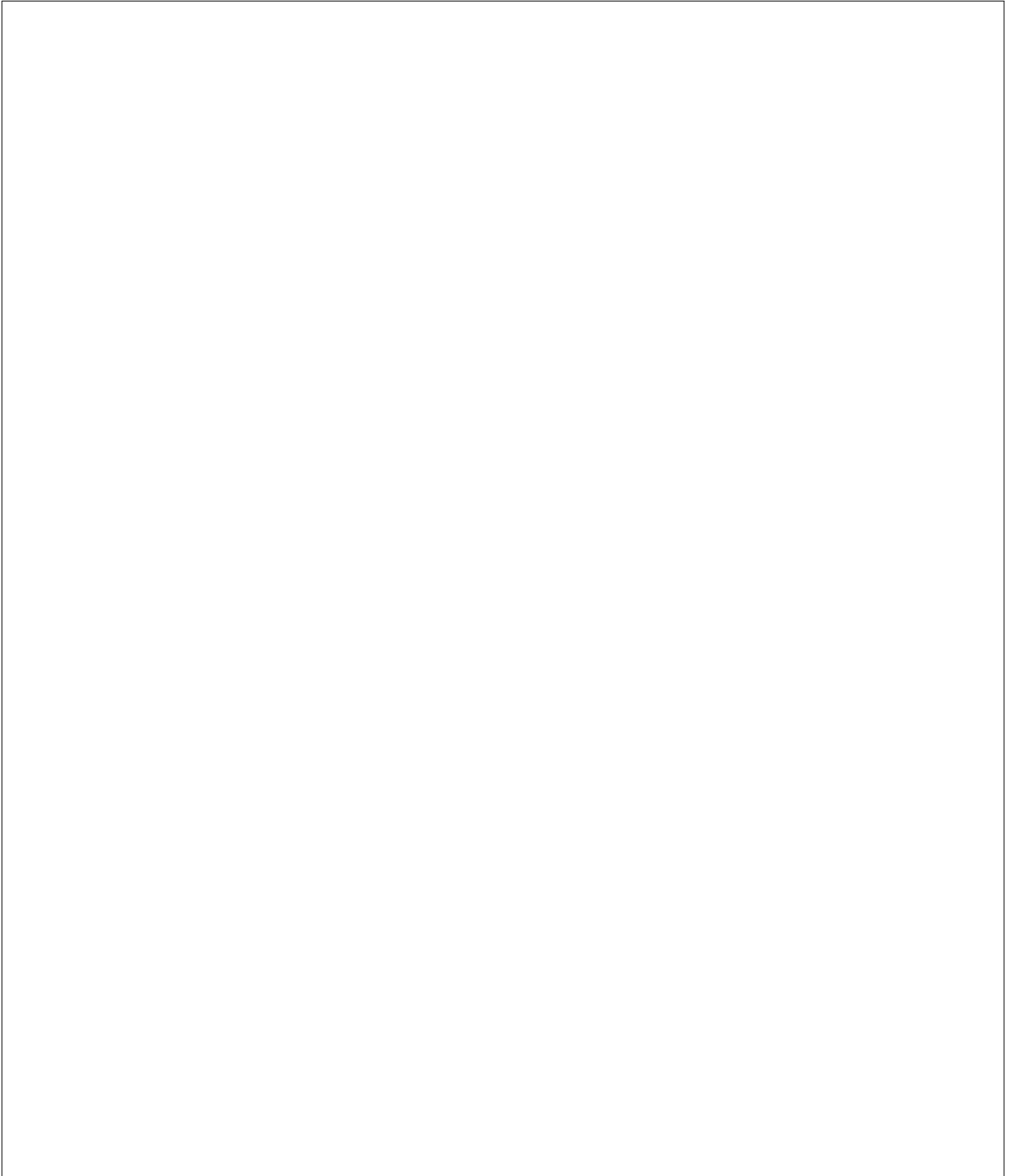
$$(2 - i)z = 9 + 8i$$

Find  $z$  and calculate  $z^3$ , give your answers in the form  $x + yi$ , where  $x, y \in \mathbb{R}$ .

**2.** [Maximum mark: 4]

Solve the inequality:

$$|x + 4| > |2x - 3|$$



**3.** [Maximum mark: 8]

Consider the polynomial

$$P(x) = x^3 + ax^2 + bx + c$$

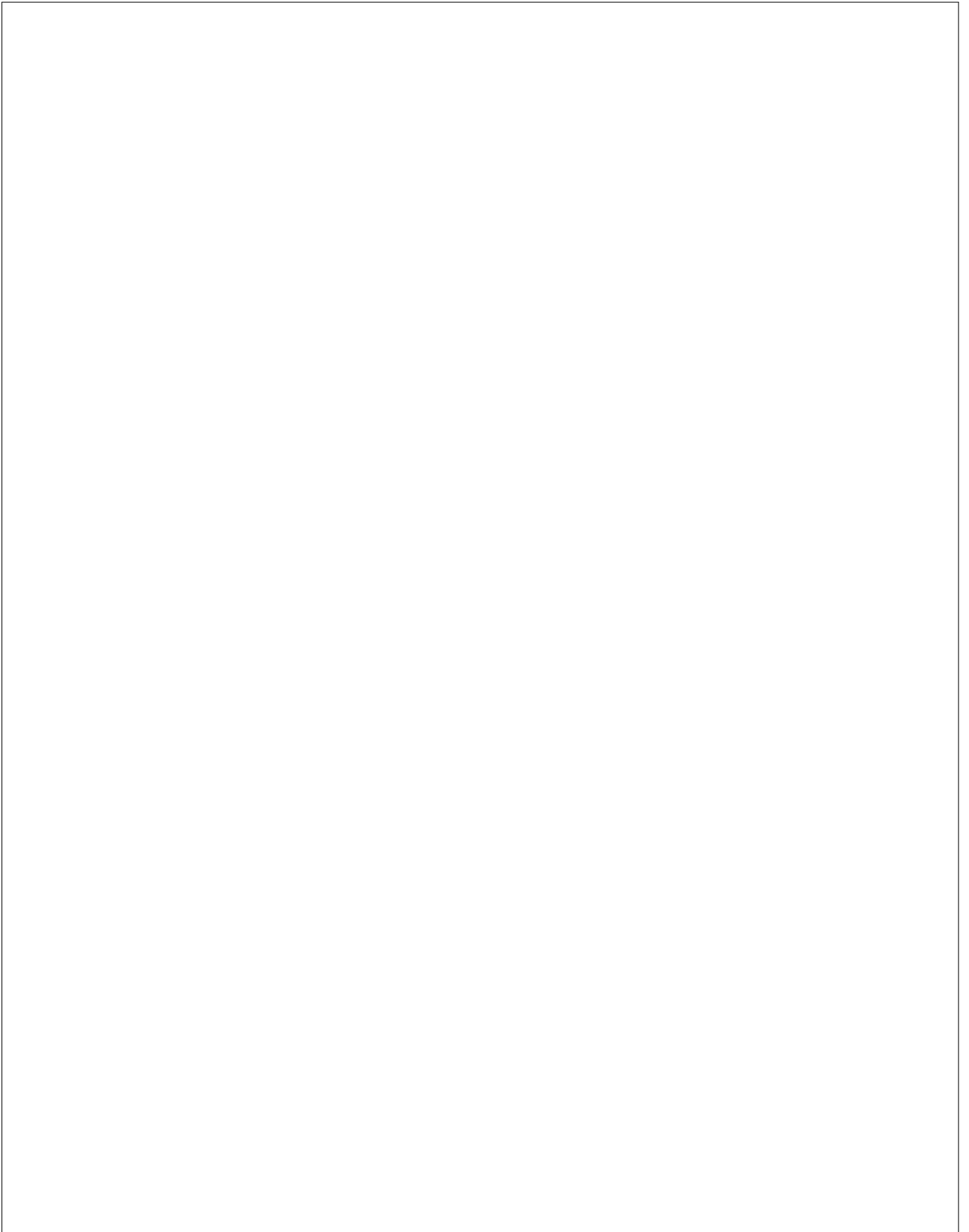
The remainders when this polynomial is divided by  $(x - 1)$  and  $(x + 1)$  are both equal to 3.  
The remainder when it is divided by  $(x - 2)$  is 15.

a) Find the values of  $a$ ,  $b$  and  $c$ .

[5 points]

b) The remainder when  $P(x)$  is divided by  $(x - p)$  is 1. Find the value of  $p$  given that  $p > 0$ .

[3 points]



4. [Maximum mark: 7]

Prove that

$$\sum_{j=1}^n j2^j = (n-1)2^{n+1} + 2$$

5. [Maximum mark: 7]

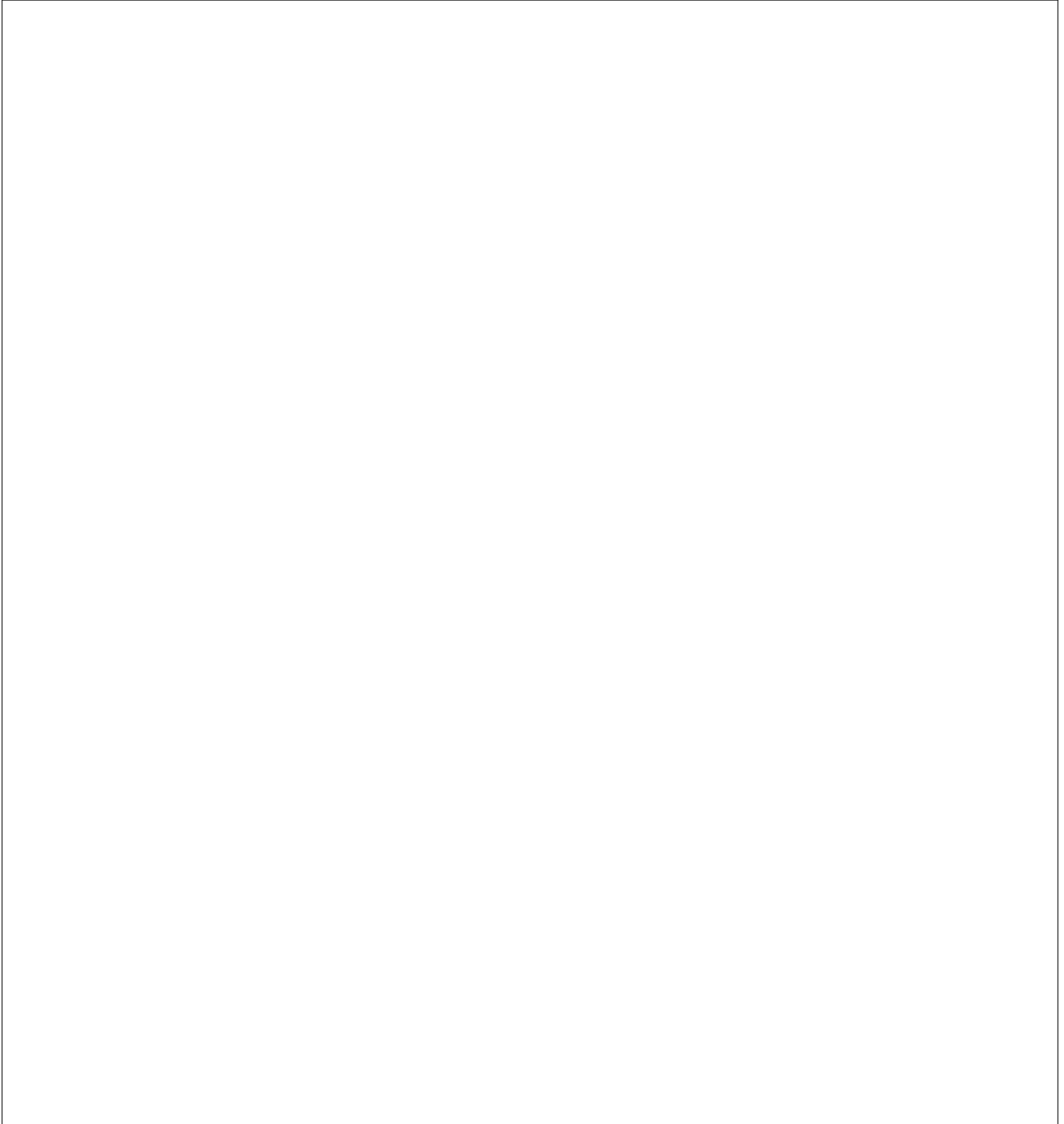
Prove that  $3^{2n+2} - 8n - 9$  is divisible by 64 for all positive integers  $n$ .

6. [Maximum mark: 9]

a) Show that when  $n^2$  is divided by 3, the only possible remainders are 0 and 1, for all  $n \in \mathbb{Z}$ . [3 points]

b) Show that if  $n^2$  is divisible by 3, then  $n$  is divisible by 3, for all  $n \in \mathbb{Z}$ . [2 points]

c) Prove that  $\sqrt{3}$  is irrational. [4 points]





7. [Maximum mark: 5]

Solve the equation:

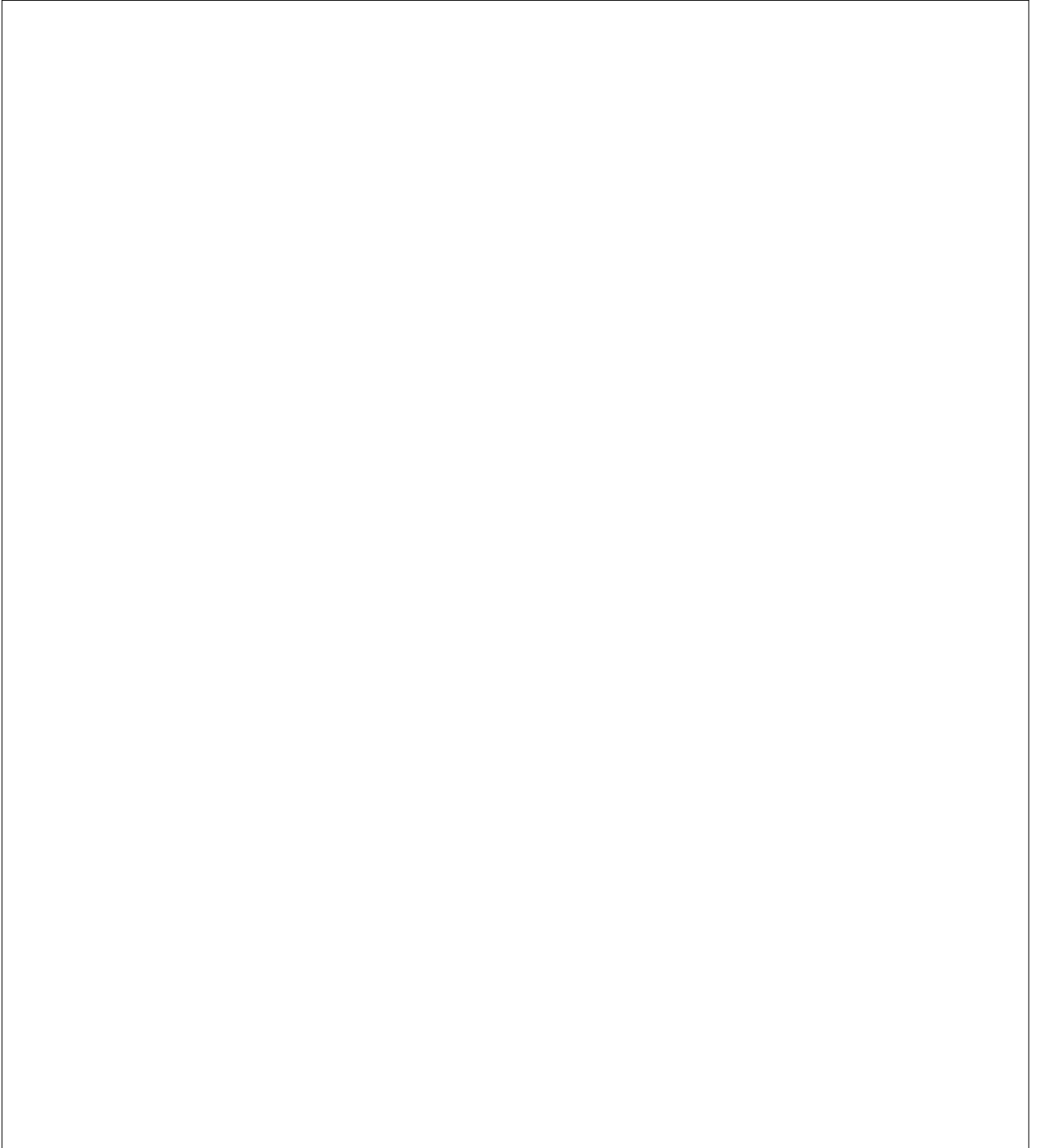
$$2(\log_9 x)^2 + \log_{\frac{1}{9}} x = \log_{\sqrt{2}} 2\sqrt{2}$$

8. [Maximum mark: 5]

Find all possible values of  $k$  for which the graph of the function

$$f(x) = (k + 1)x^2 - 2(k - 1)x + 3k - 3$$

lies entirely below the  $x$ -axis.



9. [Maximum mark: 6]

Consider the system of equations:

$$\begin{cases} 8x + 3y + az = 12 \\ x + 2z = 3 \\ 2x + y - z = b \end{cases}$$

- a) Find the values of  $a$  and  $b$  for which the system has infinitely many solutions. [4 points]
- b) For the values of  $a$  and  $b$  found in part (a) find the solutions to the system in terms of parameter  $\lambda$ . [2 points]

10. [Maximum mark: 5]

A small classroom has six one-person desks arranged in 3 rows of 2. Six students are to be seated in the room for Mathematics examination. Find the number of possible arrangements of students if 2 of the 6 students: Tomasz and Maria cannot sit next to each other in the same row and one cannot sit directly behind the other.

11. [Maximum mark: 11]

A sequence is given by  $u_1 = 7$  and  $u_{n+1} = 2u_n + 3$  for  $n \geq 1$ .

a) Using the definition above find  $u_2$  and  $u_3$ . [2 points]

b) Show that a sum of an even number and an odd number is odd and hence show that every term of this sequence is odd. [3 points]

c) Show that the formula for the  $n$ -th term of the sequence is  $u_n = 5 \cdot 2^n - 3$ . [6 points]

