INTERNATIONAL BACCALAUREATE
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MARKSCHEME

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MATHEMATICS

Higher Level

Paper 1

12 pages

1. (a) let $y = \frac{1}{x+1} \Leftrightarrow x+1 = \frac{1}{y} \Leftrightarrow x = \frac{1}{y} - 1$, thus $f^{-1}(x) = \frac{1}{x} - 1$ ***(M1)(A1)***

(b) $(f \circ g)(x) = f\left(\sqrt{x^2-1}\right) = \frac{1}{\sqrt{x^2-1}+1}$ ***(M1)(A1)***

Answers: (a) $f^{-1}(x) = \frac{1}{x} - 1$ ***(C2)***

(b) $(f \circ g)(x) = \frac{1}{\sqrt{x^2-1}+1}$ ***(C2)***

2. $4^x = 8^y \Leftrightarrow 2^{2x} = 2^{3y} \Leftrightarrow 2x = 3y$, and ***(M1)***

$x + 2y = 5 \Leftrightarrow 2x + 4y = 10$

$\Leftrightarrow 3y + 4y = 10 \Leftrightarrow y = \frac{10}{7}$, and since $x = \frac{3y}{2} \Leftrightarrow x = \frac{15}{7}$ ***(M1)(A2)***

Answer: $x = \frac{15}{7}, y = \frac{10}{7}$ ***(C4)***

3. (a) $P(-2) = (-2)^3 - 3(-2)^2 + 4(-2) + c = -28 + c$ ***(M1)(A1)***

(b) $-28 + c = -23$ therefore $c = 5$ ***(M1)(A1)***

Answers: (a) $-28 + c$ ***(C2)***

(b) $c = 5$ ***(C2)***

4. $\binom{5}{3}k^3 = \binom{5}{4}k^4 \Rightarrow 10k^3 = 5k^4$ ***(M1)(A1)***

$\Rightarrow 5k^3(2-k) = 0$ ***(A1)***

$\Rightarrow k = 2$ ***(A1)***

Answer: $k = 2$ ***(C4)***

5. (a) The values seem to repeat every 6 units. The period is 6. *(R1)(A1)*

(b) $f(41) = f(36+5) = f(5) = 4$ *(M1)(A1)*

Answers: (a) The period is 6. *(C2)*

(b) $f(41) = 4$ *(C2)*

6. (a) $k + \frac{1}{4} + \frac{1}{4} + 3k = 1 \Rightarrow 4k = \frac{1}{2} \Rightarrow k = \frac{1}{8}$ *(M1)(A1)*

(b) $E(X) = 1 \times \frac{1}{4} + 2 \times \frac{1}{4} + 3 \times \frac{3}{8} = \frac{15}{8}$ *(M1)(A1)*

Answers: (a) $k = \frac{1}{8}$ *(C2)*

(b) $E(X) = \frac{15}{8}$ *(C*

7. $\vec{a} \perp \vec{b} \Leftrightarrow (-2 \times 1) + (p \times 3)(p+4) - 1(2p-5) = 0$ *(M1)*

$\Leftrightarrow 3p^2 + 10p + 3 = 0 \Leftrightarrow (3p+1)(p+3) = 0$ *(M1)*

$\Leftrightarrow p = -\frac{1}{3}$, or $p = -3$ *(A2)*

Answer: $p = -\frac{1}{3}$, or $p = -3$ *(C4)*

8. In triangle ABL, it is clear that $\hat{L} = 20^\circ$. Using the law of sines we have *(A1)*

$\frac{AB}{\sin 20^\circ} = \frac{BL}{\sin 50^\circ} \Leftrightarrow \frac{50}{\sin 20^\circ} = \frac{BL}{\sin 50^\circ}$ *(M1)*

$\Rightarrow BL = \frac{50 \sin 50^\circ}{\sin 20^\circ} = 111.99 \text{ km} = 112 \text{ km}$ *(M1)(A*

Answer: $BL = 112$ km *(C4)*

9. (a) For $f(x)$ to be defined, $6x^2-5x-6$ must be larger than zero. *(R1)*

$6x^2-5x-6>0 \Leftrightarrow (3x+2)(2x-3)>0$ *(M1)*

$\Leftrightarrow \left\{x \in \mathbb{R} \middle| x<\frac{-2}{3} \text{ or } x>\frac{3}{2}\right\}$ *(A1)*

(b) $\mathbb{R}$ *(A1)*

Answers: (a) $\left\{x \in \mathbb{R} \middle| x<\frac{-2}{3} \text{ or } x>\frac{3}{2}\right\}$ *(C3)*

(b) $\mathbb{R}$ *(C1)*

10. $z^2=-5+12\mathrm{i} \Leftrightarrow (a^2-b^2)+(2ab)\mathrm{i}=-5+12\mathrm{i}$ *(M1)*

$\Leftrightarrow a^2-b^2=-5$, and $2ab=12$, or $ab=6$

$\Leftrightarrow a^2-\frac{36}{a^2}=-5 \Leftrightarrow a^4+5a^2-36=0$ *(M1)*

$\Leftrightarrow (a^2+9)(a^2-4)=0 \Rightarrow a=\pm 2$ and $b=\pm 3$

($a^2+9=0$ has no real solution)

The ordered pairs are then $(2,3)$ and $(-2,-3)$ *(A2)*

Answer: $(2,3)$ and $(-2,-3)$ *(C4)*

11. (a) For a quadratic equation to have real roots, the discriminant b^2-4ac must be larger than or equal to zero.

$4(m+2)^2-4m(m+2)\geq 0 \Leftrightarrow 4(m+2)(m+2-m)\geq 0 \Rightarrow m+2\geq 0 \Rightarrow m\geq -2$ *(M1)(A1)*

(b) For the quadratic equation to have two roots of opposite sign their product must be negative.

$\frac{m+2}{m}<0 \Rightarrow -2<m<0$ *(M1)(A1)*

Answers: (a) $m+2\geq 0 \Rightarrow m\geq -2$ *(C2)*

(b) $-2<m<0$ *(C2)*

12. (a) $p(A|B)=\dfrac{p(A\cap B)}{p(B)}\Rightarrow p(B)=\dfrac{\frac{1}{5}}{\frac{3}{10}}=\dfrac{2}{3}$ *(M1)(A1)*

(b) $p(B|A)=\dfrac{p(A\cap B)}{p(A)}\Rightarrow p(A)=\dfrac{\frac{1}{5}}{\frac{1}{2}}=\dfrac{2}{5}$ *(A1)*

(c) $p(A\cup B)=p(A)+p(B)-p(A\cap B)=\dfrac{2}{5}+\dfrac{2}{3}-\dfrac{1}{5}=\dfrac{13}{15}$ *(A1)*

Answers: (a) $p(B)=\dfrac{2}{3}$ *(C2)*

(b) $p(A)=\dfrac{2}{5}$ *(C1)*

(c) $p(A\cup B)=\dfrac{13}{15}$ *(C1)*

13. $(3\cos x+5)^2=(4\sin x)^2$ *(M1)*

$9\cos^2 x+30\cos x+25=16\sin^2 x$

$25\cos^2 x+30\cos x+9=0$ *(M1)*

$(5\cos x+3)^2=0$

$\cos x=-\dfrac{3}{5}$, hence $\tan x=-\dfrac{4}{3}$ *(M1)(A1)*

Answer: $\tan x=-\dfrac{4}{3}$ *(C4)*

14. $\dfrac{1}{2}\begin{pmatrix} b & -5 & 4\\ -1 & 1 & 0\\ 1 & -3 & 2\end{pmatrix}\begin{pmatrix} 1 & -1 & -2\\ 1 & 1 & -2\\ 1 & 2 & a\end{pmatrix}=\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$ *(M1)*

$\Leftrightarrow \dfrac{1}{2}\begin{pmatrix} b-1 & 3-b & 4a-2b+10\\ 0 & 2 & 0\\ 0 & 0 & 2a+4\end{pmatrix}=\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$ *(A1)*

$\Leftrightarrow b-1=2, 3-b=0, 4a-2b+10=0, 2a+4=2$

$\Leftrightarrow a=-1, b=3$ *(M1)(A1)*

Answer: $a=-1, b=3$ *(C4)*

15. For a plane to be perpendicular to two intersecting planes, it must be perpendicular to their line of intersection. Hence, if we find the cross product of normals to the two given planes, we can use it as a normal to the new plane. *(R1)*

$(6\vec{i} - 2\vec{j} + 3\vec{k}) \times (\vec{i} - 3\vec{j}) = 9\vec{i} + 3\vec{j} - 16\vec{k}$ *(A1)*

Therefore the equation of the plane is of the form $9x + 3y - 16z = d$. *(R1)*
Since the plane contains the point $(2, 2, 3)$, $d = -24$, and the equation is $9x + 3y - 16z = -24$. (Other forms of the equation are acceptable.) *(A1)*

Answer: $9x + 3y - 16z = -24$ *(C4)*

16. The largest parallelogram is a rhombus, its area is $4 \times 4 \times \sin 60° = 8\sqrt{3}$, *(R1)*
the second parallelogram is a rectangle with one side 2 and the other $2\sqrt{3}$, $\Rightarrow$ Area $= 4\sqrt{3}$, *(A1)*
the third is a rhombus again with side 2, its area is then $2\sqrt{3}$, and so on.

The sum is then an infinite geometric series with a common ratio of $\frac{1}{2}$. *(M1)*

Therefore the sum is $\dfrac{8\sqrt{3}}{1-\frac{1}{2}} = 16\sqrt{3}$. *(A1)*

Answer: $\dfrac{8\sqrt{3}}{1-\frac{1}{2}} = 16\sqrt{3}$ *(C4)*

17. $\int_1^k \frac{x}{\sqrt{3}}\sqrt{x^2-1}\,dx = 1 \Rightarrow \left[\frac{1}{3\sqrt{3}}(x^2-1)^{\frac{3}{2}}\right]_1^k = 1$ *(M1)(A1)*

$\Rightarrow (k^2-1)^{\frac{3}{2}} = 3\sqrt{3} \Rightarrow (k^2-1)^3 = 27 \Rightarrow k^2 - 1 = 3$ *(A1)*

$\Rightarrow k = 2$ (since $k \geq 1$). *(A1)*

Answer: $k = 2$ (since $k \geq 1$). *(C4)*

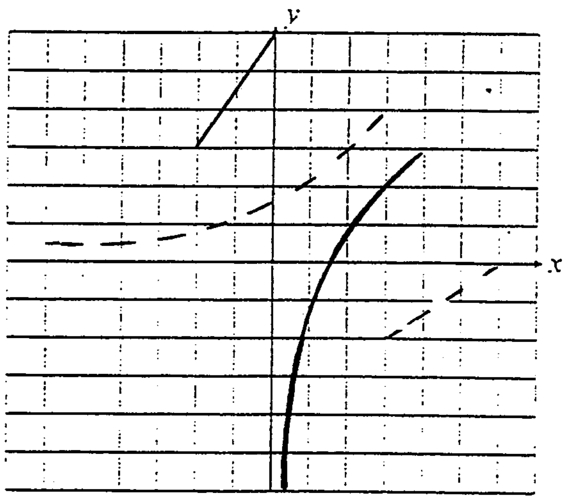
18. For the three vectors to be coplanar, then a vector perpendicular to two of them should be perpendicular to the third, hence, the cross product of two of them multiplied (scalar product) by the third should be zero. *(R1)*

$\vec{w}\cdot\vec{u}\times\vec{v}=0 \Rightarrow \left(\vec{i}+(2-t)\vec{j}+(t+1)\vec{k}\right)\cdot(-12\vec{i}+9\vec{j}+2\vec{k})=0$ *(M1)(A1)*

$\Rightarrow 8-7t=0 \Rightarrow t=\dfrac{8}{7}$ *(A1)*

Answer: $t=\dfrac{8}{7}$ *(C4)*

19. (a)

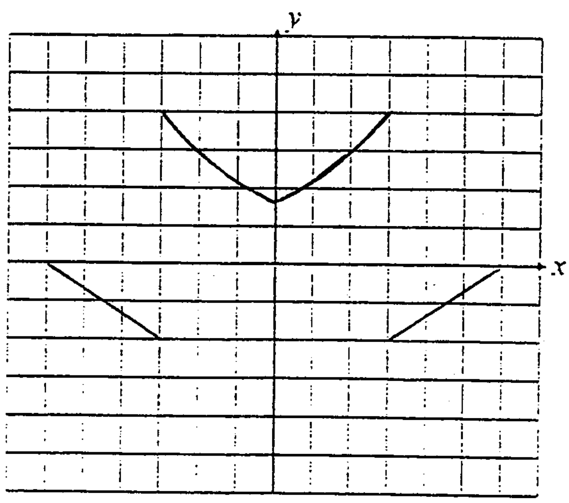


(Deduct 1 mark for each error)

(C2)

The dotted line is the original graph.

(b)



(Deduct 1 mark for each error)

(C2)