TRIGONOMETRIC FUNCTIONS AND EQUATIONS

Name:

1. (5 points) The angle α satisfies the equation

$$2\tan^2\alpha - 5\sec\alpha - 10 = 0$$

where α is in the second quadrant. Find the **exact** value of $\sec\alpha$.

2. (6 points) The obtuse angle β is such than $\tan\beta = -\frac{5}{12}$. Find the values of
   (a) $\sin\beta$
   (b) $\cos\beta$
   (c) $\sin 2\beta$
   (d) $\cos 2\beta$

3. (7 points)

   (a) (2 points) Show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

   (b) (2 points) Using substitution $t = 2\cos\theta$, show that $t^3 = 3t + 1$ becomes $\cos 3\theta = \frac{1}{2}$.

   (c) (3 points) Hence find the exact values of the roots of the equation $x^3 - 3x - 1 = 0$.

4. (5 points)

   (a) Simplify the expression $\dfrac{\sin\theta}{1-\cos\theta} + \dfrac{\sin\theta}{1+\cos\theta}$.

   (b) Hence solve $\dfrac{\sin\theta}{1-\cos\theta} + \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{4}{\sqrt{3}}$, for $0 \leq \theta \leq 2\pi$.

5. (10 points)

(a) Write down the minimum value of $\cos x$ and the smallest positive value of x (in radians) for which the minimum occurs.

(b) i. Describe two transformations which transform the graph of $y = \cos x$ to the graph of $y = 2\cos\left(x + \frac{\pi}{6}\right)$.

ii. Hence state the minimum value of $2\cos\left(x + \frac{\pi}{6}\right)$ and find the value of $x \in [0, 2\pi]$ for which the minimum occurs.

(c) The function f is defined for $x \in [0, 2\pi]$ by $f(x) = \dfrac{5}{3 + 2\cos\left(x + \frac{\pi}{6}\right)}$.

i. State, with reason, whether f has any vertical asymptotes.

ii. Find the range of f.