

Name:  
Group B  
Result:

1.

[8 points]

Calculate the following sums:

a)  $3 + 7 + 11 + \dots + 87 =$

b)  $\sum_{k=1}^9 3 \cdot 2^{k-1}$

c)  $\sum_{k=1}^{10} \frac{k+1}{5}$

d)  $\sum_{k=1}^{\infty} 12 \cdot \left(\frac{1}{3}\right)^k$

2.

[2 points]

In an arithmetic sequence the third term is equal to 12 and the tenth term is equal to  $-23$ . Find the first term and the common difference.

**3.**[4 *points*]

$2a + 1$ ,  $3a - 5$  and  $a + 1$  are the first three of an arithmetic sequence (in the given order).

(a) Find  $a$ .

(b) Find the sum of the first 20 terms of this sequence.

**4.**[4 *points*]

$3x + 5$ ,  $3x + 1$  and  $2x$  are the first three of an infinite geometric sequence (in the given order).

(a) Find the possible values of  $x$ .

(b) Find the sum to infinity of both sequences (if such sums exist).

**5.**[2 *points*]

Show that if a sequence  $u_n$  is geometric, then the sequence  $v_n = \log(u_n)$  is arithmetic.

**6.**[3 *points*]

The sum of an arithmetic sequence is given by the formula  $S_n = n^2 - 4n$ . Find the number of terms of this sequence which are smaller than 100.

**7.**

[2 points]

Consider an infinite geometric series:

$$3 + \frac{2x - 1}{3} + \frac{(2x - 1)^2}{27} + \dots$$

- (a) Find the set of possible values of  $x$  for which the series converges.
- (b) Find the sum when  $x = 1$ .