



## MATHEMATICS HIGHER LEVEL PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

2 hours

Examination	code
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## **INSTRUCTIONS TO CANDIDATES**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the *Mathematics HL and Further Mathematics HL* formula booklet is required for this paper.
- The maximum mark for this examination paper is [120 marks].

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## **SECTION A**

Answer all the questions in the boxes provided. Working may be continued below the lines, if necessary.

**1.** [Maximum mark: 5]

When the polynomial  $3x^3 + ax + b$  is divided by (x-2), the remainder is 2, and when divided by (x+1), it is 5. Find the value of a and the value of b.

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<b>2.</b> [Maximum mark: 4
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Four numbers are such that their mean is 13, their median is 14 and their mode is 15. Find the four numbers.

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Consider  $a = \log_2 3 \times \log_3 4 \times \log_4 5 \times ... \times \log_{31} 32$ . Given that  $a \in \mathbb{Z}$ , find the value of a.




[Maximum mark: 6] 4.

The equation  $5x^3 + 48x^2 + 100x + 2 = a$  has roots  $r_1$ ,  $r_2$  and  $r_3$ . Given that  $r_1 + r_2 + r_3 + r_1 r_2 r_3 = 0$ , find the value of a.




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The triangle ABC is equilateral of side 3 cm. The point D lies on [BC] such that  $BD = 1 \, \text{cm}$ . Find  $\cos D\hat{A}C$ .




[Maximum mark: 6]

Given that  $\sin x + \cos x = \frac{2}{3}$ , find  $\cos 4x$ .

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## **13.** [Maximum mark: 17]

A geometric sequence  $\{u_n\}$ , with complex terms, is defined by  $u_{n+1} = (1+i)u_n$  and  $u_1 = 3$ .

- (a) Find the fourth term of the sequence, giving your answer in the form x + yi,  $x, y \in \mathbb{R}$ . [3]
- (b) Find the sum of the first 20 terms of  $\{u_n\}$ , giving your answer in the form  $a \times (1+2^m)$  where  $a \in \mathbb{C}$  and  $m \in \mathbb{Z}$  are to be determined. [4]

A second sequence  $\{v_n\}$  is defined by  $v_n = u_n u_{n+k}$ ,  $k \in \mathbb{N}$ .

- (c) (i) Show that  $\{v_n\}$  is a geometric sequence.
  - (ii) State the first term.
  - (iii) Show that the common ratio is independent of k. [5]

A third sequence  $\{w_n\}$  is defined by  $w_n = |u_n - u_{n+1}|$ .

- (d) (i) Show that  $\{w_n\}$  is a geometric sequence.
  - (ii) State the geometrical significance of this result with reference to points on the complex plane. [5]

