SL 1.4, SL 1.7 Finances

Introduction

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Interest rates

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Fixed amount is added each period

Compound interest

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Compound interest
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Simple interest

In **simple interest** fixed amount is added each period.

If we invest 1000 PLN in a fund that pays 5% simple interest per year, it means that we will get 50 PLN (because 5% of 1000 PLN is 50 PLN) each year. So after 5 years we will have $1000 + 5 \cdot 50 = 1250PLN$

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Simple interest - exercise

Tomasz invests 20000 PLN into savings account that pays 6% simple interest per year. Find the value of his investment after 7 years.

$6\% \cdot 20000 = 120$

So he gets 120 PLN each year

After 7 years his investment will be worth

$20000 + 7 \cdot 120 = 28400$

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Simple interest - GDC

On **CASIO** you can calculate simple interest as follows. Go to TVM and then F1 simple interest. You need to put in the **P**resent **V**alue (it has to be negative, as you "lose" the money initially), the interest rate and the number of **days** (sic!).

In the previous example we would put in:

$n = 7 \cdot 365 = 2555$

$I\% = 6$

$PV = -20000$

We then get Simple Future Value = 28400

I strongly recommend that you do the simple interest calculation by hand.

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Compound interest

In **compound interest** fixed percentage is added each period.

If we invest 1000 PLN in a fund that pays 5% compound interest (compounded annually), it means that we will get 50 PLN (because 5% of 1000 PLN is 50 PLN) after the first year, but we will get 52.5 PLN (because 5% of 1050 PLN is 52.5 PLN) in the second year and the amount we receive increases as we have more money in the account.

Because the amount we have increases by a fixed percentage, we can multiply it by 1.05 each year. Note that multiplying by 1.05 denotes the increase by 5%.

So after 5 years we will have:

$$1000 \cdot 1.05^5 = 1276.28$$

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Compound interest

Tomasz invests 50000 PLN into savings accound that pays 3% annual interest compounded yearly. Calculate the value of his investment after 8 years.

$$50000 \cdot 1.03^8 = 63338.50$$

We multiplied by 1.03 to denote the 3% increase. 1.03 is to the 8th power because we want to know the value of the investment after 8 years.

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Compound interest GDC

To solve compound interest problems on GDC on **CASIO** you choose TVM (Finance) from the Menu and on **Ti-84** you need to press APPS and find Finance.

Compound interest GDC

When using GDC you need to input the following data:

- n - number of periods,
- I% - annual interest rate,
- PV - present value,
- PMT - payments,
- FV - future value,
- P/Y - payments (or periods) per year,
- C/Y - compounding per year.

Note that we use sing (+ or -) to indicate the direction of the money. For
example -1000 means that we lose (or invest) the money, +1000 means
that we get (or borrow, or withdraw) the money.

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Compound interest GDC

In our example we have

$n = 8$ (our period is a year and we want 8 years)

$I\% = 3$

$PV = -50000$ (minus sign indicates that we invest this money)

$PMT = 0$ (there are no payments)

$FV = ?$ (this doesn't matter, we need find it)

$P/Y = 1$ (there is one year in a year)

$C/Y = 1$ (it is compounded once a year)

On Casio we press F5 to find FV. On Ti-84 you need to highlight FV and press ALPHA-ENTER (Solve).

You should get FV= 63338.50

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Compound interest - exercise 1

Let's consider a different example.

Tomasz invests 20000 PLN. The bank pays 4% annual interest rate and
the interest is compounded every 6 months. Calculate the value of the
investment after 7 years.

The annual interest rate is 4%, but it is compounded twice a year, so
every 6 months we receive 2%. In 7 years there are 14 6-month periods, so
we can calculate the value of the investment as follows:

$$20000 \cdot 1.02^{14} = 26389.58$$

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Compound interest - exercise 1

Doing the same calculation using TVM on GDC:

$n = 14$ (we have 14 6-month periods)

$I\% = 4$ (this is the annual interest rate)

$PV = -20000$ (remember about the minus sign)

$PMT = 0$ (there are no payments)

$FV = ?$ (this is what we're looking for)

$P/Y = 2$ (there are 2 6-month periods in a year)

$C/Y = 2$ (the interest is compounded twice a year)

We get $FV = 26389.58$

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Compound interest - exercise 2

Tomasz invests 40000 PLN. The bank pays $r\%$ annual interest rate and compounded monthly. Calculate r if after 5 years the value of the investment is 45320.04 PLN.

We can try solving this algebraically by solving the following equation:

$$45320.04 = 40000 \cdot \left(1 + \frac{r}{12 \cdot 100}\right)^{60}$$

We should get $r = 2.50$. Think about this equation. It helps if you understand what is happening here. However it is easier to simply use TVM on GDC.

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We have:

$n = 60$ (we have 60 months in 5 years)

$I\% = ?$ (unknown)

$PV = -40000$ (remember about the minus sign)

$PMT = 0$ (there are no payments)

$FV = 45320.04$ (no minus here, we're getting this money)

$P/Y = 12$ (12 months in a year)

$C/Y = 12$ (interest is compounded 12 times a year)

We get that the interest rate is $r = 2.50$

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We get that the interest rate is $r = 2.50$.

Compound interest - exercise 3

Tomasz would like to have 100 000 PLN on his account in 10 years time. How much does he need to invest into savings account that pays an interest of 2% compounded quarterly?

Algebraically we would do it as follows

$$100000 = x \cdot (1.005)^{40}$$

We multiply by 1.005 because we earn 0.5% per quarter. The power is 40, because we have 40 quarters in 10 years. The answer we get is

$$x = 81913.89$$

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Compound interest - exercise 3

Now let's use TVM:

n = 40 (we have 40 quarters in 10 years)

I% = 2 (annual interest rate)

PV = ? (this will be negative)

PMT = 0 (there are no payments)

FV = 100 000 (we want to have this much)

P/Y = 4 (4 quarters in a year)

C/Y = 4 (compounded 4 times a year)

We get that PV = −81913.89. So he should invest 81913.89 PLN. Do not use the negative sign in your final answer.

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Payments

Note that in all examples so far we've set $PMT = 0$ as there were no payments made during the investment.

PMT is any money that is paid in or withdrawn during the investment.

Note that in the IB examination payments will always be made at the end of the period. So in CASIO press SET UP and make sure you have Payment END

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Payments

Tomasz invests 100 000 PLN into savings account that pays 3% annual interest compounded monthly. At the end of each month he withdraws 1500 PLN from his account. Calculate how much money does he have on his account after 4 years.

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Here we will go directly to TVM.

Payments

We have:

$n = 48$ (we have 48 months in 4 years)

$I\% = 3$ (annual interest rate)

$PV = -100\,000$ (must be negative)

$PMT = 1500$ (must be positive, he gets this money)

$FV = ?$ (unknown)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $FV = 38335.90$

Payments

We have:

$n = 48$ (we have 48 months in 4 years)

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$PV = $ -100 000 (must be negative)

$PMT = 1500$ (must be positive, he gets this money)

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We have:

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$PMT = 1500$ (must be positive, he gets this money)

$FV = ?$ (unknown)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $FV = 36335.99$.

Tomasz invests 50 000 PLN into savings account that pays 2.2% annual interest compounded quarterly. At the end of each quarter he puts another 5000 PLN into his account. Calculate how much money does he have on his account after 5 years.

We will use TVM. Note that this time PMT will be negative, since he puts the money into the account.

Payments 2

Tomasz invests 50 000 PLN into savings account that pays 2.2% annual interest compounded quarterly. At the end of each quarter he puts another 5000 PLN into his account. Calculate how much money does he have on his account after 5 years.

We will use TVM. Note that this time PMT will be negative, since he puts the money into the account.

Payments

We should input:

n = 20 (we have 20 quarters in 5 years)

I% = 2.2 (annual interest rate)

PV = -50 000 (must be negative)

PMT = -5000 (negative, he invests the money)

FV = ? (unknown)

P/Y = 4 (4 payments per year)

C/Y = 4 (compounded 4 times per year)

We get FV = 161198.61

Payments

We should input:

$n = 20$ (we have 20 quarters in 5 years)

$I\% = 2.2$ (annual interest rate)

$PV = $ -50 000 (must be negative)

$PMT = $ -5000 (negative, he invests the money)

$FV = ?$ (unknown)

$P/Y = 4$ (4 payments per year)

$C/Y = 4$ (compounded 4 times per year)

Payments

We should input:

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$PMT = $ -5000 (negative, he invests the money)

$FV = ?$ (unknown)

$P/Y = 4$ (4 payments per year)

$C/Y = 4$ (compounded 4 times per year)

We get $FV = 161198.61$.

Loans

Tomasz takes a loan of 120 000PLN. The loan is to be repaid in 20 years with equal monthly payments. The annual interest on the loan is 4.5% compounded monthly. Calculate the monthly repayments.

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Again we will use TVM.

We have:

$n = 240$ (we have 240 months in 20 years)

$I\% = 4.5$ (annual interest rate)

$PV = 120\ 000$ (this time it's positive as we get this money)

$PMT = ?$ (we want to calculate the repayments, they will be negative)

$FV = 0$ (the loan needs to be repaid)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $PMT = -759.18$. So Tomasz needs to pay 759.18 PLN per month to repay his loan in 20 years.

Loans

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   $n = 240$ (we have 240 months in 20 years)

  $I\% = 4.5$ (annual interest rate)

  $PV = 120\ 000$ (this time it's positive as we get this money)

$PMT = ?$ (we want to calculate the repayments, they will be negative)

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$PV = 120\ 000$ (this time it's positive as we get this money)

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$C/Y = 12$ (compounded 12 times per year)

And we get $PMT = -759.18$. So Tomasz needs to pay 759.18 PLN per month to repay his loan in 20 years.

Annuity

Tomasz invests 150 000 PLN into an annuity that pays a fixed amount of money per month. The annual interest on the annuity is 1.5$ and it is compounded monthly. Calculate how much money can Tomasz receive each month, if the annuity is to last for 15 year.

TVM on the next slide.

Annuity

Our inputs:

$n = 180$ (we have 180 months in 15 years)

$I\% = 1.5$ (annual interest rate)

$PV = -150\,000$ (negative, he invests it)

$PMT = ?$ (should be positive, as he gets this money)

$FV = 0$ (no more money on the investment)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $PMT = 931.13$. Tomasz can withdraw 931.13 PLN each month for 15 years.

Annuity

Our inputs:

$n = 180$ (we have 180 months in 15 years)

$I\% = 1.5$ (annual interest rate)

$PV = -150\,000$ (negative, he invests it)

$PMT = ?$ (should be positive, as he gets this money)

$FV = 0$ (no more money on the investment)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

Annuity

Our inputs:

$n = 180$ (we have 180 months in 15 years)

$I\% = 1.5$ (annual interest rate)

$PV = -150\ 000$ (negative, he invests it)

$PMT = ?$ (should be positive, as he gets this money)

$FV = 0$ (no more money on the investment)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $PMT = 931.11$. Tomasz can withdraw 931.11 PLN each month for 15 years.

Now we move on to some more complicated examples. These are the types of examples you may expect to appear on the exam.

Quick note on writing out the solution. If you use TVM on GDC, then as your solution you should write down your GDC input (n = ..., I% = ..., etc.). But make sure you write down the answer explicitly, for example "Tomasz needs to invest 666 PLN"

Now we move on to some more complicated examples. These are the types of examples you may expect to appear on the exam.

Quick note on writing out the solution. If you use TVM on GDC, then as your solution you should write down your GDC input (n=..., i% = ..., etc.).

Now we move on to some more complicated examples. These are the types of examples you may expect to appear on the exam.

Quick note on writing out the solution. If you use TVM on GDC, then as your solution you should write down your GDC input (n=..., i% = ..., etc.). But make sure you write down the answer explicitly, for example: "Tomasz needs to invest 666 PLN".

Exam-style question 1

Maria takes a loan of 300 000 PLN. The annual interest on the loan is 3.5% and it is compounded monthly.

(a) Calculate the monthly repayments if the loan is to be repaid in 20 years.

(b) Calculate the total interest Maria would pay for her loan.

(c) After 10 years Maria is able to increase the monthly repayments to 2500 PLN. Calculate how long it will take to repay the whole loan.

Part (a) is easy:

$n = 240$ (we have 240 months in 20 years)

$I\% = 3.5$ (annual interest rate)

$PV = 300\,000$ (positive, she gets this money)

$PMT = ?$ (should be negative as she repays it)

$FV = 0$ (loan is repaid)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $PMT = -1739.88$. Maria needs to pay 1739.88 PLN each month to repay her loan in 20 years.

Exam-style question 1a

Part (a) is easy:

$n = 240$ (we have 240 months in 20 years)

$I\% = 3.5$ (annual interest rate)

$PV = 300\ 000$ (positive, she gets this money)

$PMT = ?$ (should be negative as she repays it)

$FV = 0$ (loan is repaid)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $PMT = -1739.88$. Maria needs to pay 1739.88 PLN each month to repay her loan in 20 years

Exam-style question 1a

Part (a) is easy:

$n = 240$ (we have 240 months in 20 years)

$I\% = 3.5$ (annual interest rate)

$PV = 300\ 000$ (positive, she gets this money)

$PMT = ?$ (should be negative as she repays it)

$FV = 0$ (loan is repaid)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $PMT = -1739.88$. Maria needs to pay 1739.88 PLN each month to repay her loan in 20 years.

In part (b) we first need to calculate how much she paid back. She paid 1739.88 per month for 240 months. So she paid a total of

$$1739.88 \cdot 240 = 417571.2$$

The loan was 300 000, so the total interest was

$$417571.2 - 300000 = 117571.2$$

In part (b) we first need to calculate how much she paid back. She paid 1739.88 per month for 240 months. So she paid a total of

$$1739.88 \cdot 240 = 417571.2$$

The loan was 300 000, so the total interest was:

$$417571.2 - 300000 = 117571.2$$

Exam-style question 1c

Part (c) is done in 2 parts. First we need to see how much she still owes after 10 years:

$n = 120$ (we have 120 months in 10 years)

$I\% = 3.5$ (annual interest rate)

$PV = 300\,000$ (positive, she gets this money)

$PMT = -1739.88$ (should be negative as she repays it)

$FV = ?$ (how much she still owes)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $FV = -175048.09$. So she still need to repay 175048.09PLN

Exam-style question 1c

Part (c) is done in 2 parts. First we need to see how much she still owes after 10 years:

$n = 120$ (we have 120 months in 10 years)

$I\% = 3.5$ (annual interest rate)

$PV = 300\ 000$ (positive, she gets this money)

$PMT = -1739.88$ (should be negative as she repays it)

$FV = ?$ (how much she still owes)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $FV = -175948.09$. So she still need to repay 175948.09PLN

Exam-style question 1c

Part (c) is done in 2 parts. First we need to see how much she still owes after 10 years:

n = 120 (we have 120 months in 10 years)

$I\%$ = 3.5 (annual interest rate)

PV = 300 000 (positive, she gets this money)

PMT = -1739.88 (should be negative as she repays it)

FV = ? (how much she still owes)

P/Y = 12 (12 payments per year)

C/Y = 12 (compounded 12 times per year)

And we get $FV = -175948.09$. So she still need to repay 175948.09PLN.

Now we increase the payments:

- $n = ?$ (unknown number of months)
- $I\% = 3.5$ (annual interest rate)
- $PV = 175948.09$ (positive, she gets this money)
- $PMT = -2500$ (increased repayments)
- $FV = 0$ (loan is to be repaid)
- $P/Y = 12$ (12 payments per year)
- $C/Y = 12$ (compounded 12 times per year)

And we get $n = 78.9$. So she needs another 79 months to repay the loan. The total time to repay the loan is then 199 months (120 + 79).

Now we increase the payments:

n = ? (unknown number of months)

$I\%$ = 3.5 (annual interest rate)

PV = 175948.09 (positive, she gets this money)

PMT = -2500 (increased repayments)

FV = 0 (loan is to be repaid)

P/Y = 12 (12 payments per year)

C/Y = 12 (compounded 12 times per year)

And we get n = 79.9. So she needs another 79 months to repay the loan.
The total time to repay the loan is then 199 months (120 + 79).

Exam-style question 1c

Now we increase the payments:

$n = ?$ (unknown number of months)

$I\% = 3.5$ (annual interest rate)

$PV = 175948.09$ (positive, she gets this money)

$PMT = -2500$ (increased repayments)

$FV = 0$ (loan is to be repaid)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $n = 78.9$. So she needs another 79 months to repay the loan. The total time to repay the loan is then 199 months $(120 + 79)$.

Exam-style question 2

Maria invests 50 000 PLN into savings accounts that pays 1.5% annual interest rate compounded monthly. Additional she puts another 1000 PLN into the account at the end of each month. Tomasz also invests a certain sum into a savings account that pays 1.6% annual interest rate compounded quarterly. He makes no further payments into the account. Calculate how much he needs to invest to have the same amount of money in the account in 5 years.

We will do this in two parts. First we will calculate the amount in Maria's account in 5 years.

Exam-style question 2

Maria invests 50 000 PLN into savings accounts that pays 1.5% annual interest rate compounded monthly. Additional she puts another 1000 PLN into the account at the end of each month. Tomasz also invests a certain sum into a savings account that pays 1.6% annual interest rate compounded quarterly. He makes no further payments into the account. Calculate how much he needs to invest to have the same amount of money in the account in 5 years.

We will do this in two parts. First we will calculate the amount in Maria's account in 5 years.

We input:

$n = 60$ (60 months in 5 years)

$I\% = 1.5$ (annual interest rate)

$PV = -50\,000$ (negative, she invests this)

$PMT = -1000$ (also negative)

$FV = ?$ (unknown)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $FV = 116158.62$.

We input:

$n = 60$ (60 months in 5 years)

$I\% = 1.5$ (annual interest rate)

$PV = $ -50 000 (negative, she invests this)

$PMT = $ -1000 (also negative)

$FV = ?$ (unknown)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $FV = 116158.62$.

Exam-style question 2

We input:

$n = 60$ (60 months in 5 years)

$I\% = 1.5$ (annual interest rate)

$PV = $ -50 000 (negative, she invests this)

$PMT = $ -1000 (also negative)

$FV = ?$ (unknown)

$P/Y = 12$ (12 payments per year)

$C/Y = 12$ (compounded 12 times per year)

And we get $FV = 116158.62$.

Now we look at Tomasz:

$n = 60$ (60 months in 5 years)

$I\% = 1.6$ (annual interest rate)

$PV = ?$ (unknown, will be negative)

$PMT = 0$ (no payments)

$FV = 116158.6181$ (what Maria should have)

$P/Y = 4$ (4 periods per year)

$C/Y = 4$ (compounded 4 times per year)

And we get $PV = -91417.36$. Tomasz needs to invest 91417.36 PLN to have the same amount as Maria in 5 years.

Exam-style question 2

Now we look at Tomasz:

$n = 60$ (60 months in 5 years)

$I\% = 1.6$ (annual interest rate)

$PV = ?$ (unknown, will be negative)

$PMT = 0$ (no payments)

$FV = 116158.6181$ (what Maria should have)

$P/Y = 4$ (4 periods per year)

$C/Y = 4$ (compounded 4 times per year)

And we get $PV = -91417.36$. Tomasz needs to invest 91417.36 PLN to have the same amount as Maria in 5 years.

Exam-style question 2

Now we look at Tomasz:

$n = 60$ (60 months in 5 years)

$I\% = 1.6$ (annual interest rate)

$PV = ?$ (unknown, will be negative)

$PMT = 0$ (no payments)

$FV = 116158.6181$ (what Maria should have)

$P/Y = 4$ (4 periods per year)

$C/Y = 4$ (compounded 4 times per year)

And we get $PV = -91417.36$. Tomasz needs to invest 91417.36 PLN to have the same amount as Maria in 5 years.

Not really an exam-style question 3

Tomasz takes a loan of 200 000 PLN. The interest on the loan is 3.2% compounded quarterly. The loan is to be repaid in monthly payments of 1477.74 PLN per month.
(a) How long does it take to repay the loan?
(b) Calculate the total interest on the loan.
(b) After 5 years the bank decreases the interest on the loan to 3% how much quicker will Tomasz repay his loan given that the repayments are not changed?

This is an unusual question, because $P/Y \neq C/Y$. I don't think a question like this will be on the exam, however you should be prepared just in case.

Tomasz takes a loan of 200 000 PLN. The interest on the loan is 3.2% compounded quarterly. The loan is to be repaid in monthly payments of 1477.74 PLN per month.
(a) How long does it take to repay the loan?
(b) Calculate the total interest on the loan.
(b) After 5 years the bank decreases the interest on the loan to 3% how much quicker will Tomasz repay his loan given that the repayments are not changed?

This is an unusual question, because $P/Y \neq C/Y$. I don't think a question like this will be on the exam, however you should be prepared just in case.

Exam-style question 3a

Our input:

$n = ?$ (unknown number of months)

$I\% = 3.2$ (annual interest rate)

$PV = 200\,000$ (positive, he gets the money)

$PMT = -1477.74$ (negative, he gives the money back)

$FV = 0$ (loan is repaid)

$P/Y = 12$ (12 payments per year)

$C/Y = 4$ (compounded 4 times per year)

And we get $n == 168$. So the loan will be repaid in 168 months or 14 years.

Exam-style question 3a

Our input:

$n = ?$ (unknown number of months)

$I\% = 3.2$ (annual interest rate)

$PV = 200\ 000$ (positive, he gets the money)

$PMT = -1477.74$ (negative, he gives the money back)

$FV = 0$ (loan is repaid)

$P/Y = 12$ (12 payments per year)

$C/Y = 4$ (compounded 4 times per year)

And we get n == 168. So the loan will be repaid in 168 months or 14 years.

Exam-style question 3a

Our input:

$n = ?$ (unknown number of months)

$I\% = 3.2$ (annual interest rate)

$PV = 200\ 000$ (positive, he gets the money)

$PMT = -1477.74$ (negative, he gives the money back)

$FV = 0$ (loan is repaid)

$P/Y = 12$ (12 payments per year)

$C/Y = 4$ (compounded 4 times per year)

And we get $n = 168$. So the loan will be repaid in 168 months or 14 years.

Total interest is:

$$168 \cdot 1477.74 - 200000 = 48260.32$$

Because he pays 1477.74 PLN for 168 months for a from of 200 000 PLN

Exam-style question 3b

Total interest is:

$$168 \cdot 1477.74 - 200000 = 48260.32$$

Because he pays 1477.74 PLN for 168 months for a loan of 200 000 PLN.

Exam-style question 3c

We first calculate how much he still owes after 5 years:

$n = 60$ (60 months in 5 years)

$I\% = 3.2$ (annual interest rate)

$PV = 200\,000$ (positive, he gets the money)

$PMT = -1477.74$ (negative, he gives the money back)

$FV = ?$ (unknown)

$P/Y = 12$ (12 payments per year)

$C/Y = 4$ (compounded 4 times per year)

And we get $FV = -138560.3338$.

We first calculate how much he still owes after 5 years:

$n = 60$ (60 months in 5 years)

$I\% = 3.2$ (annual interest rate)

$PV = 200\ 000$ (positive, he gets the money)

$PMT = -1477.74$ (negative, he gives the money back)

$FV = ?$ (unknown)

$P/Y = 12$ (12 payments per year)

$C/Y = 4$ (compounded 4 times per year)

And we get $FV = -138560.3338$.

Exam-style question 3c

We first calculate how much he still owes after 5 years:

$n = 60$ (60 months in 5 years)

$I\% = 3.2$ (annual interest rate)

$PV = 200\ 000$ (positive, he gets the money)

$PMT = $ -1477.74 (negative, he gives the money back)

$FV = ?$ (unknown)

$P/Y = 12$ (12 payments per year)

$C/Y = 4$ (compounded 4 times per year)

And we get $FV = -138560.3338$.

Exam-style question 3c

Now we decrease the interest rate:

$n = 7$ (unknown)

I% = 3 (new interest rate)

PV = 138560.3338 (he still owes this)

PMT = -1477.74 (repayments are not changed)

FV = 0 (loan is repaid)

P/Y = 12 (12 payments per year)

C/Y = 4 (compounded 4 times per year)

And we get $n = 107$. So the loan takes a total of 107 months to be repaid. He repays it only one month earlier.

Exam-style question 3c

Now we decrease the interest rate:

$n = ?$ (unknown)

$I\% = 3$ (new interest rate)

$PV = 138560.3338$ (he still owes this)

$PMT = -1477.74$ (repayments are not changed)

$FV = 0$ (loan is repaid)

$P/Y = 12$ (12 payments per year)

$C/Y = 4$ (compounded 4 times per year)

And we get $n = 107$. So the loan takes a total of 107 months to be repaid. He repays it only one month earlier.

Exam-style question 3c

Now we decrease the interest rate:

$n = ?$ (unknown)

$I\% = 3$ (new interest rate)

$PV = 138560.3338$ (he still owes this)

$PMT = -1477.74$ (repayments are not changed)

$FV = 0$ (loan is repaid)

$P/Y = 12$ (12 payments per year)

$C/Y = 4$ (compounded 4 times per year)

And we get $n = 107$. So the loan takes a total of 167 months to be repaid. He repays it only one month earlier.

Make sure you go through all the examples in this presentation.

If there are any questions, ask me in person or on Teams chat.

We will do some more exam-style questions in class.