

1. A biased coin is weighted such that the probability of obtaining a head is  $\frac{4}{7}$ . The coin is tossed 6 times and  $X$  denotes the number of heads observed. Find the value of the ratio  $\frac{P(X = 3)}{P(X = 2)}$ .  
(Total 4 marks)

2. A discrete random variable  $X$  has a probability distribution given in the following table.

$x$	0.5	1.5	2.5	3.5	4.5	5.5
$P(X = x)$	0.15	0.21	$p$	$q$	0.13	0.07

- (a) If  $E(X) = 2.61$ , determine the value of  $p$  and of  $q$ .  
(4)
- (b) Calculate  $\text{Var}(X)$  to three significant figures.  
(2)  
(Total 6 marks)

3. In each round of two different games Ying tosses three fair coins and Mario tosses two fair coins.

- (a) The first game consists of one round. If Ying obtains more heads than Mario, she receives \$5 from Mario. If Mario obtains more heads than Ying, he receives \$10 from Ying. If they obtain the same number of heads, then Mario receives \$2 from Ying. Determine Ying's expected winnings.  
(12)
- (b) They now play the second game, where the winner will be the player who obtains the larger number of heads in a round. If they obtain the same number of heads, they play another round until there is a winner. Calculate the probability that Ying wins the game.  
(8)  
(Total 20 marks)

4. Tim throws two identical fair dice simultaneously. Each die has six faces: two faces numbered 1, two faces numbered 2 and two faces numbered 3. His score is the sum of the two numbers shown on the dice.

(a) (i) Calculate the probability that Tim obtains a score of 6.

(ii) Calculate the probability that Tim obtains a score of at least 3.

(3)

Tim plays a game with his friend Bill, who also has two dice numbered in the same way. Bill's score is the sum of the two numbers shown on his dice.

(b) (i) Calculate the probability that Tim and Bill **both** obtain a score of 6.

(ii) Calculate the probability that Tim and Bill obtain the same score.

(4)

(c) Let  $X$  denote the largest number shown on the four dice.

(i) Show that  $P(X \leq 2) = \frac{16}{81}$ .

(ii) Copy and complete the following probability distribution table.

$x$	1	2	3
$P(X = x)$	$\frac{1}{81}$		

(iii) Calculate  $E(X)$  and  $E(X^2)$  and hence find  $\text{Var}(X)$ .

(10)

(d) Given that  $X = 3$ , find the probability that the sum of the numbers shown on the four dice is 8.

(4)

(Total 21 marks)