Practice questions 11.3

6. The random variable *X* has probability distribution as shown in the table below.

\boldsymbol{x}	1	2	3
P(X=x)	0.4	0.3	0.3

Find the probability that the sum of two independent observations of *X* is 4.

- **7.** Seven students are randomly arranged in a line. Find the probability that Anne and Beth are standing next to each other.
- **8.** A bag contains seven caramels and one chocolate. Three children, Peng, Quinn and Raul, take turns to pick a sweet out of the bag at random. If the sweet is a chocolate, they take it; if it is a caramel, they put it back and pass the bag around.
 - (a) Find the probability that Raul gets the chocolate on his second turn.
 - (b) Find the probability that the chocolate is still in the bag when it gets to Quinn for the fifth time.

Practice questions 11.4

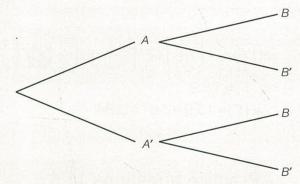
- **9.** Given that P(B) = 0.5, $P(A' \cap B') = 0.2$ and $P(B \mid A) = 0.4$, find P(A).
- **10.** All of the 100 students at a college take part in at least one of three activities: chess, basketball and singing. 10 play both chess and basketball, 12 play chess and sing, and 7 take part in both singing and basketball. 40 students play basketball, 62 play chess and 22 sing.
 - (a) How many students take part in all three activities?
 - (b) A student is chosen at random. Given that this student plays basketball and sings, what is the probability that she also plays chess?
- **2.** A discrete random variable *X* is given by $P(X = n) = kn^2$ for n = 1, 2, 3, 4. Find the expected value and variance of *X*.
- **3.** If P(A) = 0.3, $P(B \mid A') = 0.5$ and $P(A \mid B) = \frac{7}{16}$, find $P(B \mid A)$.
- **4.** The discrete random variable *Y* has probability distribution as shown in the table below.

y	1	2	3	4
P(Y=y)	0.1	0.2	p	q

Given that E(Y) = 3.1, find the values of p and q.

Practice questions 11.5

- **11.** The events A and B are such that $P(B \mid A) = 0.2$, $P(B' \mid A') = 0.3$, $P(A \mid B) = 0.4$ and P(A) = x.
 - (a) Complete the following tree diagram.
 - (b) Find the value of x.
 - (c) State, with a reason, whether the events *A* and *B* are independent.



- **12.** Given that P(B) = 0.3, $P(A \mid B) = 0.6$ and $P(A \mid B') = 0.4$, find $P(B \mid A)$.
- **13.** Every morning I either walk or cycle to school, with equal probability. If I walk, the probability that I am late is 0.2. If I cycle, the probability that I am late is 0.4. Given that I was late for school yesterday, what is the probability that I walked?
- **14.** A large box contains three different types of toys. One third of the toys are cars, one quarter are yo-yos and the rest are balloons. 20% of the cars, 30% of the yo-yos and 40% of the balloons are pink. A toy is selected at random from the box. Given that the toy is pink, find the probability that it is a balloon.

Practice questions 11.6

15. Find the expected value and the variance of the following discrete random variable.

Z	11	13	17	19	23
P(Z=z)	0.2	0.1	0.1	0.2	0.4

16. The random variable X has probability distribution as shown in the table below.

\boldsymbol{x}	1	2	3	4
P(X = x)	С	p	0.4	0.2

Given that E(X) = 2.6, find the values of c and p.

45 A random variable *X* has the probability distribution alongside:

- a Find k.
- **b** Find E(X), Var(X), and the standard deviation σ .
- \bullet Find the mode and median of X.

46 A random variable X has the probability mass function $P(x) = \frac{x^2 + kx}{50}$ for x = 1, 2, 3, 4.

- **a** Find k.
- **b** Find the mean of the distribution of X.
- c Find $P(X \ge 2)$.

48 A bag contains 1 blue ticket, 3 red tickets, and 8 yellow tickets. A player randomly selects a ticket from the bag, and receives \$40 for a blue ticket, \$20 for a red ticket, and \$5 for a yellow ticket.

- a Calculate the expected return for one trial of this game.
- **b** Given that the game costs \$15 to play, explain why it would not be advisable to play this game.

• Find the number of extra red tickets that should be added to the bag to make the game fair.

49 X has probability distribution:

x	1	2	3	4
P(X = x)	0.25	0.38	0.17	0.2

Find:

a $\mathrm{E}(X)$

b Var(X)

 $\sigma(X)$

P(X=x)

d E(X+2)

- e Var(2-3X)
- f $\sigma(2X-10)$

53 5% of all items coming off a production line are defective. The manufacturer packages the items in boxes of six, and guarantees a refund if more than two items in a box are defective.

- a On what percentage of boxes will the manufacturer have to pay a refund?
- **b** Patrick purchases 10 boxes. Find the probability that he will get a refund for exactly 1 box.

54 A hundred seeds are planted in ten rows of ten seeds per row. Assuming that each seed independently germinates with probability $\frac{1}{2}$, find the probability that the row with the maximum number of germinations contains at least 8 seedlings.

55 In a game, a player rolls a biased four-sided die. The probability of obtaining each possible score is shown in the table.

Score	1	2	3	4
Probability	1 12	k	1/4	$\frac{1}{2}$

- **a** Find the value of k.
- Let the random variable X denote the number of 2s that occur when the die is rolled 2400 times. Calculate the exact mean and standard deviation of X.

EXAM-STYLE QUESTION

5 A random variable T takes only integer values and has probability distribution function defined by:

$$f(t) = P(T = t) = \begin{cases} kt^2 & \text{if } t = 1, 2, 3\\ k(8-t)^2 & \text{if } t = 4, 5, 6, 7\\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

- **a** Find the value of k.
- **b** Calculate P(T=4), $P(T \le 4)$ and $P(T=4 | T \le 4)$.
- **c** Find E(T) and Var(T).
- **d** Determine the mode of T.

4 The probability distribution of a discrete random variable X is given by

f(x) = kx where x = 1, 2, ..., n and k is a parameter.

- **a** Show that $k = \frac{2}{n^2 + n}$ **b** Hence find E(X)

EXAM-STYLE QUESTIONS

- **3** In the mass production of light bulbs the probability that one bulb is defective is 1%. Bulbs are selected at random and put in packs of eight.
 - **a** If a pack is selected at random, what is the probability that it will contain:
 - i at least one defective bulb
 - ii not more than two defective bulbs?
 - **b** Given that a pack selected at random contains at least one defective bulb, what is the probability that it contains exactly two defective bulbs?
- **4** Let $X \sim B$ (6, 0.35). Find
 - **a** The mode a of X
 - **b** The median b of X
 - **c** P(X < 2a | X > b)

* EXAM-STYLE QUESTION

5 A random variable R follows a binomial distribution B(n, p) with mean 2 and variance 1.5. Find the values of n and p.

EXAM-STYLE QUESTIONS

- **6** In a multiple choice test there are 20 questions. For each question there is a choice of four answers and only one of these is correct.
 - a If a student guesses each answer find the probability that he gets
 - i none correct
- ii more than ten correct
- iii not more than five correct.
- **b** Calculate the mean and standard deviation of the number of correct answers.
- **c** Suppose that five students guess the answers to the test. What is the probability that at least two of them get more than ten answers correct?
- **10** Consider two independent random variables X and Y such that $X \sim B(2, a)$ and $Y \sim B(2, b)$.
 - Let *W* be the random variable that represents the product of each value of *X* with each value of *Y*.
 - a Construct a table showing the probability distribution of W.
 - **b** Hence find an expression for E(W) in terms of a and b.