1. (a) Differentiate $f(x) = \arcsin x + 2\sqrt{1-x^2}$, $x \in [-1, 1]$.

(3)

(b) Find the coordinates of the point on the graph of $y = f(x)$ in $[-1, 1]$, where the gradient of the tangent to the curve is zero.

(3)
(Total 6 marks)

2. Find the gradient of the curve $e^{xy} + \ln(y^2) + e^y = 1 + e$ at the point $(0, 1)$.

(Total 7 marks)

3. Find the gradient of the tangent to the curve $x^3 y^2 = \cos(\pi y)$ at the point $(-1, 1)$.

(Total 6 marks)

4. The cubic curve $y = 8x^3 + bx^2 + cx + d$ has two distinct points P and Q, where the gradient is zero.

(a) Show that $b^2 > 24c$.

(4)

(b) Given that the coordinates of P and Q are $\left(\frac{1}{2}, -12\right)$ and $\left(-\frac{3}{2}, 20\right)$, respectively, find the values of b, c and d.

(4)
(Total 8 marks)

5. If $y = \ln\left(\frac{1}{3}(1+e^{-2x})\right)$, show that $\frac{dy}{dx} = \frac{2}{3}(e^{-y} - 3)$.

(Total 7 marks)

6. Consider the curve with equation $x^2 + xy + y^2 = 3$.

(a) Find in terms of k, the gradient of the curve at the point $(-1, k)$.

(5)

(b) Given that the tangent to the curve is parallel to the x-axis at this point, find the value of k.

(1)
(Total 6 marks)