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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

Tuesday 2 November 2021 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 16]

A group of 1280 students were asked which electronic device they preferred. The results per age group are given in the following table.

	Age			
Preferred device	**11–13**	**14–16**	**17–18**	**Total**
Laptop	143	160	153	456
Tablet	205	224	131	560
Mobile phone	72	128	64	264
Total	420	512	348	1280

(a) A student from the group is chosen at random. Calculate the probability that the student

(i) prefers a tablet.

(ii) is $11–13$ years old and prefers a mobile phone.

(iii) prefers a laptop **given that** they are $17–18$ years old.

(iv) prefers a tablet or is $14–16$ years old. [9]

A χ^2 test for independence was performed on the collected data at the $1\,\%$ significance level. The critical value for the test is 13.277.

(b) State the null and alternative hypotheses. [1]

(c) Write down the number of degrees of freedom. [1]

(d) (i) Write down the χ^2 test statistic.

(ii) Write down the p-value.

(iii) State the conclusion for the test in context. Give a reason for your answer. [5]

2. [Maximum mark: 16]

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	**Number of applications received in year n**
1	12 300
2	12 669

(a) Calculate the percentage increase in applications from the first year to the second year. [2]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

(b) (i) Write down the common ratio of the sequence.

(ii) Find an expression for u_n.

(iii) Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. [4]

In the first year there were $10\,380$ places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

(c) Write down an expression for v_n. [2]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an $\$80$ acceptance fee.

(d) Calculate the total amount of acceptance fees paid to the university in the first 10 years. [3]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

(e) Find k. [3]

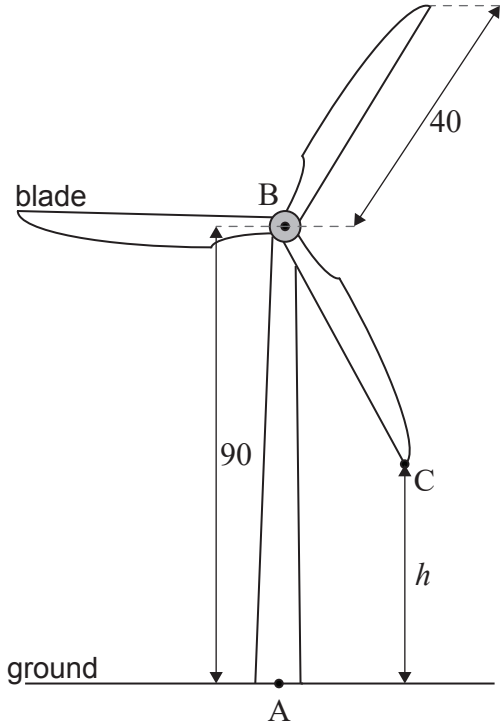
(f) State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. [2]

3. [Maximum mark: 20]

A wind turbine is designed so that the rotation of the blades generates electricity. The turbine is built on horizontal ground and is made up of a vertical tower and three blades.

The point A is on the base of the tower directly below point B at the top of the tower. The height of the tower, AB, is $90\,\text{m}$. The blades of the turbine are centred at B and are each of length $40\,\text{m}$. This is shown in the following diagram.

diagram not to scale



The end of one of the blades of the turbine is represented by point C on the diagram. Let h be the height of C above the ground, measured in metres, where h varies as the blade rotates.

(a) Find the

(i) maximum value of h.

(ii) minimum value of h. [2]

The blades of the turbine complete 12 rotations per minute under normal conditions, moving at a constant rate.

(b) (i) Find the time, in seconds, it takes for the blade [BC] to make one complete rotation under these conditions.

(ii) Calculate the angle, in degrees, that the blade [BC] turns through in one second. [3]

(This question continues on the following page)

(Question 3 continued)

The height, h, of point C can be modelled by the following function. Time, t, is measured from the instant when the blade [BC] first passes [AB] and is measured in seconds.

$$h(t) = 90 - 40\cos(72t^\circ),\ t \geq 0$$

(c) (i) Write down the amplitude of the function.

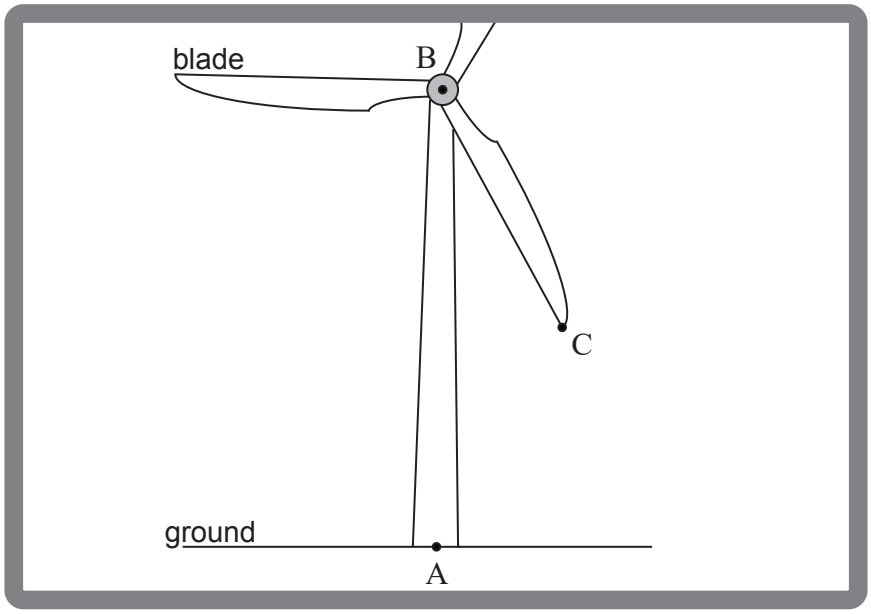
(ii) Find the period of the function. [2]

(d) Sketch the function $h(t)$ for $0 \leq t \leq 5$, clearly labelling the coordinates of the maximum and minimum points. [3]

(e) (i) Find the height of C above the ground when $t = 2$.

(ii) Find the time, in seconds, that point C is above a height of $100\,\text{m}$, during each complete rotation. [5]

Looking through his window, Tim has a partial view of the rotating wind turbine. The position of his window means that he cannot see any part of the wind turbine that is **more than $100\,\text{m}$** above the ground. This is illustrated in the following diagram.



(f) (i) At any given instant, find the probability that point C is visible from Tim's window.

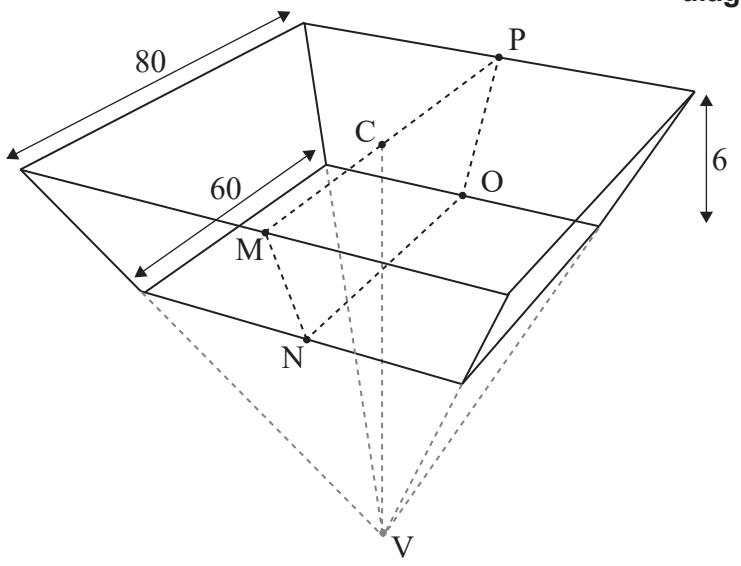
The wind speed increases. The blades rotate at twice the speed, but still at a constant rate.

(ii) At any given instant, find the probability that Tim can see point C from his window. Justify your answer. [5]

4. [Maximum mark: 14]

A large water reservoir is built in the form of part of an upside-down right pyramid with a horizontal square base of length 80 metres. The point C is the centre of the square base and point V is the vertex of the pyramid.

diagram not to scale



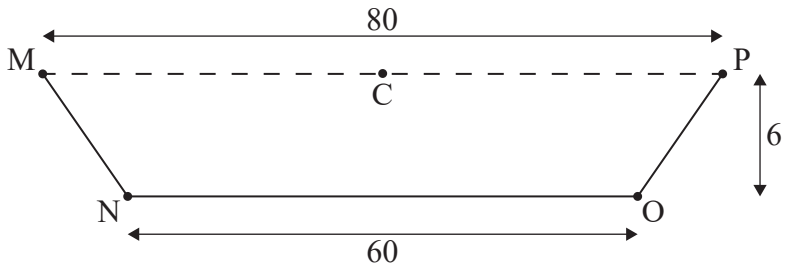
The bottom of the reservoir is a square of length 60 metres that is parallel to the base of the pyramid, such that the depth of the reservoir is 6 metres as shown in the diagram.

(This question continues on the following page)

(Question 4 continued)

The second diagram shows a vertical cross section, MNOPC, of the reservoir.

diagram not to scale



(a) Find the angle of depression from M to N. [2]

(b) (i) Find CV.

(ii) Hence or otherwise, show that the volume of the reservoir is $29\,600\,\text{m}^3$. [5]

Every day $80\,\text{m}^3$ of water from the reservoir is used for irrigation.

Joshua states that, if no other water enters or leaves the reservoir, then when it is full there is enough irrigation water for at least one year.

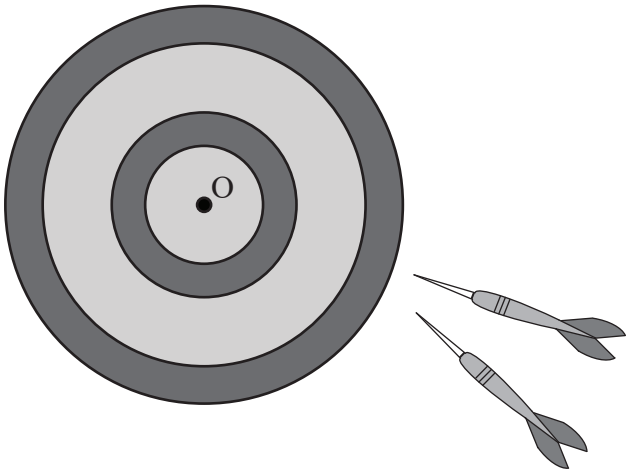
(c) By finding an appropriate value, determine whether Joshua is correct. [2]

To avoid water leaking into the ground, the five interior sides of the reservoir have been painted with a watertight material.

(d) Find the area that was painted. [5]

5. [Maximum mark: 14]

Arianne plays a game of darts.



The distance that her darts land from the centre, O, of the board can be modelled by a normal distribution with mean $10\,\text{cm}$ and standard deviation $3\,\text{cm}$.

(a) Find the probability that

(i) a dart lands less than $13\,\text{cm}$ from O.

(ii) a dart lands more than $15\,\text{cm}$ from O. [3]

Each of Arianne's throws is independent of her previous throws.

(b) Find the probability that Arianne throws two consecutive darts that land more than $15\,\text{cm}$ from O. [2]

In a competition a player has three darts to throw on each turn. A point is scored if a player throws **all** three darts to land within a central area around O. When Arianne throws a dart the probability that it lands within this area is 0.8143.

(c) Find the probability that Arianne does **not** score a point on a turn of three darts. [2]

In the competition Arianne has ten turns, each with three darts.

(d) (i) Find the probability that Arianne scores at least 5 points in the competition.

(ii) Find the probability that Arianne scores at least 5 points and less than 8 points.

(iii) Given that Arianne scores at least 5 points, find the probability that Arianne scores less than 8 points. [7]

References:

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