Name:

Mathematics IB HL Test 7
Calculus I

March 2, 2021

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Calculators are **not allowed** for this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [**72 marks**].
- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to **show all working**.
- Write your solutions in the space provided.

SECTION A

1. [Maximum mark: 4]

Find the following limits:

(i) $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x^2 + 2x - 8}$

(ii) $\lim_{x \to 0} \dfrac{x^2 - x}{\sin 3x}$

2. [Maximum mark: 5]

Show from **first principles** that the derivative of $y = \sqrt{x} + x$ is given by $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}} + 1$.

3. [Maximum mark: 6]

Find the area of the triangle enclosed by the tangent to the curve $y = \dfrac{\arctan x}{x}$ at $x = 1$ and the axes.

4. [Maximum mark: 5]

Value of $f(x)$, $f'(x)$, $g(x)$ and $g'(x)$ for some values of x are displayed in the table below:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	4	3
2	3	4	1	2
3	1	3	2	4
4	4	2	3	1

Let $p(x) = f(g(x))$ and $q(x) = f(x)g(x)$. Find $p'(1)$ and $q'(p(1))$.

5. [Maximum mark: 5]

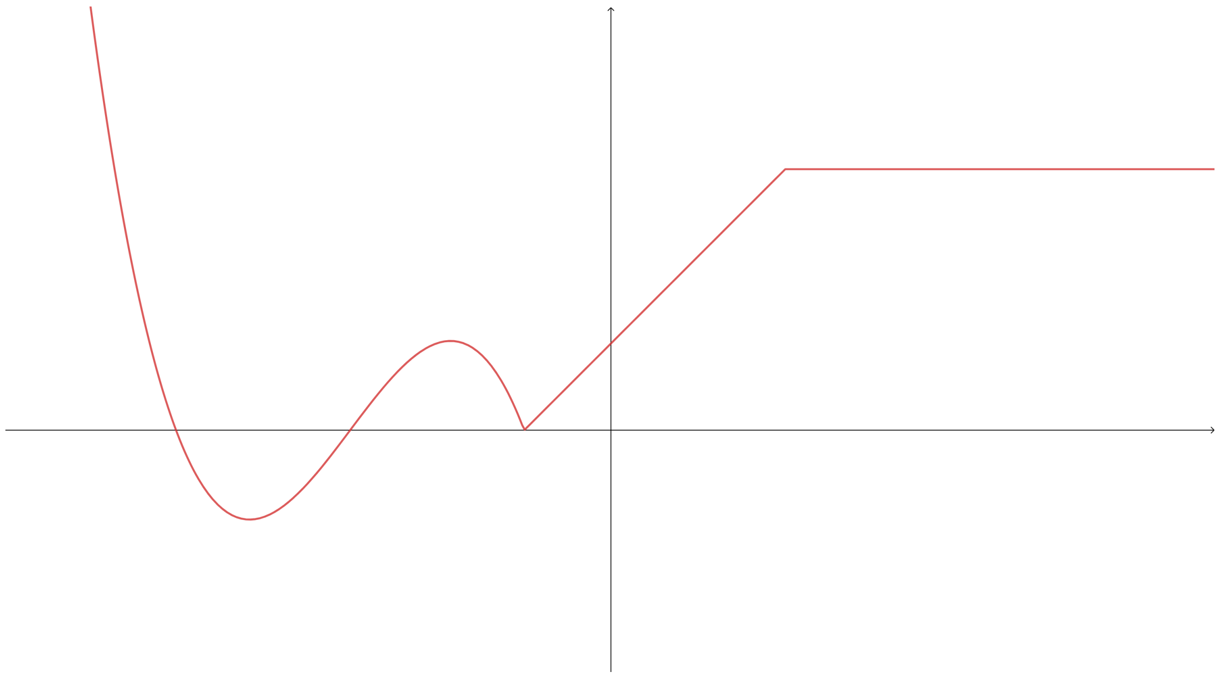
Consider the curve $x^2 + y^3 = 9$. Find the points on the curve at which the tangents are:

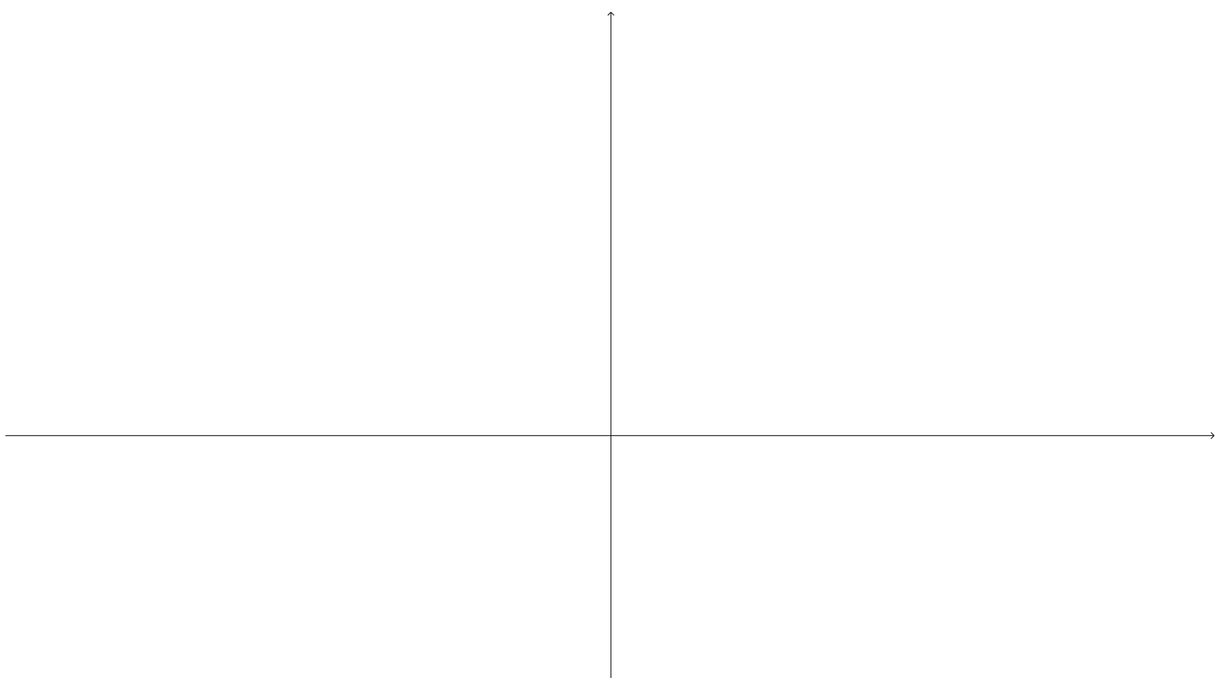
(i) horizontal,

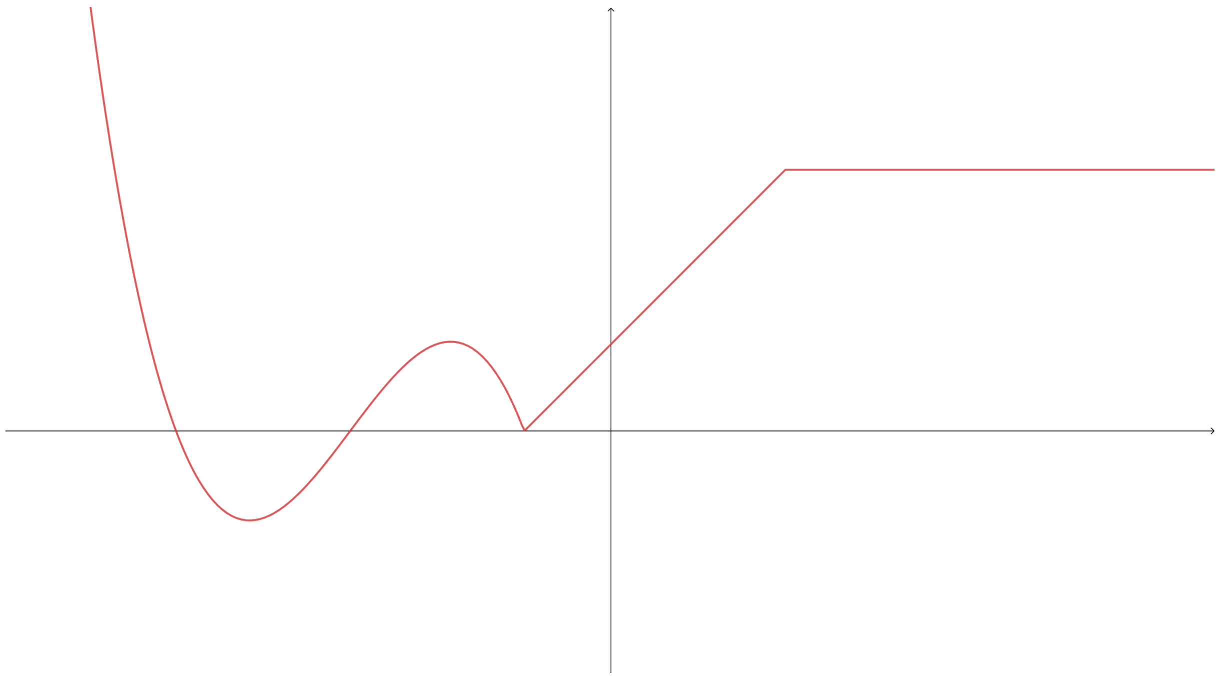
(ii) vertical.

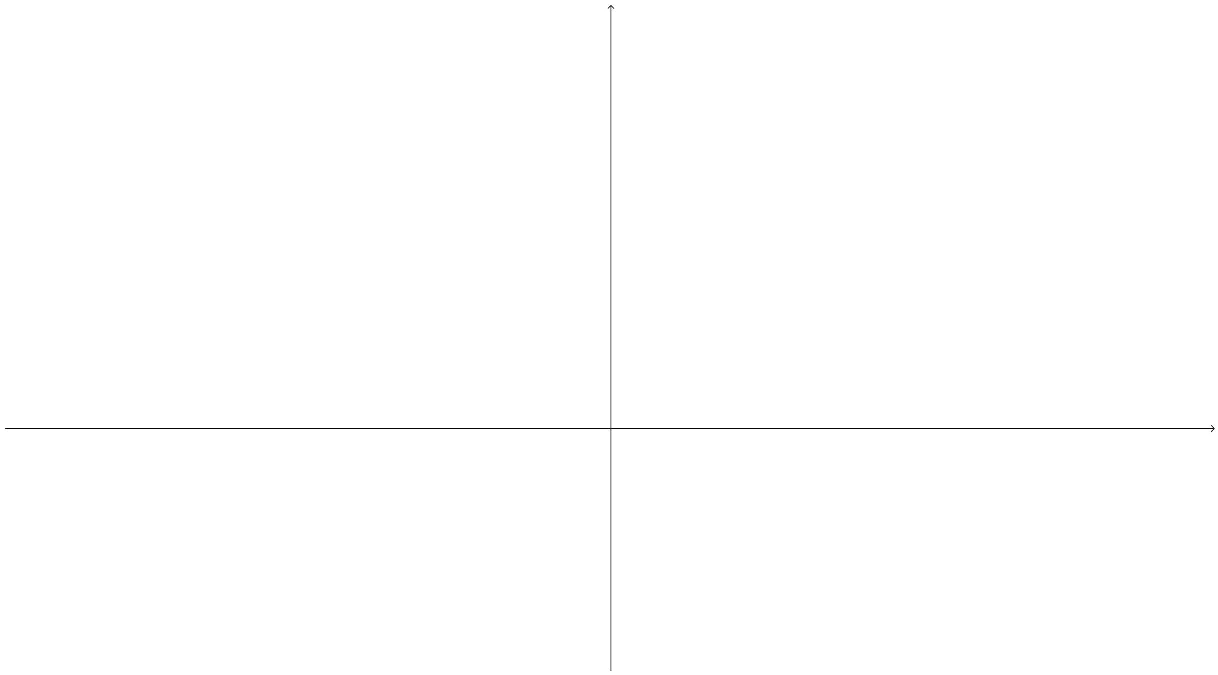
6. [Maximum mark: 8]

The diagrams below show two copies of the graph of $y = f(x)$. On the axis below draw the graph of $y = f'(x)$ and $y = F(x)$, where $F'(x) = f(x)$ and $F(0) = 0$.









7. [Maximum mark: 6]

Bolzano's Theorem states that if a continuous function is defined on the interval $[a, b]$ and $f(a)$ and $f(b)$ have opposite signs, then $f(x)$ has a zero in the interval $[a, b]$.

Consider the function $f(x) = x^3 + x^2 + 2x + 1$.

(a) Show that $f(x) = 0$ for some $x \in [-1, 0]$. [3 *points*]

(b) Show that $f(x)$ is increasing for all x, and hence deduce the number of real solutions to the equation $f(x) = 0$. Justify your answer. [3 *points*]

8. [Maximum mark: 4]

Find equations of the two possible tangents to the curve $y = x^2 + x$ from the point $(0, -4)$.

SECTION B

Answer all questions on separate sheets.

9. [Maximum mark: 13]

Consider the function $f(x) = \dfrac{x^2 + x - 2}{x - 2}$ with $x \neq 2$.

a) Find the coordinates of points where the graph $y = f(x)$ intersects the axes. [3 *points*]

b) Find all the asymptotes of the graph of $y = f(x)$ [3 *points*]

c) Find $f'(x)$ and hence find the coordinates of any stationary points and classify these points. [5 *points*]

d) Sketch the graph of $y = f(x)$. [2 *points*]

10. [Maximum mark: 16]

Let $f(x) = x\sqrt{9-x^2} + 2\arcsin(\frac{x}{3})$.

a) By considering both terms of the function, find the largest possible domain D for the function f. [2 *points*]

For the following parts it is assumed that $x \in D$.

b) Find $f'(x)$ in simplified form and hence find the x-coordinate of the maximum point of the graph of $y = f(x)$. Justify that it is a maximum. [7 *points*]

c) Show that $f''(x) = \dfrac{x(2x^2-25)}{(9-x^2)\sqrt{9-x^2}}$ and hence find coordinates of any points of inflexion of the graph of $y = f(x)$. Justify your answer. [7 *points*]