

1. Find the following indefinite integrals.

(a) $\int 10x\sqrt{5x^2 + 2} dx$

(b) $\int \frac{x^2}{(x^3 + 4)^2} dx$

(c) $\int -6x(1 - 2x^2)^3 dx$

(d) $\int 3\sqrt{x}(9 + 2\sqrt{x^3})^4 dx$

(e) $\int 6 \cdot x^3\sqrt{x^2 + 4} dx$

(f) $\int \frac{2x + 3}{(x^2 + 3x + 1)^3} dx$

(g) $\int \frac{4x}{\sqrt{x^2 + 2}} dx$

(h) $\int \frac{x^3}{(1 - x^4)^4} dx$

(i) $\int 3e^{3x}\sqrt{1 + e^{3x}} dx$

(j) $\int \frac{x + 1}{(x^2 + 2x - 1)^2} dx$

(k) $\int \frac{x^2 + 1}{\sqrt{x^3 + 3x + 1}} dx$

(l) $\int x\sqrt{3 + 4x^2} dx$

(m) $\int \frac{e^x}{\sqrt{e^x + 2}} dx$

(n) $\int \frac{e^{-2x}}{(1 - e^{-2x})^3} dx$

(o) $\int 10x^2(x^3 + 1)^4 dx$

(p) $\int (x^3 + 2)(x^4 + 8x - 3)^5 dx$

(q) $\int 2x^3\sqrt{(x^4 + 5)^3} dx$

(r) $\int \frac{\cos 2x}{\sqrt{1 - \sin 2x}} dx$

(s) $\int \cos x\sqrt{4 + 3 \sin x} dx$

(t) $\int \frac{\sec^2 4x}{(1 + 3 \tan 4x)^2} dx$

(u) $\int \frac{1 - \sin x}{\sqrt[3]{x + \cos x}} dx$

(v) $\int \sin \frac{1}{2} x \cos^3 \frac{1}{2} x dx$

(w) $\int \frac{x \cos x + \sin x}{\sqrt{1 + x \sin x}} dx$

(x) $\int \frac{(\sqrt{x} + 1)^{1/2}}{\sqrt{x}} dx$

2. Find the antiderivative of the following

(a) $2xe^{x^2+1}$

(b) $\frac{3}{\sqrt{x}}e^{\sqrt{x}}$

(c) $\sec^2 3xe^{\tan 3x}$

(d) $(2ax + b)e^{-(ax^2 + bx)}$

(e) $3 \sin \frac{1}{2} x e^{\cos \frac{1}{2} x}$

(f) $\frac{4}{x^2}e^{4+x^{-1}}$

(g) $e^x \sin(2e^x)$

(h) $\frac{e^{2x}}{(1 - e^{2x})^2}$

(i) $\frac{e^{-x}}{1 + e^{-x}}$

(j) $\frac{5}{e^{-x} + 2}$

(k) $e^{-ax}\sqrt{4 + e^{-ax}}$

(l) $\frac{e^{2x}}{1 + e^{2x}} \ln(1 + e^{2x})$

3. Find the antiderivative of the following

- | | | |
|---|--|---|
| (a) $2x \sin(x^2 + 1)$ | (b) $\frac{5}{\sqrt{x}} \sin(\sqrt{x})$ | (c) $\frac{2}{x^2} \cos\left(2 + \frac{1}{x}\right)$ |
| (d) $\sin x \sqrt{\cos x}$ | (e) $\frac{\sin 3x}{\cos 3x}$ | (f) $\frac{4 \sec^2 3x}{1 + \tan 3x}$ |
| (g) $\frac{4 \sec^2 3x}{(1 + \tan 3x)^2}$ | (h) $\frac{2}{x} \cos(\ln x)$ | (i) $\sin x \cos x \sqrt{1 + \cos 2x}$ |
| (j) $e^x \cos(e^x)$ | (k) $3x^2 e^{-x^3 + 2}$ | (l) $\cot \frac{1}{2} x \ln\left(\sin \frac{1}{2} x\right)$ |
| (m) $\sin x \sec^2 x$ | (n) $\frac{1}{e^{-x} + 2} \ln(1 + 2e^x)$ | (o) $(x^2 - 3) \sec^2\left(\frac{1}{3}x^3 - 3x\right)$ |

4. Find the antiderivative of the following

- | | | |
|---------------------------------|---------------------------------|---------------------------------|
| (a) $\frac{2}{4 + x^2}$ | (b) $\frac{3}{x^2 + 9}$ | (c) $\frac{\sqrt{5}}{5 + x^2}$ |
| (d) $\frac{1}{\sqrt{25 - x^2}}$ | (e) $\frac{1}{\sqrt{16 - x^2}}$ | (f) $-\frac{1}{\sqrt{9 - x^2}}$ |

5. Find the following indefinite integrals

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|--|---|---|
| (a) $\int \frac{3}{1 + x^2} dx$ | (b) $\int \frac{5}{\sqrt{1 - x^2}} dx$ | (c) $\int \frac{1}{\sqrt{4 - x^2}} dx$ |
| (d) $\int \frac{1}{\sqrt{9 - x^2}} dx$ | (e) $\int \frac{1}{\sqrt{1 - 4x^2}} dx$ | (f) $\int \frac{1}{\sqrt{9 - 4x^2}} dx$ |
| (g) $\int \frac{1}{\sqrt{4 - 25x^2}} dx$ | (h) $\int \frac{2}{1 + 4x^2} dx$ | (i) $\int \frac{1}{9 + 4x^2} dx$ |
| (j) $\int \frac{1}{9 + 16x^2} dx$ | (k) $\int \frac{1}{9 + 5x^2} dx$ | (l) $\int \frac{1}{\sqrt{3 - 5x^2}} dx$ |

2. Using the substitution method, find

- | | | |
|--------------------------------------|---|---------------------------------------|
| (a) $\int x \sqrt{2x - 1} dx$ | (b) $\int x^2 \sqrt{1 - x} dx$ | (c) $\int (x + 1) \sqrt{x - 1} dx$ |
| (d) $\int \sec^2 x e^{\tan x} dx$ | (e) $\int \frac{4x}{(1 - 2x^2)^2} dx$ | (f) $\int \frac{4x}{(1 - 2x^2)^2} dx$ |
| (g) $\int \frac{1}{x} (\log_e x) dx$ | (h) $\int \frac{e^{-x}}{1 + e^{-x}} dx$ | (i) $\int \frac{1}{x \log_e x} dx$ |

7. Find the following indefinite integrals

(a) $\int \frac{1}{x^2 + 6x + 10} dx$ (b) $\int \frac{1}{x^2 - x + 1} dx$ (c) $\int \frac{1}{\sqrt{1 + 4x - x^2}} dx$

(d) $\int \frac{3}{\sqrt{8 - 2x - x^2}} dx$ (e) $\int \frac{2}{\sqrt{5 + 3x - x^2}} dx$ (f) $\int \frac{x}{\sqrt{9 - x^4}} dx$

(g) $\int \frac{\arcsin x}{\sqrt{1 - x^2}} dx$ (h) $\int \frac{(\arccos x)^2}{\sqrt{1 - x^2}} dx$ (i) $\int \frac{1}{\arcsin^3 x \sqrt{1 - x^2}} dx$

1. Integrate the following expressions with respect to x :

(a) $x \sin x$ (b) $x \cos \frac{x}{2}$ (c) $2x \sin \frac{x}{2}$ (d) $x e^{-x}$
(e) $5x e^{-4x}$ (f) $\ln x$ (g) $x \ln x$ (h) $-x \cos(-5x)$
(i) $4x \sin\left(-\frac{x}{3}\right)$ (j) $\frac{x}{\cos^2 x}$ (k) $\sqrt{x} \ln x$

2. Use integration by parts to antidifferentiate

(a) $x\sqrt{x+1}$ (b) $x\sqrt{x-2}$ (c) $(x+1)\sqrt{x+2}$

3. Find

(a) $\int \cos^{-1} x dx$ (b) $\int \tan^{-1} x dx$ (c) $\int \sin^{-1} x dx$

4. Find

(a) $\int x \cos^{-1} x dx$ (b) $\int x \tan^{-1} x dx$ (c) $\int x \sin^{-1} x dx$

8. Find

(a) $\int \cos(\ln x) dx$ (b) $\int \sin(\ln x) dx$ (c) $\int x^3 \sqrt{1 - x^2} dx$

1. Find the following integrals (not all are best evaluated using the parts formula):

- (a) $\int x^2 e^x dx$ (b) $\int 3x^2 \cos(2x) dx$ (c) $\int x^3 \ln(2x) dx$
(d) $\int e^x \sin(2x) dx$ (e) $\int x^2 \cos(3x) dx$ (f) $\int e^{-2x} \cos(2x) dx$
(g) $\int 4x^3 \sin \frac{x}{2} dx$ (h) $\int \frac{1}{x} \ln x dx$ (i) $\int (\ln(3x))^2 dx$
(j) $\int \cos x \sin(2x) dx$ (k) $\int e^{ax} \cos \frac{x}{a} dx$ (l) $\int x^2 \sqrt{x+2} dx$
(m) $\int x^3 \ln(ax) dx$ (n) $\int \frac{x^2}{\sqrt{4-x^2}} dx$ (o) $\int \frac{3x^2 dx}{\sqrt{x^2-9}}$
(p) $\int \frac{x}{x^2+4} dx$ (q) $\int \frac{x^2}{x^2+4} dx$

1. Either by using a suitable substitution, or by considering the chain rule, find these integrals:

- (a) (i) $\int x(x^2+3)^3 dx$ (ii) $\int 3x(x^2-1)^5 dx$
(b) (i) $\int (2x-5)(3x^2-15x+4)^4 dx$
(ii) $\int (x^2+2x)(x^3+3x^2-5)^3 dx$
(c) (i) $\int \frac{2x}{x^2+3} dx$ (ii) $\int \frac{6x^2-12}{x^3-6x+1} dx$
(d) (i) $\int 4 \cos^5 3x \sin 3x dx$ (ii) $\int \cos 2x \sin^3 2x dx$
(e) (i) $\int 3xe^{3x^2-1} dx$ (ii) $\int 3xe^{x^2} dx$
(f) (i) $\int \frac{e^{2x+3}}{e^{2x+3}+4} dx$ (ii) $\int \frac{\cos x}{3+4 \sin x} dx$

4. Find the following using an appropriate substitution:

(a) (i) $\int x(2x-1)^4 dx$

(ii) $\int 9x(3x+2)^5 dx$

(b) (i) $\int x\sqrt{x-3} dx$

(ii) $\int (x+1)\sqrt{5x-6} dx$

(c) (i) $\int \frac{x^2}{\sqrt{x-5}} dx$

(ii) $\int \frac{4(x+5)}{(2x-3)^3} dx$

2. Use trigonometric identities before using a substitution (or reversing the chain rule) to integrate:

(a) $\int \cos^3 x \sin^2 x dx$

(b) $\int \frac{\cos^3 x}{\sin^2 x} dx$

(c) $\int \sin x \cos x e^{\cos 2x} dx$

(d) $\int \tan^4 3x + \tan^6 3x dx$

(e) $\int \frac{\sin 2x \cos 2x}{\sqrt{1+\cos 4x}} dx$

3. Find the following integrals:

(a) (i) $\int 2 \cos^2 x dx$

(ii) $\int \cos^2 3x dx$

(b) (i) $\int 2 \tan^2 \left(\frac{x}{2} \right) dx$

(ii) $\int \tan^2 3x dx$

2. By first completing the square, find the following:

(a) (i) $\int \frac{1}{x^2 + 4x + 5} dx$

(ii) $\int \frac{1}{x^2 - 6x + 10} dx$

(b) (i) $\int \frac{1}{\sqrt{8x - x^2 - 15}} dx$

(ii) $\int \frac{1}{\sqrt{2x - x^2}} dx$

(c) (i) $\int \frac{6}{x^2 + 10x + 27} dx$

(ii) $\int \frac{5}{\sqrt{-4x^2 - 12x}} dx$

3. Find the following by splitting into a polynomial and a proper fraction:

(a) (i) $\int \frac{x+1}{x+2} dx$

(ii) $\int \frac{2x+3}{x-1} dx$

(b) (i) $\int \frac{x^2+2}{x-3} dx$

(ii) $\int \frac{x^2+2x-1}{x+5} dx$

(c) $\int \frac{x^2+5x+1}{x^2+3} dx$

1. Use integration by parts to find the following:

(a) (i) $\int x \cos 2x \, dx$ (ii) $\int x \sin\left(\frac{x}{2}\right) \, dx$

(b) (i) $\int 4xe^{-2x} \, dx$ (ii) $\int xe^{4x} \, dx$

(c) (i) $\int 2x \ln 5x \, dx$ (ii) $\int x \ln x \, dx$

(d) (i) $\int x^2 \cos 3x \, dx$ (ii) $\int x^2 \sin x \, dx$

(e) $\int \frac{1}{4} x^2 e^{\frac{x}{4}} \, dx$

(f) $\int \frac{\ln x}{x^3} \, dx$

(g) $\int (\ln x)^2 \, dx$

2. Use integration by parts to find the following:

(a) $\int \arctan x \, dx$ (b) $\int \ln(2x + 1) \, dx$

1 $\int \sin^3 t \cos^2 t \, dt$

2 $\int \sin^3 t \cos^3 t \, dt$

3 $\int \sin^3 3\theta \cos 3\theta \, d\theta$

4 $\int \frac{1}{t^2} \sin^5\left(\frac{1}{t}\right) \cos^2\left(\frac{1}{t}\right) \, dt$

5 $\int \frac{\sin^3 x}{\cos^2 x} \, dx$

6 $\int \tan^5 3x \sec^2 3x \, dx$

7 $\int \theta \tan^3 \theta^2 \sec^4 \theta^2 \, d\theta$

8 $\int \frac{1}{\sqrt{t}} \tan^3 \sqrt{t} \sec^3 \sqrt{t} \, dt$

9 $\int \tan^4(5t) \, dt$

10 $\int \frac{dt}{1 + \sin t}$

● **Hint:** multiply the integrand by $\frac{1 - \sin t}{1 - \sin t}$.

$$11 \int \frac{d\theta}{1 + \cos \theta}$$

$$12 \int \frac{1 + \sin t}{\cos t} dt$$

$$13 \int \frac{\sin x - 5 \cos x}{\sin x + \cos x} dx$$

• **Hint:** find numbers a and b such that
 $\sin x - 5 \cos x = a(\sin x + \cos x) + b(\cos x - \sin x)$.

$$14 \int \frac{\sec \theta \tan \theta}{1 + \sec^2 \theta} d\theta$$

$$15 \int \frac{\arctan t}{1 + t^2} dt$$

$$16 \int \frac{1}{(1 + t^2)\arctan t} dt$$

$$17 \int \frac{dx}{x\sqrt{1 - (\ln x)^2}}$$

$$18 \int \sin^3 x dx$$

$$19 \int \frac{\sin^3 x}{\sqrt{\cos x}} dx$$

$$20 \int \frac{\sin^3 \sqrt{x}}{\sqrt{x}} dx$$

$$21 \int \cos t \cos^3(\sin t) dt$$

$$22 \int \frac{\cos \theta + \sin 2\theta}{\sin \theta} d\theta$$

$$23 \int t \sec t \tan t dt$$

$$24 \int \frac{\cos x}{2 - \sin x} dx$$

$$25 \int e^{-2x} \tan(e^{-2x}) dx$$

$$26 \int \frac{\sec(\sqrt{t})}{\sqrt{t}} dt$$

$$27 \int \frac{dt}{1 + \cos 2t}$$

$$28 \int \sqrt{1 - 9x^2} dx$$

$$29 \int \frac{dx}{(x^2 + 4)^{\frac{3}{2}}}$$

$$30 \int \sqrt{4 + t^2} dt$$

$$31 \int \frac{3e^t dt}{4 + e^{2t}}$$

$$32 \int \frac{1}{\sqrt{9 - 4x^2}} dx$$

$$33 \int \frac{1}{\sqrt{4 + 9x^2}} dx$$

$$34 \int \frac{\cos x}{\sqrt{1 + \sin^2 x}} dx$$

$$35 \int \frac{x}{\sqrt{4 - x^2}} dx$$

$$36 \int \frac{x}{x^2 + 16} dx$$

$$37 \int \frac{\sqrt{4 - x^2}}{x^2} dx$$

$$38 \int \frac{dx}{(9 - x^2)^{\frac{3}{2}}}$$

$$40 \int e^{2x} \sqrt{1 + e^{2x}} dx$$

$$42 \int \frac{e^x dx}{\sqrt{e^{2x} + 9}}$$

$$44 \int \frac{x^3}{(x + 2)^2} dx$$

$$39 \int x \sqrt{1 + x^2} dx$$

$$41 \int e^x \sqrt{1 - e^{2x}} dx$$

$$43 \int \frac{\ln x}{\sqrt{x}} dx$$

$$1 \int x^2 e^{-x^3} dx$$

$$3 \int x^2 \cos 3x dx$$

$$5 \int \cos x \ln(\sin x) dx$$

$$7 \int x^2 \ln x dx$$

$$9 \int x \cos \pi x dx$$

$$11 \int \arcsin x dx$$

$$13 \int e^{-2x} \sin 2x dx$$

$$15 \int \cos(\ln x) dx$$

$$17 \int e^{kx} \sin x dx$$

$$19 \int \sin x \sin 2x dx$$

$$21 \int \frac{\ln x}{\sqrt{x}} dx$$

$$2 \int x^2 e^{-x} dx$$

$$4 \int x^2 \sin ax dx$$

$$6 \int x \ln x^2 dx$$

$$8 \int x^2 (e^x - 1) dx$$

$$10 \int e^{3t} \cos 2t dt$$

$$12 \int x^3 e^x dx$$

$$14 \int \sin(\ln x) dx$$

$$16 \int \ln(x + x^2) dx$$

$$18 \int x \sec^2 x dx$$

$$20 \int x \arctan x dx$$

$$22 \int t \sec^2 t dt$$