1.

(a) (i) $\boldsymbol{r}_1 = \begin{bmatrix}16\\12\end{bmatrix} + t\begin{bmatrix}12\\-5\end{bmatrix}$

$t = 0 \Rightarrow \boldsymbol{r}_1 = \begin{bmatrix}16\\12\end{bmatrix}$ (M1)

$|\boldsymbol{r}_1| = \sqrt{(16^2 + 12^2)} = 20$ (A1)

(ii) Velocity vector = $\begin{bmatrix}12\\-5\end{bmatrix}$

$\Rightarrow$ speed = $\sqrt{(12^2 + (-5)^2)}$ (M1)

$= 13$ (A1) 4

(b) $\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}16\\12\end{bmatrix} + t\begin{bmatrix}12\\-5\end{bmatrix}$

$\Rightarrow \begin{bmatrix}5\\12\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}5\\12\end{bmatrix}\cdot\begin{bmatrix}6\\12\end{bmatrix} + \begin{bmatrix}5\\12\end{bmatrix}\cdot t\begin{bmatrix}12\\-5\end{bmatrix}$ (M1)

$\Rightarrow 5x + 12y = 80 + 144$ (A1)

$5x + 12y = 224$ (A1)(AG)

OR

$\frac{x-16}{12} = \frac{y-12}{-5}$ (M1)

$5x - 80 = 144 - 12y$ (A1)

$\Rightarrow 5x + 12y = 224$ (A1)(AG)

OR

$x = 16 + 12t,\ y = 12 - 5t \Rightarrow t = \frac{12-y}{5}$ (M1)

$\Rightarrow x = 16 + 12\left(\frac{12-y}{5}\right)$ (A1)

$\Rightarrow 5x = 80 + 144 - 12y$

$\Rightarrow 5x + 12y = 224$ (A1)(AG) 3

(c) $\boldsymbol{v}_1 = \begin{bmatrix} 12 \\ -5 \end{bmatrix}$ $\quad \boldsymbol{v}_2 = \begin{bmatrix} 2.5 \\ 6 \end{bmatrix}$ (M1)

$\boldsymbol{v}_1 . \boldsymbol{v}_2 = \begin{bmatrix} 12 \\ -5 \end{bmatrix} . \begin{bmatrix} 2.5 \\ 6 \end{bmatrix}$ (M1)

$= 30 - 30$

$\Rightarrow \boldsymbol{v}_1 . \boldsymbol{v}_2 = 0$ (A1)

$\Rightarrow \theta = 90°$ (A1) 4

(d) (i) $\begin{bmatrix} x \\ y \end{bmatrix} . \begin{bmatrix} 12 \\ -5 \end{bmatrix} = \begin{bmatrix} 23 \\ -5 \end{bmatrix} . \begin{bmatrix} 12 \\ -5 \end{bmatrix}$ (M1)

$\Rightarrow 12x - 5y = 23 \times 12 + 25 = 301$ (A1)

OR

$\frac{x - 23}{2.5} = \frac{y + 5}{6}$

$\Rightarrow 6x - 138 = 2.5y + 12.5$ (M1)

$\Rightarrow 12x - 276 = 5y + 25$

$\Rightarrow 12x - 5y = 301$ (A1)

(ii) $\left.\begin{matrix} 5x + 12y = 224 \\ 12x - 5y = 301 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 25x + 60y = 1120 \\ 144x - 60y = 3612 \end{matrix}\right\}$ (M1)

$169x = 4732$

$x = 28, y = (12 \times 28 - 301) \div 5 = 7$

(28, 7) (A1)(A1) 5

Note: *Accept any correct method for solving simultaneous equations.*

(e) $16 + 12t = 23 + 2.5t \Rightarrow 9.5t = 7$ (M1)

$12 - 5t = -5 + 6t \quad \Rightarrow 17 = 11t$ (M1)

$\frac{7}{9.5} \neq \frac{17}{11}$ (A1)

$\Rightarrow$ planes cannot be at the same place at the same time (R1)

OR

$\boldsymbol{r}_1 = \begin{bmatrix} 28 \\ 7 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 28 \\ 7 \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix} + t \begin{bmatrix} 12 \\ -5 \end{bmatrix}$ (M1)

$\Leftrightarrow \begin{cases} 12t = 12 \\ -5t = -5 \end{cases} \Leftrightarrow t = 1$ (A1)

When $t = 1$ $\boldsymbol{r}_2 = \begin{bmatrix} 23 \\ -5 \end{bmatrix} + \begin{bmatrix} 2.5 \\ 6 \end{bmatrix} = \begin{bmatrix} 25.5 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 28 \\ 7 \end{bmatrix}$ (A1)(R1)

2.

(a) $\overrightarrow{OB} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ (A1) (C1)

$\overrightarrow{AC} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ (A1) (C1)

(b) $\vec{OB} \cdot \vec{AC} = (10 \times (-3)) + (5 \times 6) = 0$ (M1)

Angle = 90° (A1) (C2)

[4]

3.

(a) $\vec{u} = -\vec{i} + 2\vec{j}$ $\vec{v} = 3\vec{i} + 5\vec{j}$

$\vec{u} + 2\vec{v} = 5\vec{i} + 12\vec{j}$ (A1) (C1)

(b) $|\vec{u} + 2\vec{v}| = \sqrt{5^2 + 12^2}$

$= 13$ (A1)

Vector $\vec{w} = \frac{26}{13}(5\vec{i} + 12\vec{j})$ (A1)

$= 10\vec{i} + 24\vec{j}$ (A1) (C3)

[4]

4.

(a) $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$ (A1) (C1)

(b) $\overrightarrow{OA} = \frac{1}{2}\overrightarrow{CD}$

$= \frac{1}{2}(\overrightarrow{OD} - \overrightarrow{OC})$ (A1) (C1)

(c) $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \overrightarrow{OD} - \frac{1}{2}(\overrightarrow{OD} - \overrightarrow{OC})$ (A1)

$= \frac{1}{2}\overrightarrow{OD} + \frac{1}{2}\overrightarrow{OC}$ (A1) (C2)

Note: *Deduct [1 mark] (once only) if appropriate vector notation is omitted.*

[4]

5.

Required vector will be parallel to $\begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ (M1)

$= \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ (A1)

Hence required equation is $\boldsymbol{r} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 4 \\ -5 \end{pmatrix}$ (A1)(A1) (C4)

Note: *Accept alternative answers, eg* $\begin{pmatrix} 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ -5 \end{pmatrix}$.

[4]

6.

$\boldsymbol{u} + \boldsymbol{v} = 4\boldsymbol{i} + 3\boldsymbol{j}$ (A1)

Then $a(4\boldsymbol{i} + 3\boldsymbol{j}) = 8\boldsymbol{i} + (b - 2)\boldsymbol{j}$

$4a = 8$

$3a = b - 2$ (A1)

Whence $a = 2$ (A1) (C2)

$b = 8$ (A1) (C2)

[4]

7.

$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} x-4 \\ y+1 \end{pmatrix}$ (M1) (M1)

Notes: *Award (M1) for using scalar product.*

Award (M1) for $\begin{pmatrix} x-4 \\ y+1 \end{pmatrix}$.

$2(x - 4) + 3(y + 1) = 0$ (A1)

$2x - 8 + 3y + 3 = 0$

$2x + 3y = 5$ (A1)

OR

Gradient of a line parallel to the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is $\frac{3}{2}$ (M1)

Gradient of a line perpendicular to this line is $-\frac{2}{3}$ (M1)

So the equation is $y + 1 = -\frac{2}{3}(x - 4)$ (A1)

$\Rightarrow 3y + 3 = -2x + 8$

$\Rightarrow 2x + 3y = 5$ (A1)

[4]

8.

$\begin{pmatrix}1\\2\end{pmatrix}\cdot\begin{pmatrix}6\\-8\end{pmatrix} = 6 - 16 = -10$ (A1)

$\left|\begin{pmatrix}1\\2\end{pmatrix}\right| = \sqrt{1^2+2^2} = \sqrt{5}, \left|\begin{pmatrix}6\\-8\end{pmatrix}\right| = \sqrt{6^2+8^2} = \sqrt{100} = 10$ (A1)

$\begin{pmatrix}1\\2\end{pmatrix}\cdot\begin{pmatrix}6\\-8\end{pmatrix} = \left|\begin{pmatrix}1\\2\end{pmatrix}\right|\left|\begin{pmatrix}6\\-8\end{pmatrix}\right|\cos\theta$

$-10 = \sqrt{5} \times 10\cos\theta \Rightarrow \cos\theta = \frac{-10}{10\sqrt{5}} = -\frac{1}{\sqrt{5}} \Rightarrow \theta = \arccos\frac{-1}{\sqrt{5}}$ (M1)

$\theta \approx 117°$ (A1)

[4]

9.

(a) $\begin{pmatrix}2x\\x-3\end{pmatrix}\bullet\begin{pmatrix}x+1\\5\end{pmatrix} = 0$ (M1)(M1)

$\Rightarrow 2x(x + 1) + (x - 3)(5) = 0$ (A1)

$\Rightarrow 2x^2 + 7x - 15 = 0$ (C3)

(b) **METHOD 1**

$2x^2 + 7x - 15 = (2x - 3)(x + 5) = 0$

$\Rightarrow x = \frac{3}{2}$ or $x = -5$ (A1) (C1)

METHOD 2

$x = \frac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)}$

$\Rightarrow x = \frac{3}{2}$ or $x = -5$ (A1) (C1)

[4]

10.

Angle between lines = angle between direction vectors. (M1)

Direction vectors are $\begin{pmatrix}4\\3\end{pmatrix}$ **and** $\begin{pmatrix}1\\-1\end{pmatrix}$. (A1)

$\begin{pmatrix}4\\3\end{pmatrix}\cdot\begin{pmatrix}1\\-1\end{pmatrix} = \left|\begin{pmatrix}4\\3\end{pmatrix}\right|\left|\begin{pmatrix}1\\-1\end{pmatrix}\right|\cos\theta$ (M1)

$4(1) + 3(-1) = \left(\sqrt{4^2+3^2}\right)\left(\sqrt{1^2+(-1)^2}\right)\cos\theta$ (A1)

$\cos\theta = \dfrac{1}{5\sqrt{2}} = 0.1414$ (A1)

$\theta = 81.9°$ (3 sf), (1.43 radians) (A1) (C6)

Note: *If candidates find the angle between the vectors* $\begin{pmatrix}4\\-1\end{pmatrix}$ *and* $\begin{pmatrix}2\\4\end{pmatrix}$, *award marks as below:*

Angle required is between $\begin{pmatrix}4\\-1\end{pmatrix}$ and $\begin{pmatrix}2\\4\end{pmatrix}$ (M0)(A0)

$\begin{pmatrix}4\\-1\end{pmatrix}\cdot\begin{pmatrix}2\\4\end{pmatrix} = \left|\begin{pmatrix}4\\-1\end{pmatrix}\right|\left|\begin{pmatrix}2\\4\end{pmatrix}\right|\cos\theta$ (M1)

$4(2) + (-1)\,4 = \left(\sqrt{4^2+(-1)^2}\right)\left(\sqrt{2^2+4^2}\right)\cos\theta$ (A1)

$\dfrac{4}{\sqrt{17}\sqrt{20}} = \cos\theta = 0.2169$ (A1)

$\theta = 77.5°$ (3sf), (1.35 radians) (A1) (C4)

[6]

11.

$x = 1 - 2t$ (A1)

$y = 2 + 3t$ (A1)

$\dfrac{x-1}{-2} = \dfrac{y-2}{3}$ (M1)

$3x + 2y = 7$ (A1)(A1)(A1) (C6)

[6]

12.

(a) $\overrightarrow{OB} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$ (A1)(A1) 2

(b) $\overrightarrow{AD} = \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ (M1)

$= \begin{pmatrix} 8 \\ 9 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \end{pmatrix}$ (A1)

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 9 \\ 2 \end{pmatrix} = \begin{pmatrix} 11 \\ 4 \end{pmatrix} \left(\text{or } \begin{pmatrix} 8 \\ 9 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} 11 \\ 4 \end{pmatrix} \right)$

$d = 11 \left(\text{accept} \begin{pmatrix} 11 \\ 4 \end{pmatrix} \right)$ (A1) 3

(c) $\overrightarrow{BD} = \begin{pmatrix} 11 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 12 \\ -3 \end{pmatrix}$ (A1) 1

(d) (i) $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 12 \\ -3 \end{pmatrix} \left(\text{or} \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \end{pmatrix} \right)$ (A2)

(ii) At B, $t = 0$ by observation (A1)

OR

$\begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 12 \\ -3 \end{pmatrix}$

$\Rightarrow t = 0$ (A1) 3

(e) $\begin{pmatrix} 7 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 12 \\ -3 \end{pmatrix} \Rightarrow 7 + 1 = 12t = 8$

$\Rightarrow t = \frac{2}{3}$ (A1)

***Note:** The equation* $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ *leads to* $t = 2$.

when $t = \frac{2}{3}$, $y = 7 + \left(\frac{2}{3}\right)(-3)$ (M1)

$= 7 - 2 = 5$ (A1)

ie P on line (AG)

OR

$5 - 7 = -3t = -2$

$\Rightarrow t = \frac{2}{3}$ (A1)

when $t = \frac{2}{3}$, $x = -1 + \frac{2}{3} \times 12$ (M1)

$= -1 + 8 = 7$ (A1)

ie P on line (AG) 3

(f) $\overrightarrow{CP} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 8 \\ 9 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$ (A1)

$\begin{pmatrix} -1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ -3 \end{pmatrix} = -12 + 12 = 0$ (M1)(A1)

Scalar product of non-zero vectors = 0 $\Rightarrow$ are perpendicular (R1)(AG)

OR

Geometric approach

CP: $m = 4$ (A1)

BD: $m_1 = \frac{-1}{4}$ (A1)

$mm_1 = 4 \times \left(\frac{-1}{4}\right) = -1$ (A1)

Product of gradients is $-1 \Rightarrow$ lines (vectors) are perpendicular (R1)(AG) 4

[16]

13.

B, or $\boldsymbol{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ (C3)

D, or $\boldsymbol{r} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ (C3)

Note: *Award C4 for B, D and one incorrect, C3 for one correct and nothing else, C1 for one correct and one incorrect, C0 for anything else.*

[6]

14.

METHOD 1

At point of intersection:

$5 + 3\lambda = -2 + 4t$ (M1)
$1 - 2\lambda = 2 + t$ (M1)

Attempting to solve the linear system (M1)
$\lambda = -1$ (or $t = 1$) (A1)

$\overrightarrow{OP} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (A1)(A1) (C6)

METHOD 2

(changing to Cartesian coordinates)
$2x + 3y = 13,\ x - 4y = -10$ (M1)(A1)(A1)
Attempt to solve the system (M1)

$\overrightarrow{OP} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (A1)(A1) (C6)

Note: *Award (C5) for the point* P(2, 3).

15.

(a) $\sqrt{16+9} = \sqrt{25} = 5$ (M1)(A1) (C2)

(b) $\begin{pmatrix} -2 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ (so B is (6, 7)) (M1)(A1) (C2)

(c) $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ (not unique) (A2) (C2)

Note: *Award (A1) if "* $\boldsymbol{r}$ = *" is omitted, ie not an equation.*

[6]

16.

(a) $\vec{PQ} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ A1A1 N2

(b) Using $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ A2A1A1 N4

[6]

17.

(a) Finding **correct** vectors, $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ A1A1

Substituting correctly in the scalar product $\overrightarrow{AB} \cdot \overrightarrow{AC} = 4(-3) + 3(1)$ A1

$= -9$ AG N0

(b) $\left|\overrightarrow{AB}\right| = 5$ $\left|\overrightarrow{AC}\right| = \sqrt{10}$ (A1)(A1)

Evidence of using scalar product formula M1

e.g. $\cos B\hat{A}C = \frac{-9}{5\sqrt{10}} = -0.569$ (3 s.f.)

$B\hat{A}C = 2.47$ (radians), $125°$ A1 N3

[7]