

1. In this question the vector  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  km represents a displacement due east, and the vector  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  km represents a displacement due north.

The point (0, 0) is the position of *Shipple Airport*. The position vector  $r_1$  of an aircraft *Air One* is given by

$$r_1 = \begin{pmatrix} 16 \\ 12 \end{pmatrix} + t \begin{pmatrix} 12 \\ -5 \end{pmatrix},$$

where  $t$  is the time in minutes **since** 12:00.

- (a) Show that the *Air One* aircraft

- (i) is 20 km from *Shipple Airport* at 12:00;  
(ii) has a speed of 13 km/min.

(4)

- (b) Show that a cartesian equation of the path of *Air One* is:

$$5x + 12y = 224.$$

(3)

The position vector  $r_2$  of an aircraft *Air Two* is given by

$$r_2 = \begin{pmatrix} 23 \\ -5 \end{pmatrix} + t \begin{pmatrix} 2.5 \\ 6 \end{pmatrix},$$

where  $t$  is the time in minutes **since** 12:00.

- (c) Find the angle between the paths of the two aircraft.

(4)

- (d) (i) Find a cartesian equation for the path of *Air Two*.  
(ii) Hence find the coordinates of the point where the two paths cross.

(5)

- (e) Given that the two aircraft are flying at the same height, show that they do not collide.

(4)

(Total 20 marks)

2. The quadrilateral  $OABC$  has vertices with coordinates  $O(0, 0)$ ,  $A(5, 1)$ ,  $B(10, 5)$  and  $C(2, 7)$ .

(a) Find the vectors  $\overrightarrow{OB}$  and  $\overrightarrow{AC}$ .

(b) Find the angle between the diagonals of the quadrilateral  $OABC$ .

(Total 4 marks)

3. The vectors  $\vec{i}$ ,  $\vec{j}$  are unit vectors along the  $x$ -axis and  $y$ -axis respectively.

The vectors  $\vec{u} = -\vec{i} + 2\vec{j}$  and  $\vec{v} = 3\vec{i} + 5\vec{j}$  are given.

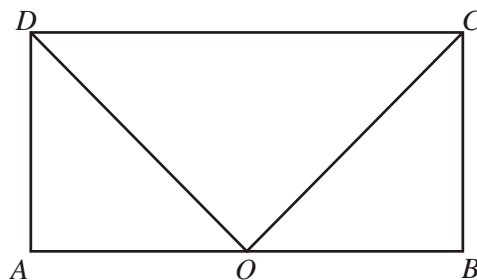
(a) Find  $\vec{u} + 2\vec{v}$  in terms of  $\vec{i}$  and  $\vec{j}$ .

A vector  $\vec{w}$  has the same direction as  $\vec{u} + 2\vec{v}$ , and has a magnitude of 26.

(b) Find  $\vec{w}$  in terms of  $\vec{i}$  and  $\vec{j}$ .

(Total 4 marks)

4.  $ABCD$  is a rectangle and  $O$  is the midpoint of  $[AB]$ .



Express each of the following vectors in terms of  $\overrightarrow{OC}$  and  $\overrightarrow{OD}$

(a)  $\overrightarrow{CD}$

(b)  $\overrightarrow{OA}$

(c)  $\overrightarrow{AD}$

(Total 4 marks)

5. Find a vector equation of the line passing through  $(-1, 4)$  and  $(3, -1)$ . Give your answer in the form  $\mathbf{r} = \mathbf{p} + t\mathbf{d}$ , where  $t \in \mathbb{R}$ .

(Total 4 marks)

6. The vectors  $\mathbf{u}$ ,  $\mathbf{v}$  are given by  $\mathbf{u} = 3\mathbf{i} + 5\mathbf{j}$ ,  $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$ .

Find scalars  $a$ ,  $b$  such that  $a(\mathbf{u} + \mathbf{v}) = 8\mathbf{i} + (b - 2)\mathbf{j}$ .

(Total 4 marks)

7. A line passes through the point  $(4, -1)$  and its direction is perpendicular to the vector  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ . Find the equation of the line in the form  $ax + by = p$ , where  $a$ ,  $b$  and  $p$  are integers to be determined.

(Total 4 marks)

8. Find the size of the angle between the two vectors  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$  and  $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$ . Give your answer to the nearest degree.

(Total 4 marks)

9. The vectors  $\begin{pmatrix} 2x \\ x-3 \end{pmatrix}$  and  $\begin{pmatrix} x+1 \\ 5 \end{pmatrix}$  are perpendicular for two values of  $x$ .

- (a) Write down the quadratic equation which the two values of  $x$  must satisfy.  
(b) Find the two values of  $x$ .

(Total 4 marks)

10. Calculate the acute angle between the lines with equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

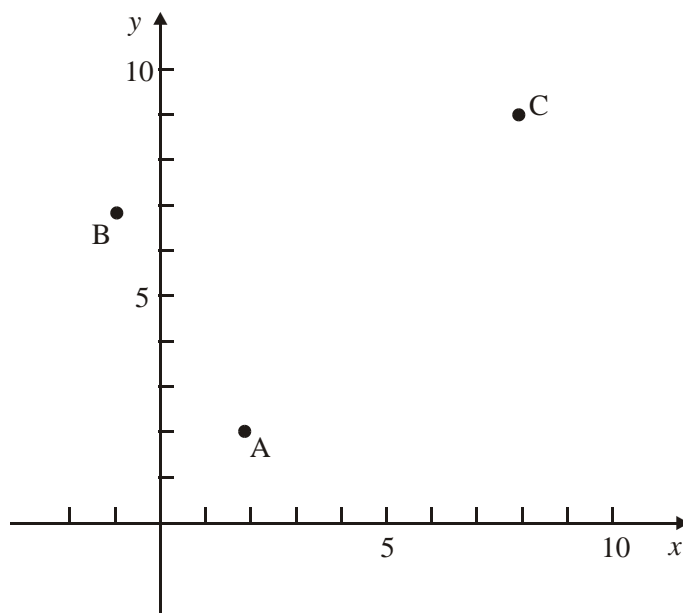
(Total 6 marks)

11. A vector equation of a line is  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ ,  $t \in \mathbb{R}$ .

Find the equation of this line in the form  $ax + by = c$ , where  $a$ ,  $b$ , and  $c \in \mathbb{Z}$ .

(Total 6 marks)

12. The diagram shows points A, B and C which are three vertices of a parallelogram ABCD. The point A has position vector  $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ .



- (a) Write down the position vector of B and of C. (2)
- (b) The position vector of point D is  $\begin{pmatrix} d \\ 4 \end{pmatrix}$ . Find  $d$ . (3)
- (c) Find  $\overrightarrow{BD}$ . (1)

The line  $L$  passes through B and D.

- (d) (i) Write down a vector equation of  $L$  in the form

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t \begin{pmatrix} m \\ n \end{pmatrix}.$$

- (ii) Find the value of  $t$  at point B. (3)
- (e) Let P be the point  $(7, 5)$ . By finding the value of  $t$  at P, show that P lies on the line  $L$ . (3)
- (f) Show that  $\overrightarrow{CP}$  is perpendicular to  $\overrightarrow{BD}$ . (4)

(Total 16 marks)

13. A vector equation for the line  $L$  is  $\mathbf{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ .

Which of the following are also vector equations for the same line  $L$ ?

A.  $\mathbf{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ .

B.  $\mathbf{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ .

C.  $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ .

D.  $\mathbf{r} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ .

(Total 6 marks)

14. The vector equations of two lines are given below.

$$\mathbf{r}_1 = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad \mathbf{r}_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

The lines intersect at the point P. Find the position vector of P.

(Total 6 marks)

15. A boat B moves with constant velocity along a straight line. Its velocity vector is given by

$$\mathbf{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}.$$

At time  $t = 0$  it is at the point  $(-2, 1)$ .

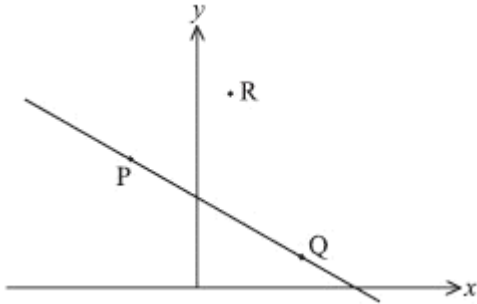
(a) Find the magnitude of  $\mathbf{v}$ .

(b) Find the coordinates of B when  $t = 2$ .

(c) Write down a vector equation representing the position of B, giving your answer in the form  $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ .

(Total 6 marks)

16. The points  $P(-2, 4)$ ,  $Q(3, 1)$  and  $R(1, 6)$  are shown in the diagram below.



- (a) Find the vector  $\overrightarrow{PQ}$ .
- (b) Find a vector equation for the line through  $R$  parallel to the line  $(PQ)$ .

**(Total 6 marks)**

17. A triangle has its vertices at  $A(-1, 3)$ ,  $B(3, 6)$  and  $C(-4, 4)$ .

- (a) Show that  $\overrightarrow{AB} \cdot \overrightarrow{AC} = -9$ .

**(3)**

- (b) Find  $\hat{BAC}$ .

**(4)**  
**(Total 7 marks)**