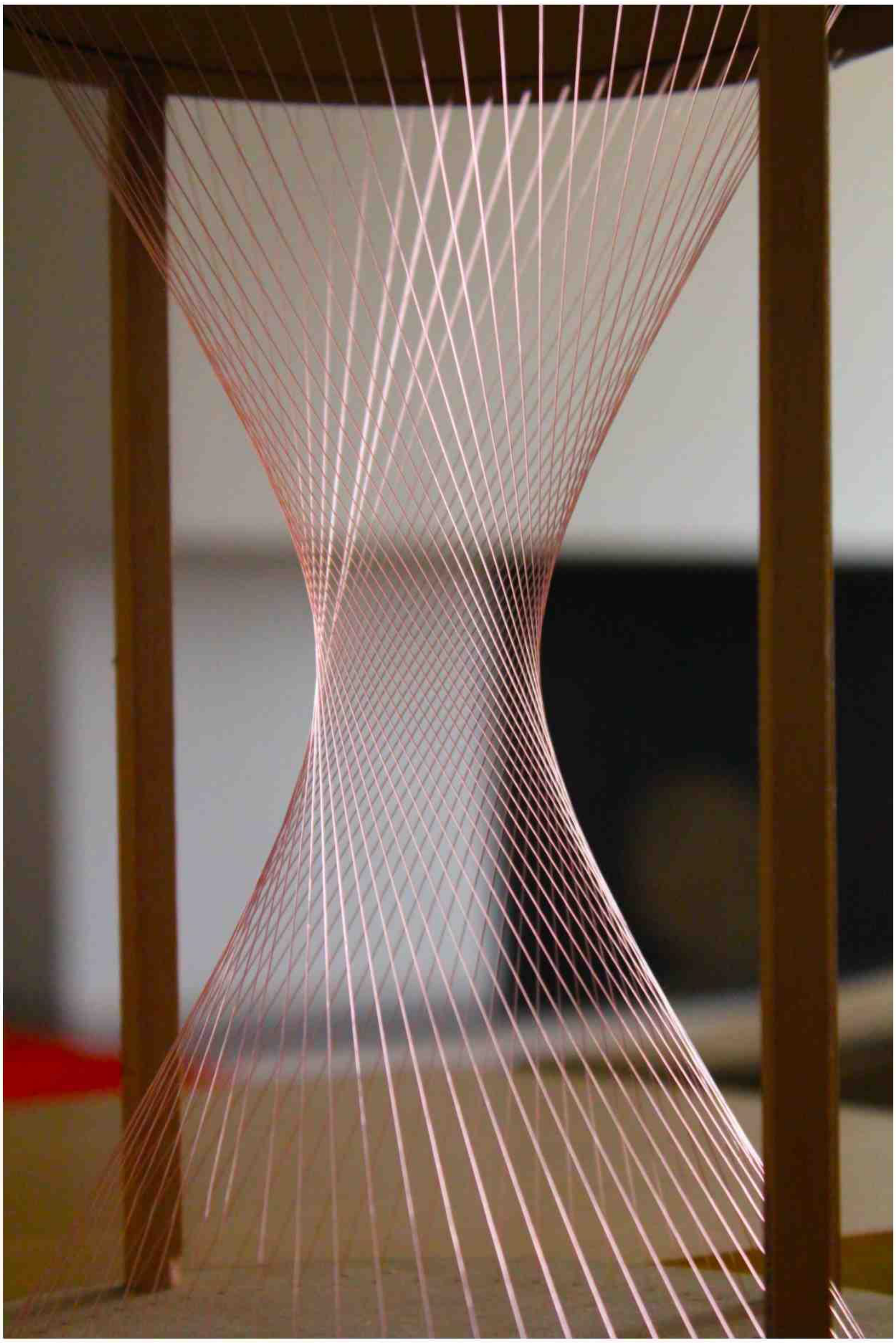
Math IA
Properties of a hyperboloid

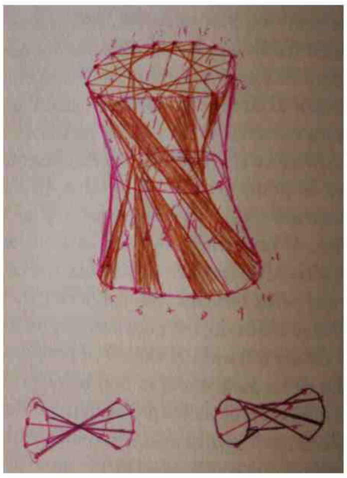


# Table of Contents


| | |
|---|---|
| Introduction | 1 |
| Hyperboloid info | 1 |
| Model | 2 |
| Radius | 5 |
| Hyperbola | 9 |
| Volume | 12 |
| Conclusion | 14 |
| Bibliography | 14 |


A: A table of contents is not necessary or required for a piece of work of this length.

# Introduction



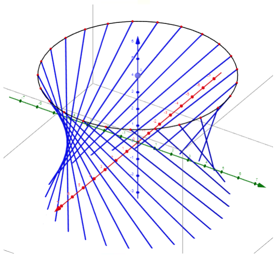
I chose the topic for my exploration in a rather coincidental manner. Being slightly distracted during my literature class, I began doodling, and found myself drawing a circular cylinder with its vertical lengths rotated relative to their corresponding circumference. Doing this, I found that by increasing the rotation, the cross sectional circumference in its center would be reduced. With this in mind, I began wondering about the possible patterns related to the properties of this shape, known mathematically as a hyperboloid, and mathematical approaches to model them.

As a prospective design student, I have always been interested in the shape and structure of visually compelling objects, and how the most abstract forms are usually linked to some sharp functionality. Besides this, by being interested in the power of mathematics of modeling the most abstract shapes found, and often unfound, around us, I chose this topic for my exploration, feeling like it would be an interesting topic in which to engage.

In this exploration, I will try to find expressions relating the rotation of one face in a hyperboloid, its central radius and volume, through different methods such as physical modeling, data regression, and deduction of functions. Furthermore, I will also evaluate the validity of this expressions, and its possible applications in real life scenarios.

A: An aim is present in the introduction.

# Hyperboloid

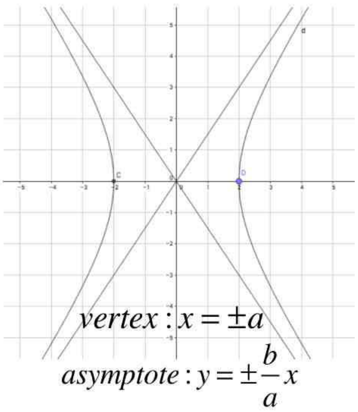


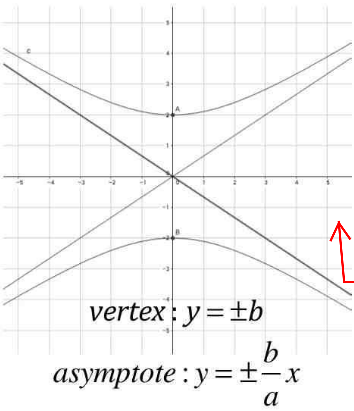
The shape studied is referred mathematically as a one-sheet hyperboloid, which has a continuous surface made up as "a surface of revolution obtained by rotating a hyperbola about the perpendicular bisector to the line between the foci" (Wolfram Alpha, 2016).

A hyperbola is a conic section which can be defined in a Cartesian plane with the general formula:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad \text{or} \quad \frac{y^2}{b^2}-\frac{x^2}{a^2}=1$$

where $a$ and $b$ are real constants. Here, the negative coefficient belongs to $x$ or $y$ depending on the axis into which the hyperbola opens (Wolfram Alpha, 2016).




C: Candidate has carried out own research for hyperbolas.

B: The axes are not labelled.

1

All hyperbolas have two vertices, which determine the domain/range of the equation, and the values of *x* or *y* for which the there is no real value.

Additionally, all hyperbolas have two linear asymptotes, from which all points will always be over or under, depending on the direction on which it opens. The gradient of one asymptote will always be the negative gradient of its pair.

For the purpose of my exploration, I will refer to the distance between the vertices and the origin, as a **central radius**, considering the context of the image formed in the center of the hyperboloid. I will also refer to the rotation of one face relative to the other in terms of degrees(°), as this is the unit used in the measuring devices available.

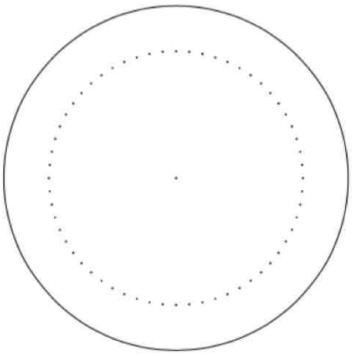
# Model

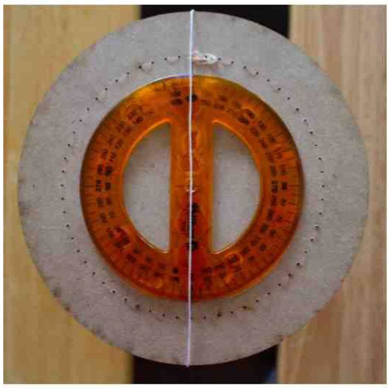
To be able to better visualize the hyperboloid, and to have a physical source from which to collect data further in my exploration, I decided to make a dynamic model of a hyperboloid, which would be able rotate one face, while keeping the opposite one fixed.

To do this, I made a vector model on AutoCAD[ (AutoCAD, 2016)], of a circle with 60 concentric points inside it, at an equal separation of 6° each, and at a distance of 7cm from its center. For the purpose of this exploration, this circle will be referred to as the **base circumference**. This circles would be the faces of the model, through which threads would go through at every point opened. Although the amount of points would be rather inconvenient later on, in terms of the amount of times the thread would need to be sewn across the model, it was a necessary commitment in order to ensure the accuracy of the model, as to few threads would have affected the shape seen when rotated.

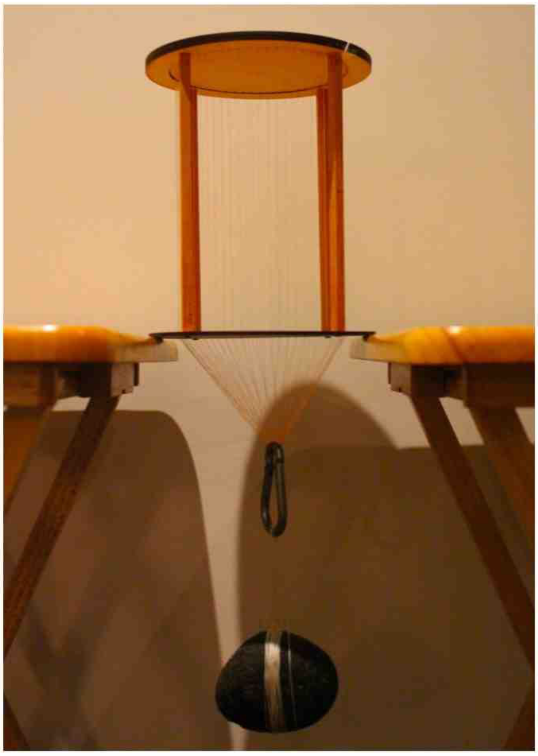
C: Authentic personal engagement, with student using technology to help build a more accurate model.

I then proceeded to cut the circumferences using a laser cutter, and build the model using fixed supports between the faces, in such way they would still allow the top face to rotate freely. Then, using a non-elastic nylon, I began sewing through the 60 holes, making a continuous line of nylon attached to a hook with a weight beneath the bottom face, and then returning to the top one. I decided to do this in order to have enough space for the weight to rise and fall according to the rotation, as a higher rotation would increase the length of the segments in between both faces, yet ensuring all nylons would have a constant and equal force downwards, so they would all tension equally, and the model could be more reliable. Finally, I attached a compass and to the top face, and a set of reference points attached to the fixed supports, to be able to measure the angle difference from an initial position. This initial position was determined to be where each thread was perpendicular to the bottom and top surface, having a difference of 0° between the top and bottom holes.

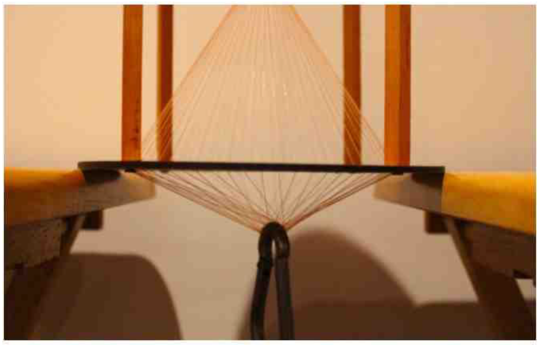


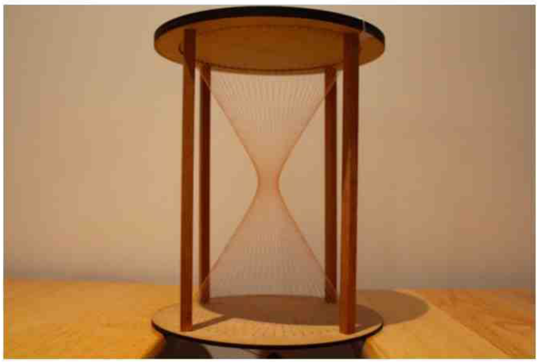


2



C: Excellent device built to ensure precision.



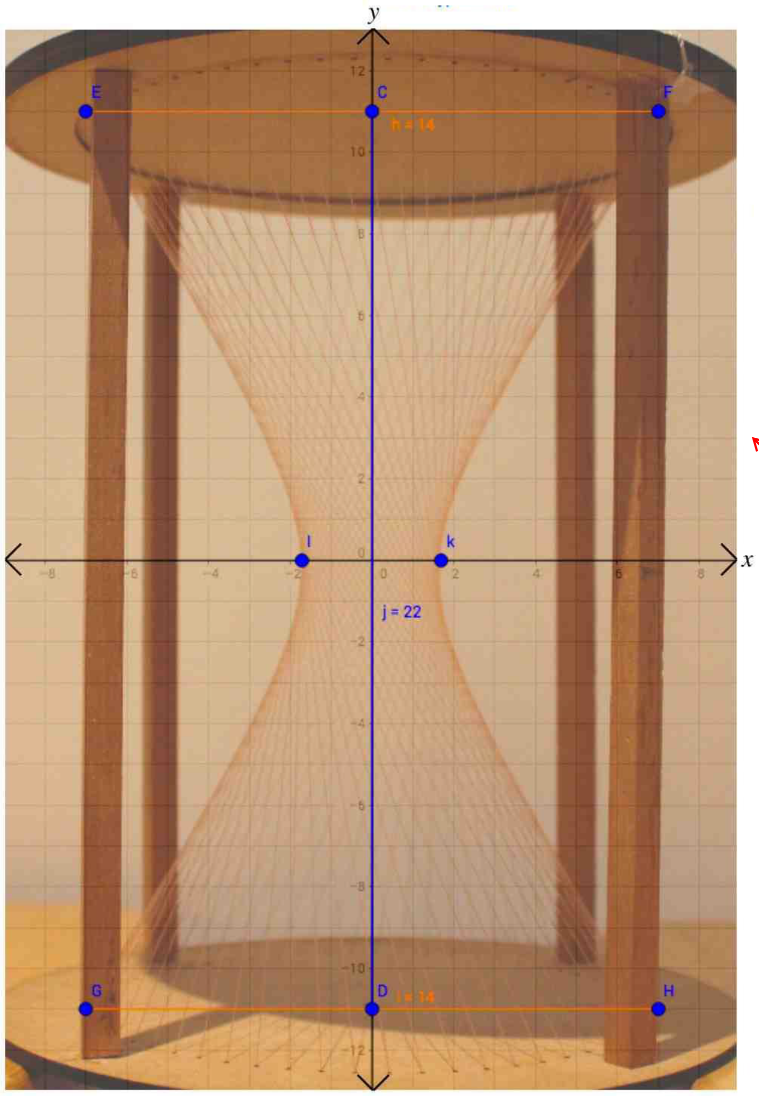


3

The model was designed in such way that its dimensions would be appropriate to ensure facilitated calculations when applying different models to its shape. The height between the faces is 22cm, and the radius from the center to the points is 7cm. Furthermore, when taking pictures of the model meant for regression, the camera was set up in a tripod as to be at the height of the central radius, and with a horizontal alignment with the center of the circumferences, as to reduce any errors caused by a change in perspective, considering the used camera has a non-orthogonal perspective, and being centered with the model is the best approximation to an orthogonal view.

Considering this, we can then project the image of the model, in such way that the curvature of the threads shape a hyperbola, having the central radius being the vertices (I and k) and aligning the x-axis and y-axis to the vertical center and the center of the circumference, respectively. This way, we would know 4 coordinates, just from the dimensions of the model: E(-7,11), F(7,11), H(7,-11), and G(-7,-11). The image could also be rotated around its center 90° and still form a hyperbola, which in this case would open towards the y-axis. This projection will later be useful when determining the central radius, and volume of the hyperbola.

D: Ongoing reflection to explain how the model can be used effectively.




B: Good use of technology.

4

# Radius

To find the central radius of the circumferences formed within the hyperbola, I first considered that if the model is seen from “above”, thus ignoring the height of the model, the segments within the base circumferences of radius 7 would create an image, in which each segment would tangent the imaginary circle, as shown in Fig. 1. From here, I then derived the shortest distance from a given tangent to the origin, which is the center of the base circumference, thus finding the radius of the central circumference.

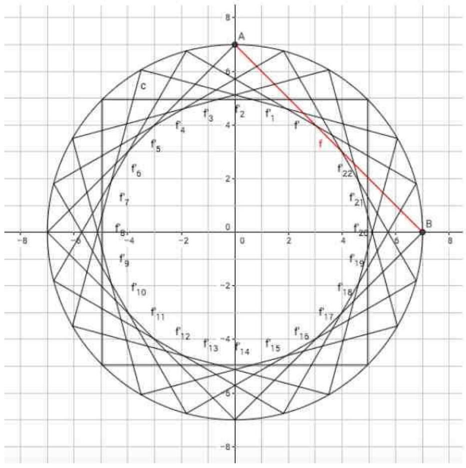



Figure 1

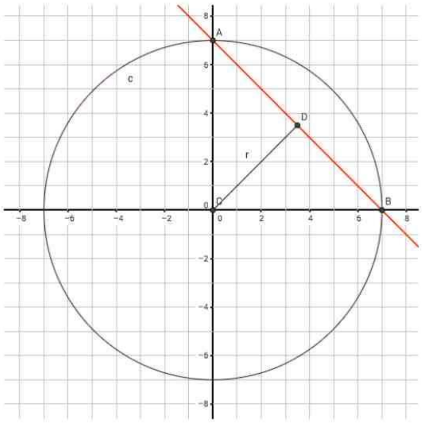



Figure 2

In Fig. 2 for example, we consider the segment resultant from a 90° rotation, joining coordinates A(0,7), and B(7,0). From these two points we can determine a linear function:

$$y = -x + 7$$

Having this function we can then consider the general formula for distance:

$$s = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

Which, considering one of coordinates is (0,0), then simplifies to:

$$s = \sqrt{{x_1}^2 + {y_1}^2}$$

Then, we replace *y*, with our linear function, simplify, and generate the function for distance between any point in the line and the origin:

$$s(x) = \sqrt{x^2 + (-x+7)^2}$$

$$s(x) = \sqrt{2x^2 - 14x + 49}$$

$$s(x) = (2x^2 - 14x + 49)^{\frac{1}{2}}$$

To find the minimum value, we then derive s*(x),* using chain rule:

B: Terminology, differentiate.

$$s'(x) = \frac{1}{2}(2x^2 - 14x + 49)^{-\frac{1}{2}}(4x - 14) \Rightarrow \frac{2x-7}{\sqrt{2x^2 - 14x + 49}}$$

And then solve for when s*’(x)=0*, as this will result in the value of *x* for which *s(x)* is minimum:

$$0 = 2x - 7 \Rightarrow x = \frac{7}{2}$$

5

With this value of *x*, when then calculate distance:

$$s\left(\frac{7}{2}\right)=\frac{7\sqrt{2}}{2}$$

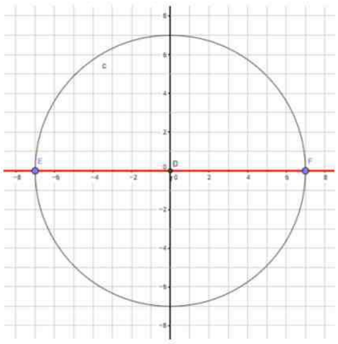
Because of this, we can say that in a function relating central radius *r* and degree of rotation:

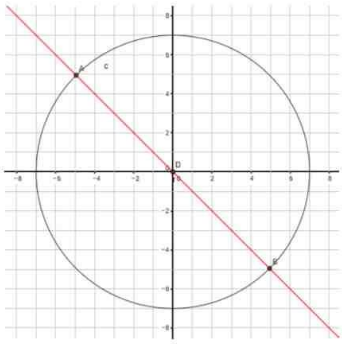
$$r(90°)=\frac{7\sqrt{2}}{2}$$

Furthermore, we can use our value of *x* to calculate the *y* component in our linear function:

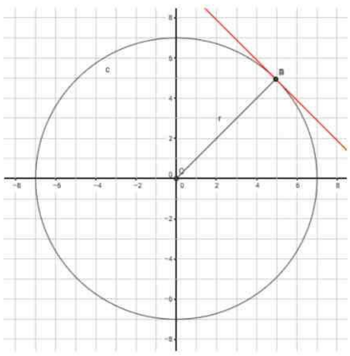
$$y=-\left(\frac{7}{2}\right)+7\Rightarrow y=\frac{7}{2}$$

With this in mind, we can see that *x*=*y*, for the resulting coordinate when calculating the minimum distance in the linear formula for 90°.





When attempting the same calculation for a rotation of 180°, I consider that the points chosen to generate the linear function don't necessarily need to lie on an axis, but can be anywhere within the circle as long as their relative rotation from each other is the one intended. With this in mind, I figure out any rotation can be expressed as a linear function:



$$y=-x+k$$

In which the gradient is always *m*=-1, and the constant *k* determines the central radius, as shown in Fig. 3. For 180° rotation, for example, a constant *k*=0 results in a radius of 0, and for 0° rotation, a constant *k*=*n* results in a radius of 7, as this is the radius of the points in the base circumference.

To find the constant *n*, we use the radius 7, and our assumption that *x*=*y*, for all linear equations with *m*=-1, which implies that:

Figure 3

$$x=y \quad \text{and} \quad 7=\sqrt{x^2+y^2}\Rightarrow 7=\sqrt{2x^2}\Rightarrow x=\frac{7\sqrt{2}}{2}$$

Assuming that *x*=*y*, for a linear equation of gradient *m*=-1, we consider that:

$$\frac{7\sqrt{2}}{2}=-\frac{7\sqrt{2}}{2}+c\Rightarrow c=7\sqrt{2}$$

With this in mind, we can consider that, in a function relating degree of rotation and linear constant *k* :

$$k(0°)=7\sqrt{2}$$
$$k(90°)=7$$
$$k(180°)=0$$

6

To find the relationship between linear constant *k* and rotation *x*, we proceed to manipulate both sides of the equations, until we get a trigonometric relationship relating the values of *x* and the desired result.

$$k(0°)=7\sqrt{2} \qquad \frac{k(0°)}{7}=\sqrt{2} \qquad \left(\frac{k(0°)}{7}\right)^2=2 \qquad \left(\frac{k(0°)}{7}\right)^2-1=1$$

$$k(90°)=7 \qquad \frac{k(90°)}{7}=1 \qquad \left(\frac{k(90°)}{7}\right)^2=1 \qquad \left(\frac{k(90°)}{7}\right)^2-1=0$$

$$k(180°)=0 \qquad \frac{k(180°)}{7}=0 \qquad \left(\frac{k(180°)}{7}\right)^2=0 \qquad \left(\frac{k(180°)}{7}\right)^2-1=-1$$

Here, we can see a trigonometric relationship between 0°, 90°, 180°, and they're respective result, for which we deduce that:

$$\left(\frac{k(0°)}{7}\right)^2-1=\cos(0°)$$

$$\left(\frac{k(90°)}{7}\right)^2-1=\cos(90°) \quad \text{and} \quad \left(\frac{k(x)}{7}\right)^2-1=\cos(x)$$

$$\left(\frac{k(180°)}{7}\right)^2-1=\cos(180°)$$

C: An original approach.

Which, when rearranged, results in:

$$k(x)=7\sqrt{1+\cos x} \qquad x\in[-180°,180°]$$

I decided to restrain the function between -180° and 180°, as the central radius will be the same if rotated clockwise or anti-clockwise, yet after 180°, the threads begin to tangle, and the radius doesn't decrease as should happen according to the function. If the threads were made of a material which could just pass through itself, such as a laser, this limitation wouldn't exist, and the central radius would return to 7 at 360°.

D: Good, meaningful and critical reflection.

Having found an expression for *k*(*x*), referred to as linear constant, we can then proceed to find an expression relating linear constant *k* and central radius. Considering that all rotations will now have the linear form of:

$$y=-x+k$$

We can deduce that all distance functions will be:

$$s=\sqrt{x^2+y^2} \Rightarrow s(x)=\sqrt{x^2+(-x+k)^2}$$

Which when derived results in:

B: Terminology.

$$s(x)=\sqrt{x^2+(-x+k)^2} \Rightarrow s(x)=\left(2x^2-2kx+k^2\right)^{\frac{1}{2}} \Rightarrow s'(x)=\frac{2x-k}{\left(2x^2-2kx+k^2\right)^{\frac{1}{2}}}$$

7

Having this, we can solve for *s'*(*x*)=0, to *find* the minimum value of *s*(x) in terms of *k*:

$$0=2x-c \Rightarrow x=\frac{c}{2}$$

And finally, solve for distance, considering *x*=*y*:

$$s(x)=\sqrt{2x^2} \Rightarrow s(x)=\sqrt{2\left(\frac{c}{2}\right)^2} \Rightarrow s(x)=\left|\frac{c}{2}\right|\sqrt{2}$$

Then we can use our function of linear constant *k(x)*, to have a final function relating rotation *x* and distance/central radius *s*:

B: This could be a typing error.

$$s(x)=\left|\frac{7\sqrt{1+\cos x}}{2}\right|\sqrt{2} \qquad x\in\mathbb{R}\left[-180°,180°\right]$$

Having determined this function, it is interesting to attempt and show its validity using the model built. By knowing that C=2$\pi$r [ (BCC Bitesize, 2014)], where C is circumference and r is radius,we can easily measure the circumference manually for each angle in the model, and then deduce the radius by dividing by 2$\pi$. Having this in mind, I proceeded to calculate the radius using the function s(x), and then measuring for the circumference manually, to then compare the values and test the validity of my function in real life.

To do this, I created a table in Excel, in which I calculated the radius manually by measuring the circumference every 30°, and then using the function for every 30°. In this point, it is necessary to convert angles from degrees to radians, considering that when using calculus radians are more appropriate. To do this, I used the formula:

D, E: Insightful and critical reflection. Candidate demonstrates thorough understanding.

$$R=\frac{D\pi}{180}$$

In which *R* is the measure in radians, and *D* is the measure in degrees.

In the following table, data was collecting, showing both conversion from degrees to radians, the expected radius accord to the formula used, the manually measured circumference, and the corresponding radius according to manual measurements of circumference.

| Angle (deg) | Angle (rad) | expected radius (function)(cm) | Circumference (manual) (cm) | Radius (manual) (cm) |
|---|---|---|---|---|
| 0 | 0,00 | 7,00 | 45,5 | 7,24 |
| 30 | 0,52 | 6,76 | 42,0 | 6,68 |
| 60 | 1,05 | 6,06 | 39,0 | 6,21 |
| 90 | 1,57 | 4,95 | 31,0 | 4,93 |
| 120 | 2,09 | 3,50 | 22,5 | 3,58 |
| 150 | 2,62 | 1,81 | 11,0 | 1,75 |
| 180 | 3,14 | 0,00 | 0,0 | 0,00 |

*For the table above, commas are used as decimal points due to local setting of Excel.

8

Seeing my results, we can see how they approximate quite a lot to the expected value. With this in mind, we can conclude that my function for radius is quite accurate, considering that measuring by hand is the most likely cause of uncertainties and errors.

Furthermore, considering that height of the hyperboloid does not affect the central radius, yet the radius of the base circumference from which the segments join *does* affect the angular coefficient, and therefore the central radius, we can express our function including the base radius *r*. We can do this, going back to the image seen from above (Fig. 1) , and considering that with a base circle of radius *r*, the points with which to calculate the linear function would be A(*0*,*r*) and B(*r*,0). From here, we can follow through the whole process, and conclude that:

$$s(x)=\left|\frac{r\sqrt{1+\cos x}}{2}\right|\sqrt{2} \qquad x\in\left[-180°,180°\right] \quad ,r\in\mathbb{R}^{+}$$

# Hyperbola

To find the volume of the hyperboloid, I first need to find the equation of the hyperbola, in order to have a base on which to perform a ration around an axis and generate a solid of revolution, method which I have chosen to find the volume.

To determine the equation of the hyperbola, I considered the way a hyperbolic expression would be applied to my model, based on the observations made in pg. 4. Here, I determined that the vertices of the hyperbola would be defined by the max/min found in the central radius. With this in mind, I decided to use my results for the central radius, as a base point on which to build the hyperbola for each rotation.

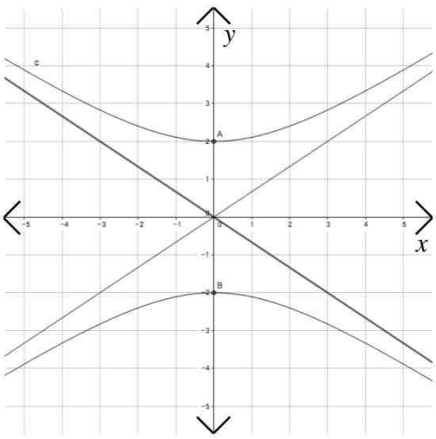



Fig. 4

Being aware that the easiest way to calculate a solid of ration is to rotate a function around the *x*-axis, I decided to express the hyperbola in a way which would allow this easily, that is, as a hyperbola that opens along the *y*-axis. For this hyperbola, the general expression is (Wolfram Alpha, 2016):

$$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$$

In a hyperbola written in this form, the vertices have a *y*-coordinate of:

$$y=\pm b$$

Knowing this, we can assume that for a hyperboloid with central radius *s*, the vertices will have coordinates of A(0,*s*) and B(0,-*s*), as shown in Fig. 4.

Because of this, the new general expression will be:

$$\frac{y^2}{s^2}-\frac{x^2}{a^2}=1$$

9

Besides this, we also know that the hyperbola will necessarily cross points E(-11,7), F(11,7), H(-11,-7), and G(11,-7), as these are the point from which the line segments emerge in our model, when viewed horizontally. Taking this into account, we can use one point (F) to solve for *a* in terms of s*:*

$$\frac{7^2}{s^2}-\frac{11^2}{a^2}=1\Rightarrow\frac{11^2}{a^2}=\frac{7^2}{s^2}-1\Rightarrow\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}=a^2$$

$$\Rightarrow a=\pm\sqrt{\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}}\Rightarrow a=\pm\frac{11}{\sqrt{\left(\frac{49}{s^2}-1\right)}}$$

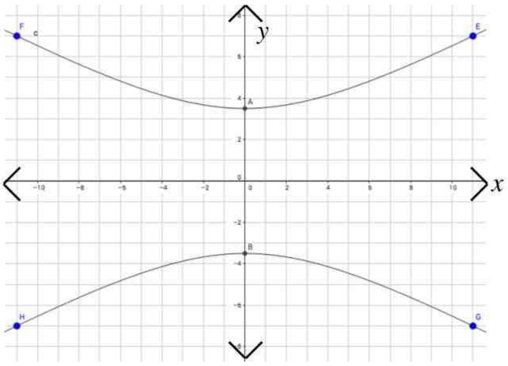
Furthermore, considering that the *a* value will be squared in the expression, we can ignore the last line above. With this information, we can then write the expression of the hyperbola in terms of the central radius *s*:

$$\frac{y^2}{s^2}-\frac{x^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}=1$$

To better show this, I decided to plot the expression using a known radius in Geogebra, with a relatively simple value; *s*(120°)=3.5

Here, we can see how the graph holds true for the vertices being $\pm s$, and the hyperbola intersecting all 4 points, which we know are true for our model.

D: More reflection to verify that the work is correct.

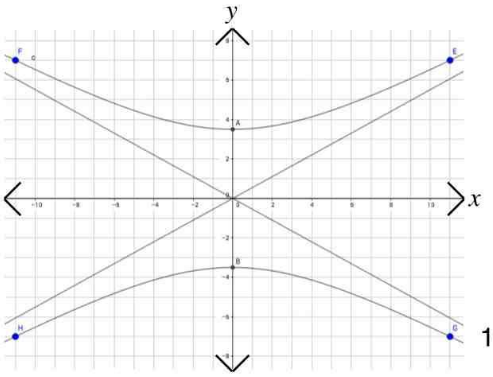



$$\frac{y^2}{3.5^2}-\frac{x^2}{\left(\frac{11^2}{\left(\frac{7^2}{3.5^2}-1\right)}\right)}=1\Rightarrow\frac{y^2}{3.5^2}-\frac{x^2}{\left(\frac{11\sqrt{3}}{3}\right)}=1$$

With this in mind, we can further explore the hyperbola, by plotting the linear asymptotes, which are known to be:

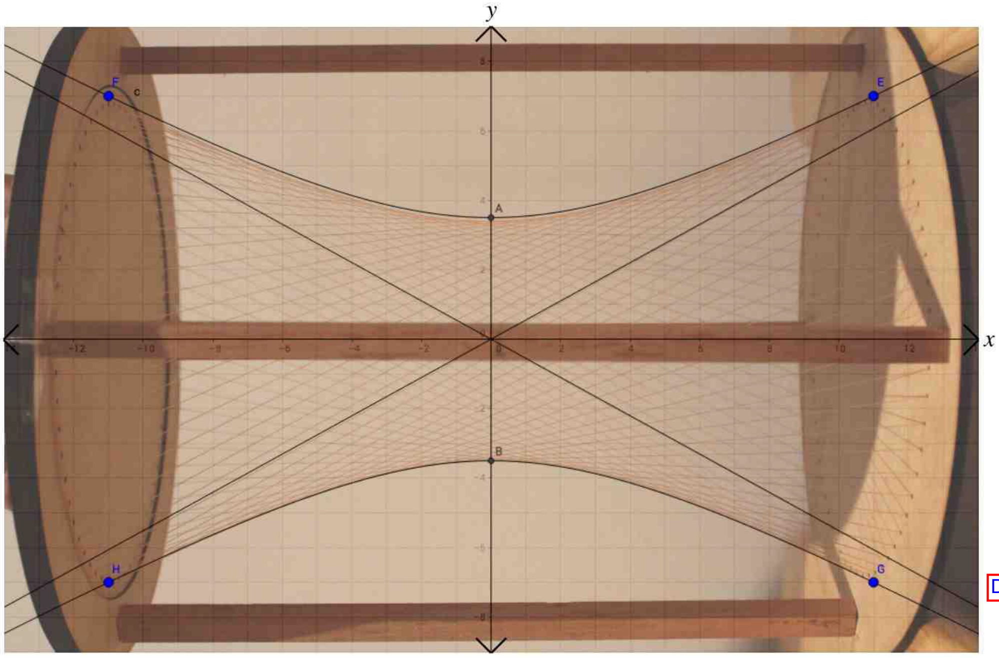
$$y=\pm\frac{b}{a}x\Rightarrow y=\pm\frac{3.5}{\left(\frac{11}{\sqrt{\left(\frac{49}{3.5^2}-1\right)}}\right)}x\Rightarrow y=\pm\frac{7\sqrt{3}}{22}x$$

based on our previous hyperbolic expression and the solution for *a* above.




10

Finally, we can also project the taken picture from the hyperboloid, and place it accordingly to match its coordinates for the known radius and height. By doing this, we can confirm the accuracy of our determined expression, by visually seeing the similarity between the model and the expression.




D: Excellent reflection.

Having confirmed the accuracy of our hyperbola, we can elaborate a final equation of hyperbola relating rotation *x*, and hyperbolic shape, by replacing the radius *s* with the expression for rotation and central radius (*s(x)*):

$$\frac{y^2}{\left(\left|\frac{7\sqrt{1+\cos\theta}}{2}\right|\sqrt{2}\right)^2}-\frac{x^2}{\left(\frac{11^2}{\left(\frac{7^2}{\left(\left|\frac{7\sqrt{1+\cos\theta}}{2}\right|\sqrt{2}\right)^2}-1\right)}\right)}=1 \qquad \theta\in\left]-180°,180°\right[$$

B: The student ensures that there is no confusion with the variables.

* *x* will be changed to $\theta$ from now on so we don't have two *x* in the expression representing different things.

It is worth noting that for the interval of $\theta$ for which the equation exists, $\pm180°$ is not included, considering that it would imply a denominator value of zero, for which a fraction would be undefined.

11

Furthermore, we can even consider the formula for an unknown height *h*, and a base circumference radius *r*, as done before with the radius. With this, the solution for *a*, would have constant values as to adjust the hyperbola to the known points:

$$\frac{y^2}{\left(\left|\frac{r\sqrt{1+\cos\theta}}{2}\right|\sqrt{2}\right)^2}-\frac{x^2}{\left(\frac{h^2}{\left(\frac{r^2}{\left(\left|\frac{r\sqrt{1+\cos\theta}}{2}\right|\sqrt{2}\right)^2}-1\right)}\right)}=1 \qquad r,h\in\mathbb{R}^+ \quad \theta\in\left]-180°,180°\right[$$

This expression would be of great power in calculating the form of a hyperbola from reestablished specifications, assuming the expressions above are all correct. However, it does have the limitation of being a hyperbola which opens along the y-axis, and considering that realistically a hyperbola is most likely to be used when opening along the x-axis, it would need to be adjusted to be fully valid and useful in a real life scenario. Luckily, the change needed would only be a 90° rotation along the origin, which can be done by switching the *x* and *y* components in the above expression.

# Volume

To find the volume, I first have to transform the hyperbolic expression described above into a function, which can then be rotated 360° about the x-axis. To do this, I rearranged the expression to solve for *y*:

$$\frac{y^2}{s^2}-\frac{x^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}=1\Rightarrow\frac{y^2}{s^2}=1+\frac{x^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}\Rightarrow y=\pm\sqrt{s^2+\frac{s^2x^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}}$$

Because the expression needs to be transformed to a function, the $\pm$ needs to be ignored, yet when rotated the shape will remain the same. Additionally, because *s*, is the central radius, we can consider it a constant for each rotation in the hyperboloid, so we will replace it with the rotation function later on.

Having the resulting function of:

$$f(x)=\sqrt{s^2+\frac{s^2x^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}}$$

We then consider it's 360° solid of revolution about the x-axis, by using the formula:

$$V=\pi\int_a^b(f(x))^2\,dx$$

where *a*, and *b*, are the *x*-values for which it is bound.

12

Considering that the model has a height of 22cm, and is divided symmetrically by the y-axis, it would be bound at ±11:

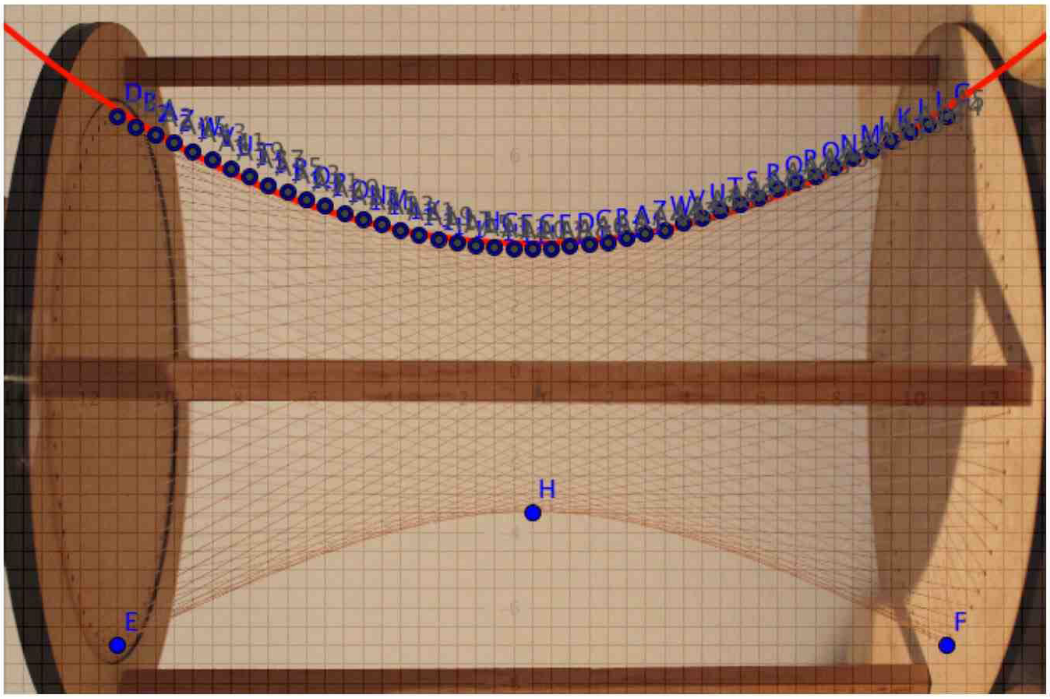
$$V=\pi\int_{-11}^{11}\left(\sqrt{s^2+\frac{s^2x^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}}\right)^2 dx \Rightarrow \pi\int_{-11}^{11} s^2+\frac{s^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}x^2\,dx \Rightarrow \pi\left[s^2x+\frac{s^2}{3\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}x^3\right]_{-11}^{11}$$

Considering this, we can then solve the volume of revolution of a hyperbola with a known central radius; $s(120°)=3.5$

B: Terminology, volume of hyperboloid. A hyperbola has no volume.

$$V=\pi\int_{-11}^{11}\left[3.5^2x+\frac{3.5^2}{3\left(\frac{11^2}{\left(\frac{7^2}{3.5^2}-1\right)}\right)}x^3\right]_{-11}^{11} \Rightarrow V=\pi\left(3.5^2(-11)+\frac{3.5^2}{3\left(\frac{11^2}{\left(\frac{7^2}{3.5^2}-1\right)}\right)}(-11^3)\right)-\left(3.5^2(11)+\frac{3.5^2}{3\left(\frac{11^2}{\left(\frac{7^2}{3.5^2}-1\right)}\right)}(11^3)\right) \Rightarrow V\approx 1693{,}3cm^3$$

Lastly, we can check the accuracy of this result, by using a different method to get the function with which to do a solid of revolution. For this, we can use Geogebra to plot a set of points in a picture of the model I made, adjusted to fit the coordinates of the real dimensions in a Cartesian plane, and then use curve fitting to find a quadratic equation that best fits the image.




13

Having done this when the model was at a rotation of 120°, we can then compare the values given from the expected volume according solid of revolution of the deduced function, and the volume according to the solid of revolution resulting from Geogebras line fitting.

| Angle (degrees) | Formula-based volume ($cm_3$) | Regression based volume ($cm_3$) | Difference |
|---|---|---|---|
| 120 | 1693,3 | 1713,4 | 20,1 |

Here, we can see how both values for volume when rotated 120° are very close, which further confirms the accuracy of the deduced function.

# Conclusion

In my exploration, I investigated the properties of a hyperboloid model I made, and found functions and expressions relating the rotation of one face and the central radius, as well as the shape and volume of the hyperboloid. This findings were confirmed by comparing with values of the actual model, and were mostly seen to be true, at least when the rotation was in between -180 and 180 degrees. With this in mind, we can see how my functions and expressions are useful in real life scenarios, as not only they apply for my model, but can calculate the wanted properties in a hyperboloid of any height and base circumference radius.

With this in mind, we can see how this has implications in fields like architecture and civil engineering, were building in the shape of hyperboloids, such as nuclear plant chimneys, have to be precisely modeled, with reestablished dimensions. However, it is important to remember my model is based on threads, and in real life, the walls of a building would change the results quite significantly. Regardless of this, my models and expressions are some quite good approximations, with which I am quite happy.

Besides enjoying the process thoroughly, I am happy to say I have learned quite a lot in this exploration, and it very satisfying to know that I might be able to use my findings if I decide to study architecture or engineering. For this, I´d like to thank everyone involved in the making of this exploration, whose endless support and motivation were key to achieving the final result.

# Bibliography

AutoCAD. (2016). Retrieved from http://www.autodesk.com/products/autocad/overview

BCC Bitesize. (2014). *Circles.* Retrieved 05 20, 2016, from Circles: http://www.bbc.co.uk/schools/gcsebitesize/maths/geometry/circlesrev2.shtml

Wolfram Alpha. (2016). *Hyperbola.* Retrieved 05 20, 2016, from Hyperbola: http://mathworld.wolfram.com/Hyperbola.html

Wolfram Alpha. (2016). *One-Sheeted Hyperboloid.* Retrieved 05 23, 2016, from One-Sheeted Hyperboloid: http://mathworld.wolfram.com/One-SheetedHyperboloid.html

14